
SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
FIAT LVX
MDCCCLXVIII
EX LIBRIS

ag

PHYSICS

FOR

SECONDARY SCHOOLS

BY

CHARLES F. ADAMS, A.M.
HEAD OF THE DEPARTMENT OF PHYSICS,
DETROIT CENTRAL HIGH SCHOOL

NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

PHYSICS.
W. P. I

PREFACE

PHYSICS deals with phenomena in which every child is interested; it treats of subjects with which all men and women have more or less to do in practical life; and its successful study requires clear thinking and accurate expression. The study of physics should, therefore, be interesting and practical, and it should afford the best of mental discipline. The subject should be so presented that the pupil's interest is maintained not merely because he is entertained by a wonder-working magician, but because he is conscious of successfully exerting his mental powers in mastering what he desires to know.

This book is the outgrowth of many years of experience in teaching physics to both boys and girls. I have made a constant study of the difficulties which beset young pupils in their attempts to understand the subject. It has been my chief aim to write a teachable book. I have endeavored especially to present the subject with such simplicity and clearness of expression and fullness of illustration that the average secondary school pupil will readily comprehend it.

A large number of problems have been included in the text for the purpose of giving the teacher a latitude of choice. They have been selected with great care, especially with a view to emphasizing and illustrating the principles involved, the numbers being so chosen that the actual arithmetical work shall be easy.

I believe that the material for laboratory work should be contained in a separate book. Such material overloads

a text-book; and, moreover, the teacher should have greater range and freedom in the prosecution of that work than would be possible if he were limited to such material as could be presented in a book of this character.

The text can be mastered by ordinary classes in a school year, and it meets the requirements of the College Entrance Examination Board and the New York State Syllabus.

I wish here to acknowledge my deep obligations to those who have read the proof or the manuscript for their valuable criticisms and suggestions, especially to Dr. Karl E. Guthe, of the State University of Iowa, Iowa City, Iowa; to Dr. Paul R. Heyl, of the Central High School of Philadelphia, Pennsylvania; to Mr. C. E. Spicer, of the Joliet Township High School, Joliet, Illinois; to Mr. Herbert C. Wood, of the East High School, Cleveland, Ohio; and to Mr. C. H. Slater, of the William McKinley High School, St. Louis, Missouri. I assume all responsibility, however, for any errors that may remain in the text.

CONTENTS

PAGE

INTRODUCTION 7

CHAPTER

I. MECHANICS AND PROPERTIES OF MATTER 15

i. Motion and force 15
ii. Accelerated motion 20
iii. Measurement of force 30
iv. Composition and resolution of forces 37
v. Gravitation 48
vi. Centrifugal force 58
vii. The pendulum 62
viii. Work 69
ix. Energy 73
x. Machines. Principle of moments 80
xi. Friction 96
xii. States of matter 101
xiii. Molecular forces and motions in gases 105
xiv. Molecular forces and motions in solids 112
xv. Molecular forces and motions in liquids 115
xvi. Surface tension and capillarity 118
xvii. Transmission of pressure by fluids 124
xviii. Pressure in fluids due to gravity 129
xix. The barometer 139
xx. Boyle's law 144
xxi. The siphon and pumps 148
xxii. Buoyancy and specific gravity 152

II. WAVE MOTION AND SOUND 162

i. Vibratory motion 162
ii. Waves — wave motion 166
iii. Sound — its origin and transmission 175
iv. Velocity of sound 180
v. Resonance 183
vi. Interference of sound 188
vii. Characteristics of sound — intensity 192
viii. Pitch — musical scale 195
ix. Quality of sound 200
x. Musical instruments — strings 204
xi. Wind instruments — pipes 209

III. LIGHT 213

i. Nature of light 213
ii. Transmission of light 215
iii. Intensity of light 221
iv. Reflection of light 225

CHAPTER		PAGE
v.	Refraction of light	227
vi.	Deviation and dispersion	235
vii.	Color	244
viii.	Kinds of spectra	249
ix.	Interference and diffraction	255
x.	Images formed by plane mirrors	259
xi.	Images formed by spherical mirrors	266
xii.	Lenses and images formed by lenses	274
xiii.	Optical instruments	282
IV.	HEAT	292
i.	Temperature and thermometers	292
ii.	Expansion by heat	297
iii.	Heat and energy	306
iv.	The steam engine	310
v.	Measurement of heat	316
vi.	Transmission of heat	322
vii.	Change of state	331
V.	ELECTRICITY AND MAGNETISM	349
	PART I. ELECTROSTATICS	349
i.	Electrical charges	349
ii.	Induced charges	354
iii.	Electrical distribution	359
iv.	Potential	361
v.	Electrical condensers	365
	PART II. MAGNETISM	369
i.	Magnets	369
ii.	Terrestrial magnetism	373
iii.	Magnetic induction	375
iv.	Magnetic field	377
	PART. III. CURRENT ELECTRICITY	380
i.	The voltaic cell	380
ii.	Magnetic effects of the current	391
iii.	Galvanometers	406
iv.	Resistance	410
v.	Ohm's law	412
vi.	Current induction	424
vii.	The dynamo and electric motor	430
viii.	Currents induced by currents	437
ix.	Telephone	445
x.	Chemical effects of the electric current	448
xi.	Energy of the current — heat effects	456
xii.	Röntgen rays — wireless telegraphy	465
	APPENDIX	475
	INDEX	481

PHYSICS FOR SECONDARY SCHOOLS

INTRODUCTION

1. Physics is the science of matter and energy. Each of these is as important as the other. We know nothing of matter except through the agency of energy and nothing of energy except through the agency of matter.

Physics is one of the exact sciences. In its investigations constant use is made of mathematics, and the most refined and accurate instruments known to man are often required. It may be said to be a science of measurements.

2. Unit of measure. — Every measurement is a comparison. The thing with which the measured quantity is compared is the *unit* of measure. If you measure the length of the table by your pencil, you may say the table is ten pencils long. You have compared the length of the table with that of the pencil, and the length of the pencil is the unit of measure. A unit is the first essential in all measurements. The magnitude of any quantity is the ratio of that quantity to the unit.

A unit must always be of the same nature as the thing measured. A unit with which length is measured must possess length. We must use a volume to measure a volume, a force to measure a force, an area to measure an area. If we wish to measure heat, we must select a certain definite quantity of heat for a unit with which to measure other quantities of heat. Generally, but not always, a name is given to the quantity selected for the unit.

Sometimes quantities are measured indirectly by measuring some other quantity which is proportional to the first quantity. For example, we may measure the strength of a man by measuring what he can do.

3. Standard and legal units. — The selection of a unit is wholly an arbitrary matter. A man may pick up a stick and use it in measuring the length of a log; the length of the stick becomes his unit of length for the time being. He might keep the stick, give a name to its length, and make others of equal length, persuading other people to use them. Thus in time it might become an established unit. When a unit has a fixed and definite value and is used by many people, it becomes a *standard unit.* Units sometimes become established by custom or use; but if a law is enacted by a government adopting any unit, the unit so adopted becomes a *legal unit.*

4. Systems of units. — There are two systems of units in use in the United States and Great Britain, the English, and the French or metric. The English system is used for general purposes, while the metric is used for scientific purposes in these countries. In other countries the metric system is used almost exclusively for all purposes. The great advantage of the metric system lies in the fact that it is a decimal system and that the various units bear simple relations to one another.

5. Fundamental and derived units. — Units selected and defined without reference to other units are termed *fundamental* units. Those used in science are the units of length, mass, and time.

Nearly all quantities with which physical science deals may be expressed in terms of these fundamental units. Thus the unit of area is a square, one unit in length, as one square inch ; and the unit of volume may be a cube whose edge is one unit in length, as the liter, which is a

cube whose edge is 10 cm. long. Thus we see that the unit of area and the unit of volume may be derived from the unit of length. Such units as may be defined and expressed in terms of the fundamental units are called *derived* units.

6. Absolute units. — Whenever any quantity is expressed wholly in terms of the units of length, mass, and time, it is said to be expressed in *absolute* units. In all scientific work the centimeter, the gram, and the second are taken as the fundamental units; and therefore all lengths are reduced to centimeters, all masses to grams, and time to seconds, to express a quantity in absolute units. This system of measurement is often called the C.G.S. (Centimeter-Gram-Second) system.

7. The unit of length. — In the metric system the unit of length is the *meter*. The meter is the distance between two lines on a platinum-iridium bar (at 0°C.) which is kept at the International Bureau of Weights and Measures in France and is called the International Prototype Meter. It is an exact copy of the original meter in the Palace of the Archives of the French government. There is a similar bar in the Bureau of Standards at Washington, called the National Prototype Meter.

The centimeter, the 100th part of a meter, and the millimeter, the 1000th part of a meter, are the subdivisions of the meter usually used in physics.

The *yard*, the unit of length in the English system, is legally defined in the United States as $\frac{36}{39.37}$ of a meter. An inch is $\frac{1}{39.37}$ of a meter and a meter is 39.37 inches. The foot and the inch are to be considered as fractions of the yard.

8. Definition of mass. — Mass has been defined as the *quantity of matter* in a body. This has been the accepted definition of mass for many years, but it is not considered

satisfactory chiefly because of the uncertainty existing about the nature of matter. We do not know what matter really is and cannot define it in any satisfactory way. We shall, however, use for the present the old definition of mass.

9. The unit of mass. — The mass of a body is usually determined by weighing it, that is, by comparing it with certain standard masses or "weights." In the metric system, the standard mass is the *kilogram*. It is a cylinder of platinum-iridium kept at the International Bureau of Weights and Measures and called the International Prototype Kilogram. It is an exact equivalent of the original kilogram of the Palace of the Archives of the French government. A similar cylinder in the Bureau of Standards at Washington is called the National Prototype Kilogram. The kilogram has the same weight as a cubic decimeter or liter of water at its temperature of greatest density, 4° C. It has therefore a certain relation to the unit of length.

The *gram*, which is the thousandth part of the kilogram, is the unit of mass in the metric system.

The unit of mass in the English system is the avoirdupois pound. The pound is a certain piece of metal kept by the British government. The United States government has no standard pound mass worthy of the name, but defines the pound in terms of the kilogram. One pound equals $\frac{1}{2.2046}$ of a kilogram or a kilogram equals 2.2046 lb.

In 1875 most of the nations of the civilized world agreed to establish and maintain a scientific and permanent International Bureau of Weights and Measures.

Standard meters and kilograms were prepared by this bureau with the greatest possible accuracy. After they had been compared with those of the Palace of the Archives of France and with one another,

one of each was selected for the international prototypes, and the rest, called national prototypes, were distributed by lot among the several nations maintaining the bureau. Those of the United States were received by President Harrison with considerable ceremony on Jan. 2, 1890.

The *second*, the unit of time used in physics, is $\frac{1}{86400}$ of the mean solar day.

10. Weight. — If a mass of anything is held in the hand, a force is felt pulling it toward the earth. This force tending to pull the mass to the earth is the *weight* of the mass; or, more accurately, the weight of a body is the force that attracts the body to the earth.

For example, the weight of a pound mass is the pull which the earth exerts upon it. The mass itself (the matter in it) is called a pound, and the weight of the mass (the pull of the earth) is also called a pound. Likewise the weight of a kilogram mass is called a kilogram, and the weight of a gram mass is called a gram. Thus all the terms, *kilogram*, *gram*, *pound*, *ounce*, etc., have two meanings, each being the name for a mass and the weight of that mass. This makes it difficult to distinguish between weight and mass. It is still more difficult because in ordinary language it is customary to use the word *weight* in the same sense in which the word *mass* is used in physics. Thus, an iron mass is called an iron weight, and we always say a "set of weights" when strictly speaking we should say a "set of masses." Weight means a force, and mass means a quantity of matter.

11. The weight of a body variable. — The attraction of the earth for bodies upon it differs slightly at different places, being least at the equator and increasing as the poles are approached. If the kilogram mass of the Bureau of Standards at Washington were taken to the equa-

tor, its weight there would be less than at Washington, although its weight would be called a kilogram at both places or wherever it might be. Hence the weight of any mass is not a constant, but a variable quantity, depending upon the place in which it happens to be.

If a wire were just strong enough to withstand a pull of 100 lb. at Havana, would it withstand a pull of 100 lb. at New York? Why?

12. Units of weight. — The weights of the units of mass are taken as the units of weight and have the same names. For example, the unit of weight in the English system is the *pound*, not the mass of metal called a pound, but the pull of the earth upon that metal. Therefore, when we use the word *pound* or *gram*, we may mean a mass or we may refer to a weight. The beginner will find it helpful if for a time he will say "pound mass" and "gram mass" when using such words as pound and gram to signify mass; and "pound weight" and "gram weight" when using them to express weight or force.

13. Relation of weight to mass. — Under the same conditions weights of bodies are proportional to their masses. This means that if one body weighs three times as much as another at the same place, its mass is also three times as great. It is because of this principle that masses can be compared and measured by comparing their weights, and upon this principle the practice of *weighing* things rests. It follows from the definition of the unit of weight given above (§ 11) and from the proportionality of mass and weight, that the weight of a body and its mass are expressed by the same number; if the mass of a body is 5 lb., its weight is also 5 lb.

14. The density of a substance is the number of units

of its mass per unit volume. For example, a cubic foot of water weighs 62.4 lb. or has a mass of 62.4 lb.; hence its density is 62.4 lb. per cubic foot. Its density may also be stated in ounces per cubic inch, or in grams per cubic centimeter, being 0.578 oz. per cubic inch, or 1 g. per cubic centimeter. Since the metric system is in almost universal use in science, the density of a substance is usually stated in grams per cubic centimeter, and it is to be so taken unless stated to the contrary. Thus, when the density of iron is given as 7.8, it is understood as 7.8 grams per cubic centimeter.

15. Relation between mass, volume, and density. — If we divide the number that represents the mass of a body in grams by the number that represents its volume in cubic centimeters, we obtain the number that represents the mass in grams in one cubic centimeter, which is by definition the density of the body; therefore, briefly the *density of a body equals its mass divided by its volume.*[1] Letting m, v, and d represent respectively the mass, volume, and density of a substance, this relation may be expressed algebraically by the equation

$$d = \frac{m}{v}, \text{ or } m = v \times d, \text{ or } v = \frac{m}{d}.$$

16. Specific gravity is the ratio of the density of a substance to the density of a standard substance, water being the standard for solids and liquids, and air for gases. Thus when we say that the specific gravity of iron is 7.8, we mean that it is 7.8 times as dense as water. In English units the density of water is 62.4 lb. per cubic foot; therefore the density of iron is (7.8×62.4) 486.7 lb. per cubic foot. In C.G.S. units the density of water is 1; there-

[1] The common use of brief expressions like this is justified by its convenience. The reader must keep constantly in mind the fact that terms like *density*, *mass*, *volume*, etc., when used in equations, stand for numbers.

fore the density of iron is (7.8 × 1) 7.8. Thus it is seen that with C.G.S. units the specific gravity of a solid or liquid and its density are expressed by the same number. For this reason the term *specific gravity* is falling more or less into disuse, density taking its place. The specific gravity of a substance is the same with all systems, but the density of a substance is expressed by different numbers in different systems.

Problems

1. A piece of marble 2 ft. square and 5 ft. long weighs 3370 lb. What is its density? Its specific gravity?

2. A mass of iron having a density of 486 lb. per cu. ft. weighs 20 tons. What is its volume?

3. A cube of copper 6 in. on a side weighs 69 lb. What is its density (*a*) in ounces per cubic inch, (*b*) pounds per cubic foot, and (*c*) in grams per cubic centimeter?

4. A glass globe 18 cm. in diameter held 3.95 g. of air. What was the density of the air?

5. A cylinder 4 cm. deep and 6 cm. in inside diameter held 1538 g. of mercury when exactly full. What is the density of mercury?

6. A gallon of hydrogen weighs 0.339 g. What is its density?

7. If 4.5 liters of carbon dioxide gas weigh 8.883 g., what is its density? What is its specific gravity, the density of air being 0.001293?

8. If coal gas has a specific gravity of 0.4, what is its density in grams per cubic centimeter?

9. The density of hydrogen is 0.0000896. What is its specific gravity?

10. The density of gold is 19.3 g. per cubic centimeter. What is the weight of a cubic foot of it? (This problem should be solved without reducing any metric units to English units or English to metric units.)

CHAPTER I

MECHANICS AND PROPERTIES OF MATTER

I. MOTION AND FORCE

17. Mechanics treats of the two effects of force on matter: (1) change of motion, called **acceleration**, and (2) change of size or shape, called **strain**.

Dynamics treats of the first, or the motive effect of force.

18. Motion is continuous change of position. Since the position of a body cannot be given except with reference to some other point or position, so change of position of a body can be conceived only with reference to some other body; that is, all motion is relative. For example, an object on the deck of a moving ship may be at rest relatively to the ship itself and to other bodies in the ship, while it is in motion relatively to the water or the shore. A body on the earth may be at rest with reference to the earth's surface, while at the same time with reference to the earth's axis or to the sun it is moving through space at a very rapid rate.

19. Velocity is the rate of motion of a body. It is measured by the number of units of distance passed over divided by the number of units of time taken for the journey, or $v = \frac{s}{t}$. For instance, if a train goes 90 mi. in 3 hr., it is said to travel at the rate of 30 mi. per hour; its velocity is 30 mi. per hour ($\frac{90}{3} = 30$). Its velocity may also be said to be 44 ft. per second —

$$\frac{90 \times 5280}{3 \times 60 \times 60} = 44.$$

20. Uniform motion. — The motion of a body may be uniform or variable. It is uniform when the body passes over equal spaces in equal and consecutive units of time; it is variable when the spaces traversed in equal intervals of time are unequal. When the motion of a body is uniform, its velocity is termed *constant*.

21. Average and actual velocity. — In the example given above the train may be imagined to traverse the 90 mi. with a constant velocity, or to vary its speed greatly, at times going slowly, at times going very fast, and at times standing still at stations. In either case, whether having uniform or variable motion, its velocity is 30 mi. per hour. In the first case, however, with uniform motion, 30 mi. per hour is its actual velocity at each instant throughout the three hours; in the second case, 30 mi. per hour is its *average velocity* for the whole time, while it is possible that for no length of time during the three hours was its velocity actually 30 mi. per hour.

The formula, $v = \frac{s}{t}$, applies to both cases, giving the actual velocity when the body has uniform motion, and the average velocity when the motion is variable.

Problems

1. A man walked 20 mi. in $4\frac{1}{2}$ hours. What was his velocity?

2. A body having uniform motion passed over 50 m. in 20 sec. What was its velocity in meters per second? In centimeters per second?

3. How long will it take a body having a velocity of 22 ft. per second to go a mile?

4. Express a velocity of 60 mi. per hour in feet per second.

5. By means of the answer to the last problem reduce a velocity of 30 mi. per hour to velocity in feet per second. Also reduce 20, 15, 12, 10, 90, and 40 mi. per hour to feet per second.

6. What is the velocity in centimeters per second of a body that moves over a space of 60 Km. in 3 hr.?

7. Express a velocity of 5 cm. per second in kilometers per hour.

22. Newton's laws of motion. — In his *Principia* (1687) Sir Isaac Newton enunciated three laws of motion which still stand as the fundamental laws of dynamics.

I. *Every body continues in its state of rest or of uniform motion in a straight line except as it is compelled to change that state by some applied force.*

II. *Change of motion is proportional to the force applied and takes place in the direction in which the force acts.*

III. *To every action there is always an equal and opposite reaction; or the mutual actions of any two bodies are always equal and opposite in direction.*

23. Inertia. — The first law of motion is sometimes called the law of *inertia*, which may be defined as the inability of a body to change its own state of rest or of motion.

It is impossible to prove this law perfectly by experiment, because we know of no body that is free from the action of a force; but the heavenly bodies furnish the best illustrations we have of the first law of motion. The motion of the earth around the sun century after century is an example. There is no need of a force to keep the earth moving, it continues to move because of its inertia; but the constant attraction of the sun is necessary to keep it from moving in a straight line and to make it travel in its orbit around the sun. We are all more or less familiar with the fact that bodies do not move themselves, or stop themselves, or change the direction of their motion themselves. Force and time are always required to overcome the resistance of a body to a change of motion. It is because of its inertia that a

body cannot be set in motion or stopped instantaneously; because of its inertia also a body resists a change of direction of its motion.

Illustrations of inertia are very common. The backward movement of a passenger when a vehicle is started suddenly, and his forward movement when it is stopped suddenly, the breaking of a swiftly moving stick when it strikes some solid object near its center, the removal of snow from a shovel by suddenly arresting the motion of the shovel, and the flying of mud from a carriage wheel, are familiar illustrations of inertia.

Experiment. — Let a heavy mass of iron be suspended as shown in Figure 1. If a sudden downward jerk is given by the handle, the string will be broken below the ball; but if a steady pull is given the string above the ball will be broken. In the first case, the inertia of the ball does not allow it to move so as to break the upper string before the lower string is broken; but in the second case, the force, being slowly increased, has time to overcome this inertia and the upper one breaks because it bears the weight of the ball in addition to the force exerted by the hand.

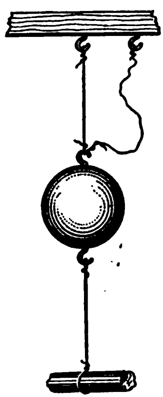

FIG. 1. — Apparatus to illustrate inertia.

24. Force. — In the preceding pages we have frequently had occasion to use the word *force*, a term with which we are all more or less familiar. We get our primitive idea of force from muscular exertion or by having our muscular exertion opposed. When we see an inanimate body producing an effect exactly like that produced by the muscular exertion of an animal, we say the body is exerting force. A force is said to be *constant* as long as it acts with unvarying intensity.

The definition of force usually given in physics is derived from Newton's first law of motion; namely,—

Force is any action that changes or tends to change the motion of a body.

25. The third law of motion.—In order that a body may exert force it must meet with resistance, and the force exerted is never greater than the resistance encountered. Thus, it is hardly possible to exert a great force on a feather floating in the air, or upon a light, hollow ball, because such a body offers little resistance. A swiftly moving body, such as a cannon ball, exerts no force except as it encounters some object that offers resistance.

Force is therefore an action occurring always between two bodies or between two parts of one body, and the action of one body is always exactly equal to the opposition or reaction of the other. Action and reaction are equal and opposite.

Forces, then, always exist in pairs. Thus, to break a string, stretch an elastic, or squeeze a lemon you must exert two equal and opposite forces. Such a thing as a single force acting alone is unknown. Usually, however, we give our attention to only one of the forces, ignoring the other.

26. Stress.—A pair of forces that constitute an action and reaction is called a *stress*. The two forces are two parts of one stress. If the two forces act away from each other, as in the breaking of a string, the stress is called a *tension;* but if they act toward each other, as in crushing anything, the stress is called a *pressure*.

27. Illustrations of the third law of motion are very common. If anything is pressed by the finger, the finger is pressed an equal amount in the opposite direction. A horse drawing a wagon is pulled backward by a force exactly equal to that which he exerts forward, and if it were not for the action and the reaction between him and

the ground, he could not move the wagon at all, any more than a man could lift himself over the fence by his boot straps.

A revolving lawn sprinkler illustrates this law. As the water flows from the side of the pipe, the pipe is pushed backward with as much force as the water is thrown forward.

II. ACCELERATED MOTION

28. Acceleration is a term used to denote the *rate of change of velocity*, or, it is the change of velocity per unit of time. It follows from this definition that acceleration equals the total change of velocity divided by the time, or $a = \frac{v}{t}$, a denoting acceleration, v total change of velocity, and t the time in which the change occurs. This definition applies to retarded motion as well as to increasing velocity, but in that case the acceleration is considered negative.

Problem illustrating acceleration. — A train moving at first at 15 mi. per hour gained in speed so that at the end of 10 min. it was going at the rate of 45 mi. per hour. What was the acceleration?

Solution. — Since the velocity changed from 15 to 45 mi. per hour, the total change was 45 minus 15, or 30 mi. per hour; and since this change occurred in 10 min., the change in one minute was one tenth of 30 mi. per hour, or 3 mi. per hour. That is, the train changed its velocity 3 mi. per hour every minute; hence, by definition, its acceleration was 3 mi. per hour per minute.

Let us also express the acceleration in feet per second.

15 mi. per hour = 22 ft. per second and 45 mi. per hour = 66 ft. per second. The total change of velocity = 66 − 22 = 44 ft. per second. 10 min. = 600 sec. Since the change of velocity of 44 ft. per second occurred in 600 sec., the change of velocity in one second was $\frac{1}{600}$ of 44 ft. per second, which is 0.073 ft. per second. Hence, its velocity changed 0.073 ft. per second every second, or the accelera-

tion was 0.073 ft. per second per second. This is sometimes expressed thus, 0.073 ft. per second^2.

Ans. 3 mi. per hour per minute, or 0.073 ft. per second per second, or 0.073 ft. per second^2.

It is to be observed that to express acceleration correctly time must be expressed twice.

Problems

1. A body started from rest and in 15 sec. had a velocity of 600 cm. per second. What was the acceleration?

Ans. 40 cm. per second^2.

2. A body was moving with a velocity of 200 cm. per second and 15 sec. later its velocity was 800 cm. per second. What was its acceleration? *Ans.* 40 cm. per second^2.

3. A body starts from rest and is given an acceleration of 500 cm. per second^2. What is its velocity after 2 min.?

4. A street car 30 sec. after starting had a velocity of 15 mi. per hour. What was its acceleration? Give the answer in miles per hour per second and in feet per second per second.

5. An automobile had an acceleration of 4 mi. per hour per second. What was its speed 6 sec. after starting?

6. A car moving at the rate of 12 mi. per hour was given a negative acceleration by the brakes of $\frac{1}{2}$ mile per second^2. How long did it take to stop the car?

7. A ship changed its velocity from 8 mi. per hour to 12 mi. per hour in 15 min. What was its acceleration?

8. At 40° N. latitude falling bodies have an acceleration of about 980 cm. per second^2. What velocity has a falling body at the end of the fifth second of its fall?

9. An automobile going at the rate of 10 mi. per hour was stopped in 22 sec. What was the acceleration? *Ans.* $-\frac{2}{3}$ ft. per second^2.

10. What do you understand by the expression 20 ft. per second^2? 20 ft. per second per minute? 20 ft. per hour per minute?

11. An acceleration of 22 ft. per second^2 is how much acceleration per second per minute?

12. How many times greater is the velocity of a freely falling body at the end of the twelfth second of its fall than at the end of the third second? Why?

13. When a body is thrown upward, its acceleration is – 9.8 m. per second². For how long a time will a body rise which starts upward with a velocity of 4900 m.?

14. A train having a speed of 60 Km. per hour was stopped by the brakes in 2 min. What was its acceleration in centimeters per second²?

29. Uniformly accelerated motion. — It follows from the first and second laws of motion that change of motion will be uniform or constant when the force causing it is constant. A *constant* force causes a *constant acceleration;* that is, the change of velocity is the same in each unit of time. When this is the case, the moving body is said to have *uniformly accelerated motion.*

30. Formulas for uniformly accelerated motion. — Since by definition a is the change of velocity in one unit of time, in t units of time the change of velocity will be $t \times a$ or at. Therefore,

$$(1) \quad v = at.$$

This is only another way of stating what has already been stated in the paragraph on acceleration. Observe that in this formula v does not represent velocity, but total *change* of velocity. When, however, the initial velocity of a body is zero, that is, when it starts from rest, v also represents *final* velocity; because initial velocity plus total change of velocity equals final velocity, or,

$$0 + at = at = v.$$

When the initial velocity of a body is zero and its final velocity is at, then its average velocity is

$$(0 + at) \div 2 = \tfrac{1}{2}\, at.$$

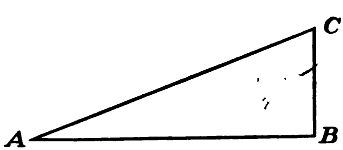

FIG. 2. — Graphical illustration of average velocity.

This may be illustrated graphically by a triangle (Fig. 2). In moving from A toward B, the distance from the line AB to the

line AC is a constantly increasing quantity; at A it is zero and at B it is BC. The average height of the triangle is $\frac{1}{2}(0 + BC) = \frac{1}{2}BC$. Likewise the velocity of a body moving with uniformly accelerated motion is a constantly increasing quantity; and if it is zero at first and at at the end, its average velocity is $\frac{1}{2}(0 + at) = \frac{1}{2}at$.

The space s traversed by a body moving with uniformly accelerated motion in a given time t equals its average velocity multiplied by the time, or $\frac{1}{2}at \times t = \frac{1}{2}at^2$. Therefore,

$$(2) \quad s = \tfrac{1}{2}at^2.$$

These two formulas, $v = at$ and $s = \frac{1}{2}at^2$, are the fundamental formulas for uniformly accelerated motion. From them, by eliminating t, a third may be derived. (The student should perform the operation and derive the formula.)

$$(3) \quad s = \frac{v^2}{2a}, \text{ or } v = \sqrt{2as}.$$

The distance traversed by a body during the fifth second of its motion may be found by subtracting the distance passed over in four seconds from that traversed in five seconds; and in general, the distance traversed in any particular unit of time is found by subtracting the distance passed over in $t - 1$ units of time, from that traversed in t units of time.

Let s' be the space traversed in any one unit of time. Then $s' = \frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 = \frac{1}{2}at^2 - \frac{1}{2}a(t^2 - 2t + 1) = \frac{1}{2}a(t^2 - t^2 + 2t - 1) = \frac{1}{2}a(2t - 1)$. Hence

$$(4) \quad s' = \tfrac{1}{2}a(2t - 1)$$

We have then four important formulas for uniformly accelerated motion:

$$(1) \quad v = at,$$

$$(2) \quad s = \tfrac{1}{2}at^2,$$

$$(3) \quad s = \frac{v^2}{2a}, \text{ or } v = \sqrt{2as},$$

$$(4) \quad s' = \tfrac{1}{2}a(2t - 1).$$

31. Falling bodies. — The force of gravity for comparatively small distances near the earth's surface is a constant force, and hence the motion of a freely falling body (the resistance of the air being neglected) is uniformly accelerated; and the same laws and formulas apply to falling bodies as to any other case of uniformly accelerated motion.

It is customary, however, to represent acceleration due to the force of gravity by the letter g instead of by a, and the formulas given in the last paragraph when applied to falling bodies are stated as follows:

$$(1)\quad v = gt,$$
$$(2)\quad s = \tfrac{1}{2}\,gt^2,$$
$$(3)\quad s = \frac{v^2}{2\,g},\ \text{or}\ v = \sqrt{2\,gs},$$
$$(4)\quad s' = \tfrac{1}{2}\,g(2\,t - 1).$$

The value of g varies on the surface of the earth from 978 cm. or 32.09 ft. per second^2 at the equator to 983.2 cm. or 32.27 ft. per second^2 at the poles. At Washington, D.C., it is 980.1 cm. or 32.155 ft. per second^2. For purposes of illustration and in problems in this book we shall use the numbers 980 and 32 to express the value of g.

32. Influence of the air and of mass on falling bodies. — It is to be observed that the quantity mass, or m, does not appear in any of the above formulas. This implies that the mass of a body does not affect the rate at which it falls, and that all bodies light or heavy fall at the same rate. It is true that the earth attracts a two-pound mass with twice as much force as it does a one-pound mass; but it must be remembered that the two-pound mass has twice as much inertia to be overcome as the one-pound mass.

That remarkable man, Galileo (1564–1642), who was

the first to investigate experimentally the laws of uniformly accelerated motion (1589–1591), proved that the mass of a body does not affect its velocity in falling by dropping a one-pound shot and a one-hundred-pound shot together from the top of the celebrated leaning tower of Pisa; both struck the ground at the same time. Everyday experience, however, teaches us that a light body such as a feather or a bit of paper will not fall under ordinary conditions so rapidly as a stone or a bullet.

Experiment. — A long glass tube (Fig. 3), sometimes called a "guinea and feather tube," closed at each end by a brass cap, is used to show the effect of the resistance of the air and the effect of mass on a falling body. One of the caps has a stopcock by which the tube can be attached to the air pump and the air exhausted from it. The tube contains a coin or bullet and a feather or bits of paper. If the tube is quickly inverted before exhaustion of the air, the coin falls much more rapidly than the feather; but after the air has been taken from the tube, little or no difference in the rapidity of the fall of the different bodies can be observed.

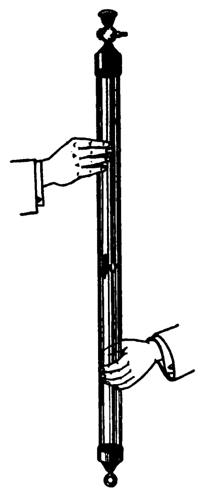

Fig. 3. — Guinea and feather tube.

33. Experiments in uniformly accelerated motion. — A freely falling body moves too rapidly to be easily experimented upon, hence various devices have been used to reduce the rate of acceleration without changing the character of the motion. Galileo's device consisted of a marble on a smooth inclined plank.

34. The Atwood machine is an apparatus much used to illustrate the laws of uniformly accelerated motion. It consists of a pulley A (Fig. 4) supported on the top of a pillar B about 7 ft. tall. The pulley, which is about 6 in. in diameter and very light, is made to run

with as little friction as possible. A light silk cord passes over its grooved rim, to each end of which equal masses K and L are attached. These masses exactly balance each other and no motion is produced; but when a small additional weight R, called a rider, is placed upon the mass K, it begins to fall slowly.

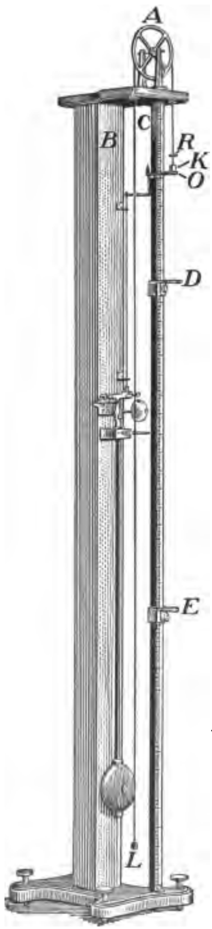

FIG. 4.—Atwood machine, to illustrate the laws of uniformly accelerated motion.

Suppose the masses K and L to be 300 g. each and the mass of the rider to be 10 g.; then there are 610 g. ($300 + 300 + 10 = 610$) whose inertia is to be overcome, but there is only the weight of the rider to do it; that is, a 10 g. weight to move a 610 g. mass. Its motion will be uniformly accelerated, but only $\frac{1}{61}$ (somewhat less because of friction and the inertia of the pulley) as great as that of a freely falling body. The motion is uniformly accelerated because the force causing it is constant.

A rod C graduated in centimeters serves to measure the distances traversed by the mass K. Near the top of this rod is a hinged shelf O to support the mass K before it begins to fall. The shelf D or the ring shelf E may be placed at any desired position on the rod C. A pendulum beating seconds is connected electrically with the shelf O and arranged to drop the shelf at its first tick. A place is found by repeated trials for the shelf D where the mass K will strike it exactly at the

next tick of the pendulum; then the shelf *D* is placed where the mass *K* will strike it at the end of two seconds, three seconds, etc., the distances being measured on the graduated rod.

Whatever the distance traversed in the first second, the mass will fall four times as far in two seconds, nine times as far in three seconds, sixteen times as far in four seconds, and so on. Similar results may be obtained with the marble and plank.

A cash car arranged to run down an inclined wire is an excellent modification of Galileo's device and a very good substitute for an Atwood machine. A No. 10 wire 30 ft. or more long should be used. This should be made taut by a turn-buckle, one end being about 2 ft. higher than the other. A heavy pendulum connected with a telegraph sounder or an electric bell should be used to measure the time intervals. The car may be arranged so as to be released electrically and its position at the end of an interval registered electrically.

35. The acceleration or the velocity acquired at the end of any given second is determined by the use of the ring shelf *E*. This shelf will allow the mass *K* to pass through it, but not the rider *R*. If it is placed so that *K* will reach it exactly at the end of the first second, the mass *K* and the rider *R* during the first second will fall with uniformly accelerated motion; but at the end of the first second the rider will be left behind on the ring shelf and the mass *K* will move on during the second second with *uniform motion* (Newton's first law), since no unbalanced force is acting upon it. The distance traversed during this second second with no force acting will be the velocity the mass *K* had acquired at the end of the first second.

In a similar manner the velocity acquired during the first two seconds can be measured. For example, to find the velocity at the end of the third second, place the

ring shelf where the rider *R* will be removed at the end of the third second and find the distance *K* goes during the fourth second without the rider. This distance will be equivalent to the velocity at the end of the third second. Should the mass *K* with the rider fall 10 cm. the first second, the velocity at the end of that second will be found to be 20 cm. per second; at the end of the second second, 40 cm. per second; and at the end of the third second, 60 cm. per second.

36. Let s = number of centimeters traversed in the whole time.
Let t = number of seconds body is falling.
Let a = acceleration in centimeters per second.^{2}
Let v = velocity at the end of any given second, or total change of velocity in centimeters per second.
Let s' = number of centimeters traversed in any given single second.

TABLE I, LAW FOR s

t	s	s FACTORED	
1 second	5 cm.	$5 \times 1 = 5 \times 1^2 = \frac{1}{2} at^2$	
2 seconds	20 cm.	$5 \times 4 = 5 \times 2^2 = \frac{1}{2} at^2$	
3 seconds	45 cm.	$5 \times 9 = 5 \times 3^2 = \frac{1}{2} at^2$	$s = \frac{1}{2} at^2$
4 seconds	80 cm.	$5 \times 16 = 5 \times 4^2 = \frac{1}{2} at^2$	
5 seconds	125 cm.	$5 \times 25 = 5 \times 5^2 = \frac{1}{2} at^2$	

The second column contains results such as may be obtained by the use of the Atwood machine. The third column contains the results of the second column factored by the space passed over during the first second. Observe that the other factor in each case is the square of the corresponding time. It is shown in Table III that 5, the space passed over in the first second, equals $\frac{1}{2} a$. Substituting $\frac{1}{2} a$ for 5 and t for the other factors gives the formula $s = \frac{1}{2} at^2$ in each case. This formula is therefore general.

37. The numbers given in column 2 of Table I and obtained by experiment are used as a basis for Table II.

To obtain s', or the space traversed in any single second, the third for example, the space traversed in the first two seconds is subtracted from the space traversed in the first three seconds, as 45 cm. − 20 cm. = 25 cm. This is shown in the second column of Table II. Observe that

TABLE II, LAW FOR s'

t	s'	s' Factored	
1st second	$5 - 0 = 5$ cm.	$5 \times 1 = \frac{1}{2}a(2t-1)$	
2d second	$20 - 5 = 15$ cm.	$5 \times 3 = \frac{1}{2}a(2t-1)$	
3d second	$45 - 20 = 25$ cm.	$5 \times 5 = \frac{1}{2}a(2t-1)$	$s' = \frac{1}{2}a(2t-1)$
4th second	$80 - 45 = 35$ cm.	$5 \times 7 = \frac{1}{2}a(2t-1)$	
5th second	$125 - 80 = 45$ cm.	$5 \times 9 = \frac{1}{2}a(2t-1)$	

when these values of s' are factored by $\frac{1}{2}a$, the second factors form a series of odd numbers. For example, the distance traversed in the fourth second is 5×7, 7 being the fourth odd number. The distance traversed in the tenth second would be $5 \times$ the tenth odd number. The tenth odd number is $2 \times (10 - 1) = 19$. Substituting $\frac{1}{2}a$ for 5 and $(2t-1)$ for the odd number in each case gives the formula $s' = \frac{1}{2}a(2t-1)$ which applies to all cases and is therefore general.

38. The numbers for Table III are either obtained from Table II as shown in column 2 of Table III, or they may be obtained directly by experiment with the Atwood machine as explained in § 35.

TABLE III, LAW FOR v

t	v	v Factored	
End of 1st sec.	$15 - 5 = 10 =$	$10 \times 1 = at$	
End of 2d sec.	$25 - 5 = 20 =$	$10 \times 2 = at$	$v = at$
End of 3d sec.	$35 - 5 = 30 =$	$10 \times 3 = at$	
End of 4th sec.	$45 - 5 = 40 =$	$10 \times 4 = at$	

The force causing the accelerated motion can do no more in one second than another, no more in the second or third second than it did in the first. What did it do in the first? It caused the body to fall 5 cm., but in the second second the body fell 15 cm. (Table II), only 5 cm. of which was due to the force acting on it; the remainder must have been due to the velocity acquired during the first second. In the third second it traversed 25 cm., only 5 cm. of which was due to the force acting during the third second; the remainder, 20 cm., must have been due to the velocity acquired during the first two seconds. The factor 10 answers to the definition of acceleration and the other factors in each case correspond to the time. Hence, substituting a for the factor 10 and t for time, $v = at$ in each case. The formula is therefore general.

Problems

1. The Washington monument is 169 m. high. In what time will a stone fall from top to bottom? With what velocity will it strike the ground?

2. If a body falls 313.6 m. in 8 sec., what is its acceleration? What is its velocity at the end of 6 sec.? How far does it fall during the last 4 sec.?

3. If a stone is thrown upward vertically to a height of 144 ft., how long will it take it to return to its starting point?

4. A body is rolling up a smooth incline at the rate of 25 cm. per second and loses velocity at the rate of 5 cm. per second. How far up the plane will it move before coming to rest?

5. A ball starting from rest and rolling down a smooth inclined plane for 4 sec. moved 14 cm. during the last second. What was its acceleration? How far did it move during the 4 sec.?

6. How far does a freely falling body fall during the last half of the first second of its fall?

7. How far does a freely falling body move during the first half of the ninth second of its fall? In the last half of the ninth second?

III. MEASUREMENT OF FORCE

39. The second law of motion. — Every rider of a bicycle knows that it takes more force to acquire speed on a smooth level road than it does to maintain the speed after

it is gained, and that it requires more force to attain speed quickly than it does to do it slowly. After the rider is under way little force is necessary to maintain the speed, just enough in fact to balance the opposing forces such as the resistance of the air and the friction of the moving parts of the bicycle. Every one is also familiar with the fact that it requires force to stop a body and more force to do it quickly than to do it slowly. Facts such as these teach us that it is not motion, but *change of motion*, which requires force. The student must remember that the word *change* in the second law of motion is a most important one.

40. Momentum. — The quantity of motion which a moving body possesses depends upon its mass and its velocity, being measured by the product of the mass by the velocity. This product is called the *momentum* of the body. Thus the momentum of a 12 Kg. mass moving with a velocity of 4 m. per second is $12 \times 4 = 48$ units; or, since 12 Kg. = 12,000 g. and 4 m. = 400 cm., its momentum is $12{,}000 \times 400 = 4{,}800{,}000$ units. No special names have been given to the various units of momentum. Of course in comparing momenta we must use the same units of mass and velocity throughout the problem.

41. Measurement of force. — Motion can be changed only in quantity and in direction. Therefore, when the second law of motion states that change of motion is proportional to the force applied, a part of its meaning is that *change in quantity of motion or change of momentum is proportional to the force applied.* The second law is sometimes stated in this way in modern text-books.

Since change of momentum is proportional to the force applied, it follows that force can be measured by the change of momentum it produces; for if the change of momentum is twice or three times as great in one case as in another,

the force causing the change must also be twice or three times as great.

For instance, if a force which causes a unit change of momentum in a second is called a unit of force, then that force which causes a change of momentum of ten units in a second is a force of ten units. Force measured by the change of momentum it produces is measured dynamically. A unit of force based on this method of measuring force is called a *dynamic* unit. Force may also be measured by comparing it with the attraction of the earth for a unit of mass, as the pound or the gram. A unit based on this method of measuring force is called a *gravitational* unit.

42. A dynamic or absolute unit of force is the force which can produce a unit change of momentum in a unit of time.

The *dyne* is a dynamic or absolute unit of force; it is the force which can produce a unit change of momentum in a unit of time, when the units used in calculating the momentum are the centimeter, the gram, and the second. For example, a force which acting for one second on a one-gram mass gives it a velocity of one centimeter per second is a dyne, because the momentum being zero at first and one at the end of the second, the *change of momentum* is one unit. Likewise, a force which acting for one second on one tenth of a gram mass gives it a velocity of 10 cm. per second is a dyne, because the change of momentum is one unit ($\frac{1}{10} \times 10 = 1$).

Why is a force which, acting for one second on a mass of one fiftieth of a gram, gives it a velocity of 50 cm. per second a dyne?

Problem. — How great is a force which in four seconds changes the velocity of a 12 g. mass from 8 to 48 cm. per sec.?

SOLUTION. — Since the body at first has a velocity of 8 cm. per second, its momentum is $12 \times 8 = 96$ units; and since its velocity at

the end is 48 cm. per second, its momentum is then $12 \times 48 = 576$ units. The change of momentum is therefore $576 - 96 = 480$ units. This change occurs in 4 seconds; the change in one second is $480 \div 4 = 120$ units. Hence the force by definition must be 120 dynes.

43. A gravitational unit of force is the weight of a unit mass, as the weight of a kilogram, gram, pound, or ounce. For example, if a boy is said to be pulling a sled with a force of fifty pounds, the force is expressed by a gravitational unit. The meaning is that the boy exerts fifty times as much force on the sled as gravity exerts on a pound mass. Since the force of gravity varies slightly with latitude and with elevation above sea level, it follows that a pound of force does not have exactly the same value at all places. This is true of all gravitational units of force. A dynamic unit, however, has the same value everywhere. A dyne of force is the same at the moon as at the earth, at the equator as at the pole.

Each unit of force has its equivalent in other units of force. Dynes may be reduced to grams, kilograms, or pounds, and pounds may be reduced to dynes, just as yards may be reduced to meters or centimeters to inches.

A dyne is an exceedingly small force. At places on the earth's surface where $g = 980$ cm. per second^2, 1 Kg. = 980,000 dynes and 1 pound = 444,525 dynes.

44. Classification of units of force.

$$\text{Units of force}\begin{cases}\begin{array}{r}\text{dynamic or}\\\text{absolute}\end{array}\left\{\text{dyne (metric)}\right. \\[2ex] \text{gravitational}\begin{cases}\left.\begin{array}{l}\text{kilogram}\\\text{gram}\end{array}\right\}\text{(metric)}\\ \left.\begin{array}{l}\text{pound}\\\text{ounce}\\\text{ton, etc.}\end{array}\right\}\text{(English)}\end{cases}\end{cases}$$

45. Value of a gram weight in dynes. — If a body is allowed to fall freely at the equator, the force of gravity will cause it to have at the end of the first second a velocity of 978 cm. per second ($g = 978$ cm. per second^2). If the mass which falls is a gram, then the change of momentum in a second is $1 \times 978 = 978$ units. Therefore, the force of gravity acting on a gram mass at the equator equals 978 dynes; but the force of gravity acting on a gram mass is by definition a gram, therefore a gram weight of force at the equator equals 978 dynes. The value of g at 40° N. Lat. is about 980 cm. per second^2, hence, at that latitude one gram equals about 980 dynes.

Dynes may be reduced to grams at any place by dividing the number of dynes by the value for g at that place. Conversely, gravitational units of force at any place may be reduced to dynamic units by multiplying by the value of g for that place. That is,

$$m \text{ grams} = mg \text{ dynes}.$$

46. Formula for calculating force in dynamic units. Since momentum = mass × velocity, change of momentum = mass × change of velocity, and change of momentum per unit of time = mass × change of velocity per unit of time, which is mass × acceleration. Therefore,

Change of momentum per unit of time = mass × acceleration.

But since change of momentum per unit of time is a measure of force (§ 43), *force = mass × acceleration*, or

$$f = ma.$$

This formula gives the value of a force in dynamic units, in dynes, in the C.G.S. system.

By substituting $\frac{v}{t}$ for a in the above formula we obtain

$f = m\frac{v}{t}$, or $ft = mv$. Since v = the total change of velocity, this formula means that *force* × *time it acts* = *the total change of momentum produced.*

47. Force measured statically.—As we have seen, to measure a force dynamically is to measure it by the motion it produces or destroys. But force is perhaps more often measured statically, that is, by balancing one force by another so that no motion is produced. An elastic spring is often used for this purpose. When an elastic spiral spring is elongated by a force, the elongation is proportional to the force; hence the elongation may be used to measure the force. A force of three pounds will stretch such a spring three times as much as one pound. The butcher's scale and the common spring balance are illustrations of this principle. The spring balance (Fig. 5) when used to measure forces is called a dynamometer. It may be graduated at pleasure in pounds, grams, dynes, or other units of force.

FIG. 5. — Spring balance, a dynamometer.

Problems

1. A car having a mass of 270 Kg. was started from rest and in 3 min. was given a velocity of 720 m. per hour. Assuming that the track was level and that there was no friction, calculate the force propelling the car.

SOLUTION.—720 m. per hour = 20 cm. per second; 270 Kg. = 270,000 g.; and 3 min. = 180 sec. The velocity, and hence the momentum of the car at first, was zero; at the end of 3 min. the momentum was $270{,}000 \times 20 = 5{,}400{,}000$ units. The change of momentum was $5{,}400{,}000 - 0 = 5{,}400{,}000$ units. Since this change occurred in 180 sec., the change of momentum per second was $\frac{1}{180}$ of 5,400,000 or 30,000 units.

Since by definition a dyne is the force that can cause a change of one unit of momentum per second, to produce a change of 30,000 units requires 30,000 dynes of force. Therefore the force propelling the car was 30,000 dynes. The same result is easily obtained by the formula $f = ma$.

2. A 60-Kg. mass was made to acquire a velocity of 2 m. per second in 2 min. What force acted on the mass? *Ans.* 100,000 dynes.

3. A 4000-lb. car has a velocity of 5 ft. per second. What is its momentum? It is made to go faster and faster until its velocity is 11 ft. per second. What is the change of momentum? If the change occurs in 4 min., what is the change per second?

4. A boat weighing 500 Kg. and having a speed of 15 m. per sec. is brought to rest in 11 sec. How much force was used in stopping the boat?

5. A baseball weighing 250 g. and having a velocity of 1200 cm. per second was stopped by the hands of the catcher in $\frac{1}{40}$ of a second. How much force was used in stopping the ball? How many times as much force would be used to stop it in $\frac{1}{100}$ of a second?

6. A 20 g. bullet having a velocity of 600 m. per second penetrating a sandbank was stopped in $\frac{1}{300}$ of a second. What was the resistance offered to the bullet by the sand?

7. A body falling to earth has at the end of the first second a velocity of 980 cm. per second and at the end of the third second a velocity of 2940 cm. per second. What is the change of momentum between the end of the first and the end of the third second, the mass of the body being 10 g.? What is the force acting on the body?

8. What change of momentum will one dyne produce in one second acting on a 1g. mass? On a 2 g. mass? On a 10 g. mass? On a 100 g. mass?

9. What change of velocity will 1 dyne produce in 1 sec. when acting on a mass of 1 g.? On a mass of 2 g.? On a mass of 10 g.? On a mass of 100 g.?

10. What change of momentum will 50 dynes produce when acting on a 10 g. mass for 2 sec.? For 4 sec.? For 12 sec.?

11. A body slides down a smooth inclined plane in 16 sec., acquiring a velocity of 600 cm. per second. Find the acceleration and the length of the plane.

In the following problems the motion is understood to be uniformly accelerated.

12. A body started from rest and moved over 128 ft. in 4 sec. What was the acceleration?

13. A body traversed 80 ft. during the first second of its motion. What was its acceleration?

14. If a body passed over 96 cm. in 4 sec., how far would it go in 8 sec.?

15. How much space has been traversed by a body that has gained a velocity of 500 cm. in 5 sec.?

16. If a body traversed 720 cm. during the 13th second of its motion, how far did it go during the first second?

17. A body having an acceleration of 36 cm. per second^2 started from rest and passed over 128 cm. What was the time required and what was its final velocity?

18. The velocity of a body after it had traversed 250 m. was 20 m. per second. What was the time required to traverse the distance?

19. A mass of 2500 g. passed over 1000 cm. in 10 sec. What was the force acting on the mass to cause this motion?

20. A mass of 2 g. passed over 4410 cm. in 3 sec. What force acted on the body?

21. A gram mass falling freely traversed 490 cm. during the first second of its fall. How great a force was acting on the body?

22. A gram mass falling freely traversed during the first two seconds of its fall 1960 cm. How great is the force of gravity acting on the mass?

23. A mass of 50 g. being acted upon by a constant force passed over 96 cm. in 4 sec. What was the magnitude of the force?

24. A body acquired a velocity of 80 cm. per second in going 1200 cm. What was the time required to cover the distance?

25. If a body has a negative acceleration of 980 cm. per second^2 and is moving at the rate of 10,290 cm. per second, how long will it be before it comes to rest?

IV. COMPOSITION AND RESOLUTION OF FORCES

48. Further discussion of the second law of motion. — This law implies, since it makes no statement to the con-

trary, that a force acting on a body already in motion has the same effect as when acting upon a body at rest. This is illustrated by the fact that a body thrown horizontally falls to the earth just as quickly as one dropped vertically. Likewise, when a body is falling vertically, the force of gravity has the same effect during the last second of its fall as during the first second; the change of velocity is the same in each second.

Experiment. — Fasten a strip of wood about 4 in. long across the end of a yardstick, forming a T, and cut out two similar notches in the upper corners of this crosspiece to hold marbles or bullets. Support the T in a horizontal position (Fig. 6) and place marbles in the two

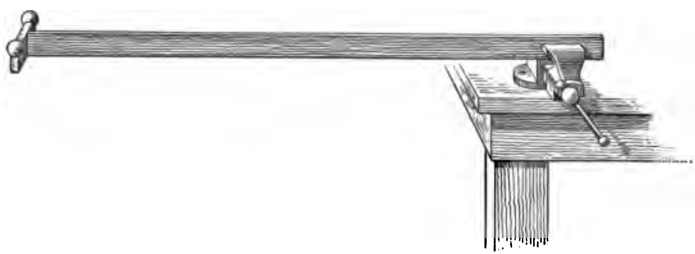

FIG. 6. — Apparatus to illustrate the second law of motion.

notches. A horizontal blow near the crosspiece will send one of the marbles off horizontally, while the other will drop vertically to the floor. The two will be heard to strike the floor at the same time.

Again, the second law of motion implies that if two or more forces act upon a body at the same time, each produces its own change of motion in its own direction regardless of the others. The principles of the composition and resolution of forces and motion which are about to be discussed are based upon the second law of motion.

49. Graphic representation of forces. — Every force always acts at some one point, in some definite direction, and with a certain intensity. Every force must have

these three characteristics, and hence they are the three essential elements of force. They are called (1) point of application, (2) direction, (3) magnitude.

A straight line also has these three characteristics or elements, point of beginning, direction, and length or magnitude; hence a straight line may be used to represent a force, one end of the line representing the point of application, the direction of the line the direction of the force, and the length of the line the magnitude of the force.

For example, the two lines AB and AC (Fig. 7) may represent two forces acting at the point A, one in the direction AC and the other in the direction AB; and if one eighth of an inch be taken to represent a force of 1 lb., the line AC represents a force of 6 lb., and AB one of 8 lb., AB being one inch long and AC three fourths of an inch long.

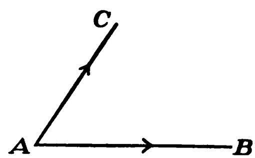

FIG. 7.—Lines representing forces.

50. Composition of forces.— Several forces may act upon a body at the same time, producing a certain effect, but there is often a possible single force which would have the same effect as the several forces if substituted for them.

The process of finding a single force that may be substituted for two or more forces and have the same effect is termed the *composition of forces*, and the single force that may be substituted for the other forces and have the same effect is called the *resultant;* the several forces for which the resultant may be substituted are called the *components.*

When forces have a common point of application, they are said to be *concurring* forces. There are three cases to be considered with two concurring forces:

First, when two concurring forces act in the same straight line and in the same direction, their resultant is their sum.

Second, when two concurring forces act in the same straight line but in opposite directions, the resultant is their difference and has the direction of the greater.

Third, when two concurring forces act at an angle to each other, the resultant is found by the principle of the parallelogram of forces.

51. The parallelogram of forces. — *If the two adjacent sides of a parallelogram are used to represent two forces acting at an angle to each other, the concurrent diagonal of this parallelogram will represent their resultant.*

Let AB (Fig. 8) be a force of 35 dynes, and AC a force of 30 dynes, the angle between them, BAC, being 60°. Let 1 mm. represent one dyne, then the line AB must be 35 mm. long and AC 30 mm. long. If now the parallelogram $ABDC$ is completed, and the diagonal AD, which is concurrent with AB and AC, be drawn, this diagonal will completely represent the resultant of the two component forces AB and AC. The magnitude of this resultant can be found by measuring the length of AD in millimeters. It is about 56.3 mm. long, hence the resultant of the two is about 56.3 dynes.

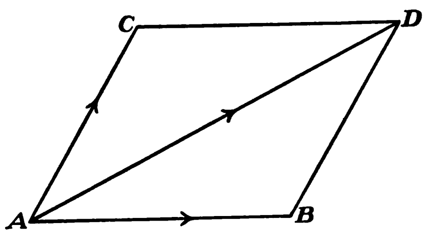

FIG. 8. — Two forces acting at an angle of 60°.

Again let two forces, one of 40 and one of 30 dynes, act at right angles to each other. These forces being represented by AB and AC (Fig. 9), AD represents their resultant. Its magnitude may be determined by

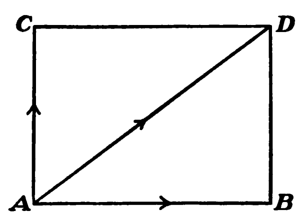

FIG. 9. — Two forces acting at right angles.

measuring the length of AD, but since $AD = \sqrt{\overline{AB}^2 + \overline{AC}^2}$ its value is easily calculated. When the angle between the two components is not a right angle, the magnitude of the resultant must usually be found by construction unless the student has a knowledge of trigonometry.

The resultant of two forces can never be either greater than their sum or less than their difference. It may have any value between these two limits.

52. Composition of more than two forces. — If three concurring forces act at the same time, their resultant may be determined by first finding the resultant of any two of them, and then by using this resultant and the third force as two components finding a second resultant. By a continuation of this process the resultant of any number of forces may be found.

Let AB, AC, and AD (Fig. 10) be three forces having a common point of application at A. AE, the resultant

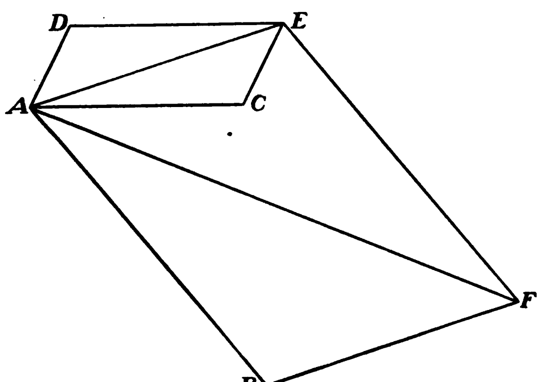

FIG. 10. — The resultant of three forces.

of AC and AD, may first be found by constructing the parallelogram $ACED$, then by using AE as a

component in connection with *AB*, the resultant *AF* is found. *AF* is therefore the resultant of *AB*, *AC*, and *AD*.

53. Experimental illustration of parallelogram of forces. — Join the hooks of three spring balances together by a cord (Fig. 11) and hang two of the balances *B* and *C* upon hooks near the top of the blackboard. Fasten the third balance *D* to another hook at such a position as to cause each balance to register several units of force. Mark the point *A* and record the forces indicated by each balance. After removing the balances draw lines from the point *A* toward each hook (Fig. 12) and lay off on these lines *AG*, *AH*, and *AL* to represent the forces. Evidently any two of these forces as *AG* and *AH* exactly counteract the third force *AL*; but a force *AK* equal and opposite to *AL* would alone counteract it. Hence *AK* is by definition the resultant of *AG* and *AH*, since it could be substituted for them and have the same effect. If now the lines *GK* and *HK* be drawn, the quadrilateral *AGKH* will be found to be a parallelogram in which *AG* and *AH* represent the two components and *AK* their resultant. Observe that while *AK* is equivalent to *AG* and *AH* in effect, it is not equal to their sum.

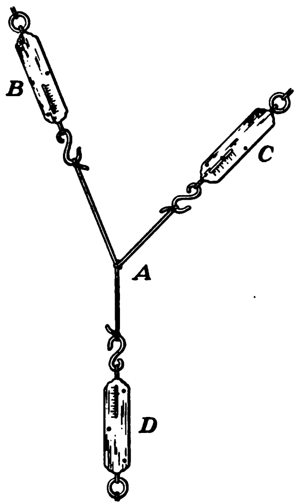

FIG. 11. — Experimental demonstration of the parallelogram of forces.

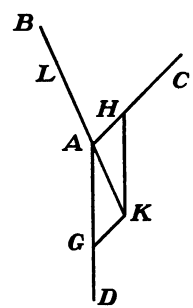

FIG. 12. — Graphical representation of the forces measured in Figure 11.

54. Equilibrant. — A single force that exactly counteracts two or more forces is called their *equilibrant*. It is always exactly equal to and opposite in direction to their

resultant. For example, AL (Fig. 12) is the equilibrant of AG and AH, and AH of AG and AL.

55. Composition of parallel forces. — Let A and B (Fig. 13) represent two forces parallel to each other and acting in the same direction, but having different points of application on a bar CD, and let them be exactly balanced by the force F whose point of application is at E. F is then by definition the equilibrant of A and B. R, the resultant of A and B, must be equal and opposite to F and have the same point of application. *The resultant of two parallel forces acting in the same direction is equal to their sum, and the distances from its point of application to the points of application of the two forces are inversely proportional to the intensities of the two forces.* For example (Fig. 13), $R = A + B$, and $A : B = DE : CE$.

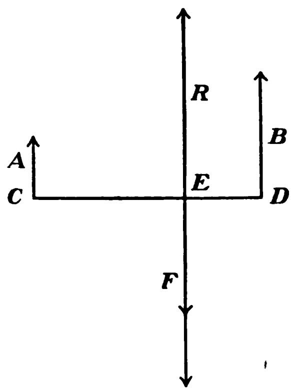

Fig. 13. — Resultant and equilibrant of two parallel forces.

Experiment. — Suspend a rod CD (Fig. 14) 24 in. long from two spring balances A and B and place a weight F of 8 lb. upon the rod. If the point E is 6 in. from D and 18 in. from C, the balance B will register 6 lb. and A 2 lb. If E is 9 in. from D, B will register 5 lb. and A 3 lb. In each case the sum of the two forces will be 8 lb. and the *force* A : *force* $B = ED : EC$.

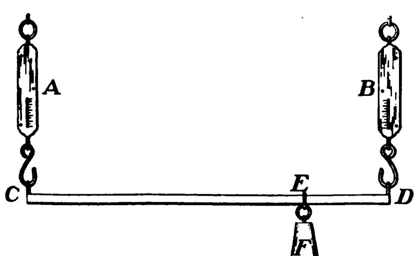

Fig. 14. — Composition of parallel forces.

56. Resolution of forces. — The process of finding two or more forces that can be substituted for a single force and have the same effect is called the ***resolution of forces.*** This is the converse of the composition of forces. The solution of the problem consists in constructing a parallelogram upon a line representing the single force, the given line being the diagonal of the parallelogram.

Let AB (Fig. 15) represent the given force to be resolved into two component forces. By constructing the parallelogram $ACBD$ upon AB as a diagonal, the two components AC and AD which are equivalent in effect to AB are found.

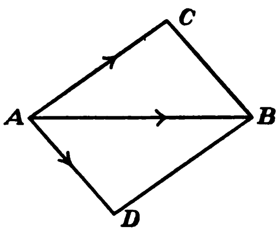

Fig. 15. — Resolution of a force into two components.

If nothing is stated as to the direction or the magnitude of the two components, then an indefinite number of parallelograms can be constructed on AB and an indefinite number of pairs of components of AB can be found which will be equivalent to it. If, however, the directions of the two components, or their magnitudes, or the magnitude and direction of one of them, are specified, then only one solution of the problem is possible.

57. Applications of resolution of forces. — It very often happens that the direction of the force and the direction of the motion produced by it are not the same, and it is desirable to determine what portion of the force is effective in the direction of the motion. For example a boy in pulling a sled exerts a force obliquely upward in the direction AC (Fig. 16) while the sled moves horizontally in the direction AB. How much of the force AC is effective in producing motion in the direction AB is easily determined by the resolution of forces. The force AC

must be resolved into two components, one of which *shall have no effect* in producing motion in the line *AB* and

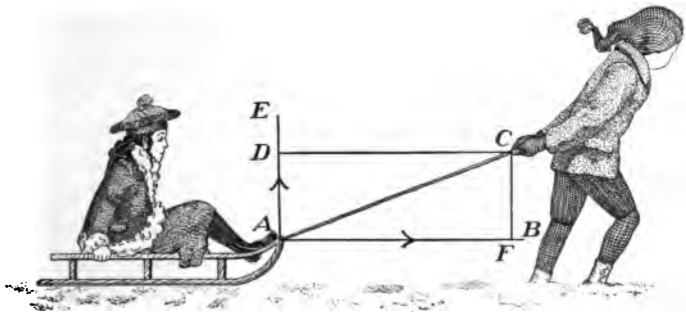

FIG. 16. — To find what portion of a force is useful in a given direction.

hence is eliminated from the problem, and another *all of which* must be effective in the line *AB*. One of the components must therefore have the direction *AE* at right angles to *AB*, because a force in that direction only will have no effect in moving the sled horizontally, either forward or backward; and the other must lie in the direction of *AB* because only such a force will be wholly effective in that direction. Completing the parallelogram *AFCD*, we have the two components *AD* and *AF*, *AF* representing that portion of the force exerted by the boy which is useful in moving the sled, while *AD* represents that part of the force *AC* which tends to lift the sled.

If a force is to have no effect in any particular direction, it must act at right angles to that direction. Thus, *AF* has no effect in the direction *AD*, and *AD* has no effect in the direction *AF*. Components which act at right angles to each other are called ***rectangular components***.

58. The inclined plane, which consists of a smooth, plane surface, making an acute angle with the horizontal plane, affords a practical illustration of the resolution of forces.

Let A (Fig. 17) represent a barrel upon a plank, one end of which rests in a wagon and the other on the ground. How great is the force which tends to make it roll down the plank? Since gravity acts vertically, the weight of the barrel acts in the direction represented by AD. Let AE represent this force. By completing the parallelogram $AGEH$, AE can be resolved into two components AH and AG. The latter has no effect in moving the barrel either up or down the plane, being perpendicular to it; but it is all used in producing pressure on the plane. AH, being parallel to the plane, produces no pressure, but is wholly effective in moving the barrel down the plane. To hold it in position a force equal and opposite to AH must be used.

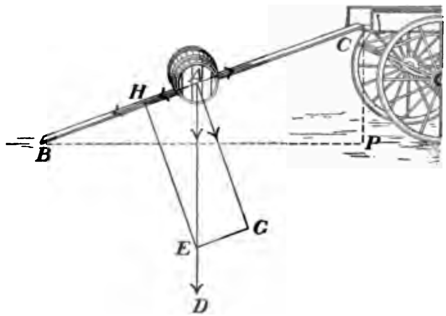

Fig. 17. — Showing what component of the weight of the barrel must be used to keep it on the inclined plane.

The triangles CPB and AHE (Fig. 17) are similar, and therefore $AH : AE = CP : CB$. *Hence, the force necessary to hold a body in place on the plane, when it acts parallel to the plane, is to the weight of the body as the height of the plane is to its length.*

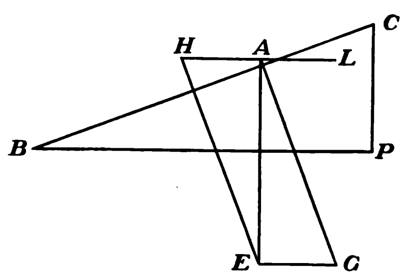

Fig. 18. — Force acting parallel to the base of the inclined plane.

If the force holding

the barrel in place acts parallel to the base of the plane, as AL (Fig. 18), the weight AE may be resolved into two components, AG and AH. AH is the component that must be overcome by AL. Since the two triangles AEH and PBC are similar, $AH : AE = CP : BP$. *Therefore the force holding the body in place on the plane, when it acts parallel to the base of the plane, is to the weight of the body as the height of the plane is to its base.*

59. Composition and resolution of motions and velocities. — A line can be used to represent the direction and magnitude of a motion or of a velocity as well as of a force; and motions, velocities, and accelerations can be compounded and resolved by the same methods used for the composition and resolution of forces. For example, a vessel sailed 100 miles toward the northeast; how far east and how far north did it go? Let AB (Fig. 19) represent the motion of the ship, then it is evident that AC represents the eastward and AD the northward component of its motion. Again, suppose a man to row at the rate of three miles per hour at right angles to the current of a river which flows at the rate of four miles per hour. His actual velocity will be represented by the diagonal of a parallelogram whose sides represent the component velocities.

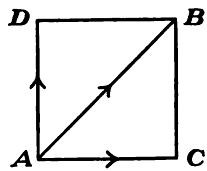

FIG. 19. — Resolution of a motion into two components.

60. Composition of uniform motion with uniformly accelerated motion. — If a body is thrown out horizontally from a high building, its actual motion (the resistance of the air being disregarded) will be the resultant of uniform motion in a horizontal direction and uniformly accelerated motion in a downward direction. Let AB (Fig. 20) represent the horizontal component and AC the vertical component of its motion, the numbers 1, 2, 3, etc.,

representing the seconds of time. The actual path of the body will be represented by the curved line *AD*, *a*

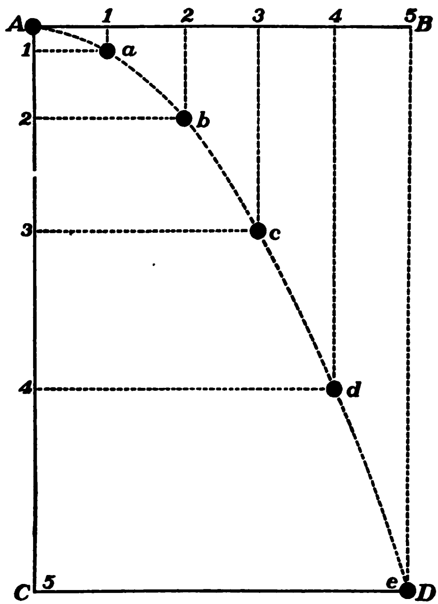

Fig. 20.—Composition of a uniform horizontal motion with a uniformly accelerated downward motion.

being its position at the end of the first second, *b* at the end of the second second, *c* at the end of the third, etc.

V. GRAVITATION

61. Gravitation is the force of attraction existing between all bodies of matter at all distances.

It is universal; every star, for example, attracts every other star. The earth attracts every star and is in turn attracted by every star. But not only does gravitation act between the heavenly bodies at enormous distances, but it

also acts between small bodies upon the earth. A book attracts all other bodies and is attracted by them. Cavendish, in 1789, by a celebrated experiment proved the existence of this attraction between two small masses of lead.

Nothing is known of the real nature of gravitation or its cause. Sir Isaac Newton was the first to establish the universality of gravitation both as to distance and all matter, and it was he who first formulated the law of gravitation.

62. Newton's law of gravitation. — *The attraction between two bodies varies directly as the product of their masses and inversely as the square of the distance between their centers of mass.*

The gravitational attraction between the earth and bodies upon it which causes bodies to fall to the earth is given the special name of *gravity.*

The weight of a body is the force of gravity acting between that body and the earth.

63. Laws of weight. — The laws of weight are merely special applications of the general law of gravitation to the force of gravity. They may be stated as follows: (1) The *weight of a body varies directly as its mass.*[1] (2) The *weight of a body varies inversely as the square of the distance between its center of gravity and the center of the earth.* This law does not apply to bodies below the surface of the earth. If the weight of a body on the surface of the earth be represented by w and its distance from the center

[1] According to Newton's law of gravitation the weight of a body would vary as the product of its mass and the mass of the earth; but in comparing the weight of one body with that of another by a proportion the mass of the earth would appear in two terms of the proportion and could be canceled. Hence the mass of the earth need not be considered in the law of weight.

of the earth (4000 miles) by d, and if the weight of another body above the surface of the earth be represented by w_1 and its distance from the center of the earth by d_1, this law may be expressed algebraically as follows:

$$w \propto \frac{1}{d^2}, \text{ or } w : w_1 = d_1^2 : d^2.$$

(3) The *weight of a body below the surface of the earth varies directly as its distance from the center of the earth.*[1] It follows from the last two laws that a body weighs most at the surface of the earth, being less either above or below it. Using the same letters as in the previous paragraph, this law may be expressed as follows:

$$w \propto d, \text{ or } w : w_1 = d : d_1.$$

64. Equilibrium. — When two or more forces so act upon a body as exactly to counteract the effect of each other, they are said to be in equilibrium. Forces that are in equilibrium, therefore, produce no change of motion, that is, no acceleration. If the body upon which they act is at rest and remains at rest, the forces acting upon it are in equilibrium; if a body is in motion and forces act upon it in such a way as to produce no change of motion, the forces are in equilibrium.

A body is in equilibrium when all the forces acting upon it are in equilibrium. A body at rest is in equilibrium; likewise a body having uniform motion in a straight line is in equilibrium, because no matter how many forces are acting upon it, they must exactly counteract the effect of one another, else there would be a change either in the magnitude or the direction of the motion.

65. Center of gravity. — A body is composed of a great number of particles each one of which is pulled toward

[1] This law assumes that the density of the earth is uniform throughout. Since the density is not absolutely uniform, the law is not rigidly exact.

the center of the earth by the force of gravity. Although these forces converge toward the center of earth, yet for bodies of ordinary size they are practically parallel because the center of the earth is so far distant. A single force GF (Fig. 21) that could take the place of all these forces would be their resultant, and its magnitude would equal the weight of the body.

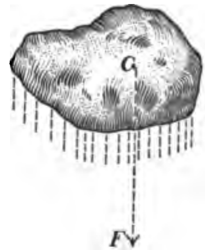

FIG. 21.—Center of gravity or center of mass.

The *center of gravity* of a body is the point of application of the resultant of all the parallel forces of gravity acting on its particles. Since the weight of each particle is proportional to its mass, the center of gravity of a body is also its *center of mass* and its *center of inertia.*

If the weight of body were a single force applied at its center of gravity it would have the same effect as the many parallel forces acting on the particles of the body, hence the weight of a body may be considered as concentrated at its center of gravity.

66. Support of a body. — A body may be supported (1) on one point or pivot; (2) on two or more points in a straight line, that is, on an axis such as a knife edge; or (3) on three or more points not in the same straight line, that is, on a base. The base of a body is the figure included by a string drawn tightly around the points of support. For example the base of a chair is the quadrilateral of which its feet are the four corners.

A body is supported when its center of gravity is supported. The supporting force, or the resultant of the supporting forces when there are two or more, must be the equilibrant of the forces of gravity acting on all the particles of the body. It will therefore be exactly equal

and opposite to their resultant, that is, it will be a vertical line passing through the center of gravity.

67. Line of direction. — A vertical line passing through the center of gravity of a body toward the center of the earth is called its *line of direction.* A body is supported in equilibrium only when its line of direction passes through its support. This is true whether the support be a pivot, an axis, or a base; and whether the support be above, below, or at the center of gravity.

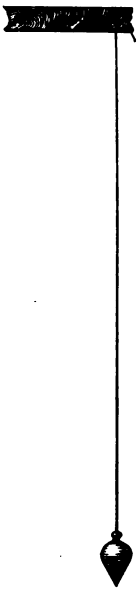

FIG. 22. — Plumb line.

When a body falls freely, its center of gravity follows the line of direction. Thus if a crooked stick is whirling while it is falling, its center of gravity will traverse a straight line, while the paths of other points in the stick may be very irregular.

68. Locating the center of gravity. — Let a body be suspended by a cord. A line coinciding with the cord and extending through the body will pass through its center of gravity and be vertical. It is on this principle that the use of the plumb line depends. This (Fig. 22) consists of a conical mass of lead (plumbum) or other metal suspended by a cord. The cord indicates the vertical direction.

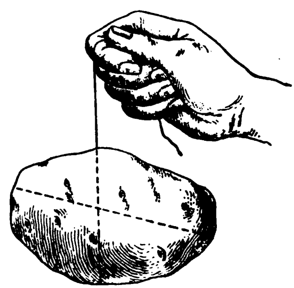

FIG. 23. — Finding the center of gravity of a body.

If a body (Fig. 23) be suspended from two different points by a cord, two lines passing through its center of gravity may be found, and thus its center of gravity may be

located; for since it lies in both of the lines, it must be at their intersection.

Experiment. — Bore two holes A and B (Fig. 24) in a thin board of uniform thickness and suspend the board by one of these holes upon a wire held horizontally. Suspend a plumb line from the same wire in front of the board and draw a line on the board just back of the plumb line and parallel to it, starting exactly underneath the wire. In the same way draw another line after suspending the board by the other hole B. The intersection of these two lines C will indicate the center of gravity of the board. Bore a hole at C and test the accuracy of the method by suspending the board by the hole C. It should rest in any position.

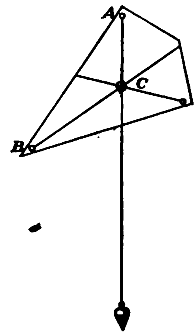

Fig. 24. — Finding the center of gravity of a board.

69. States of equilibrium with reference to gravity. — The center of gravity of a body always tends to take the lowest possible position. This fact gives rise to three states of equilibrium, *stable*, *unstable*, and *neutral* (Figs. 25, 26, 27).

A body is in stable equilibrium when it is so supported that a slight displacement tending to overturn it raises its

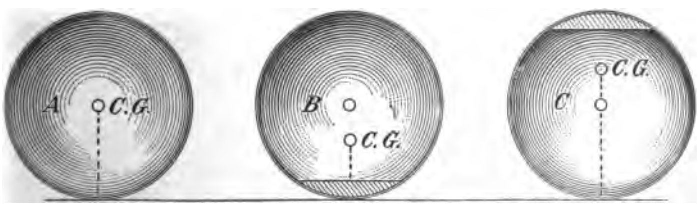

Fig. 25. — Three states of equilibrium. *A*, neutral; *B*, stable; *C*, unstable.

center of gravity; the body tends to return to its original position. A body is always in stable equilibrium when its support is above its center of gravity; also when its support is a base, whether it is above or below, or passes through its center of gravity.

A body is in unstable equilibrium when it is so supported that a slight displacement tending to overturn it lowers its center of gravity, and the body tends to fall

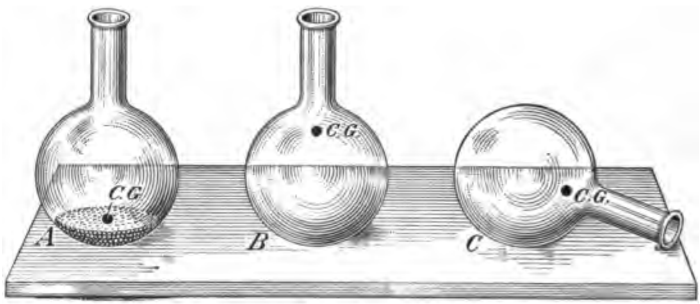

FIG. 26.—Three states of equilibrium. *A*, stable; *B*, unstable; *C*, neutral.

farther away from its original position. The support must be a point or an áxis below the center of gravity. A cone balanced on its apex and a cube balanced on one of its edges are illustrations of this kind of equilibrium.

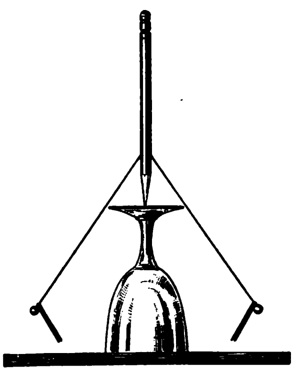

FIG. 27.—A pencil, balanced on its point, in stable equilibrium.

A body is in neutral equilibrium when it is supported in such a way that a slight displacement tending to overturn it does not change the height of its center of gravity; it tends neither to return nor to move farther away from its original position. A body is always in neutral equilibrium when its support is a point at its center of gravity or an axis passing through it. A perfectly balanced bicycle wheel, a sphere *A* (Fig. 25) whose center of gravity is at its center of volume resting on a level

surface, and an empty flask C (Fig. 26) on its side are illustrations of neutral equilibrium. If the center of gravity of a sphere is not at its center of volume, B and C (Fig. 25), then it may be in stable or in unstable equilibrium.

Stable equilibrium is illustrated in Figures 26 and 27. Figure 26, A, represents a round-bottomed flask containing about 500 g. of shot held in place by wax which has been melted about it. This brings the center of gravity so low that it must be raised to overturn the flask. Figure 27 represents a pencil into which two hat pins have been thrust. The stability of the pencil may be increased by loading the pins with pieces of bent wire as shown in the figure.

70. Stability. — Bodies in stable equilibrium may have different degrees of stability. The degree of the stability of a body depends (1) on its weight, (2) on the height of its center of gravity, and (3) on the size of its base. In order to overturn a body, it is necessary to turn it on one of its edges as i (Fig. 28) until the line of direction cd of its center of gravity falls outside of the base. The center of gravity must pass through an arc cc' of which i is the center, and be raised through a vertical distance equal to ec'.

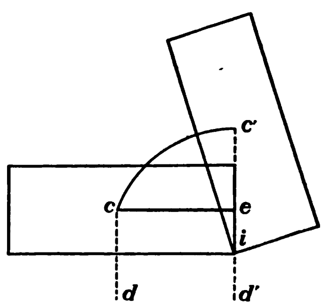

FIG. 28. — Diagram to illustrate the measurement of the degree of stability of a body.

When the weights of different bodies are the same, the vertical distance ec' through which their centers of gravity must be lifted to overturn them is the measure of their stability, as illustrated in Figure 29.

It is obvious that if two bodies are alike in every re-

spect except weight, the lighter one is the more easily overturned.

Suppose the three bodies *A*, *B*, and *C* (Fig. 29) to have the same weight, *A* and *B* having bases of the same size but differing in the height of their centers of gravity, and *B* and *C* having the same height of centers of gravity but differing in the size of their bases.

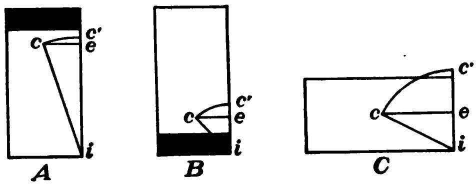

FIG. 29.—Diagram illustrating what stability depends upon.

It is evident from a comparison of *A* and *B* that the lower the center of gravity, other things being equal, the greater the stability of the body, because the distance *ec'* is greater.

From a comparison of *B* and *C* it is evident that the larger the base of a body, other things being equal, the greater its stability.

Problems

1. How great is the equilibrant of two forces, one of 900 dynes and one of 1600 dynes, acting perpendicularly to each other?

2. If a mass of 100 lb. is suspended 4 ft. from one end of a bar and 6 ft. from the other end, how much pressure is exerted at each end of the bar on its supports?

3. A body is projected horizontally with a velocity of 10 ft. per second from a tower 256 ft. high. How far from the foot of the tower will it strike the ground?

4. If a force of 600 dynes is resolved into two equal rectangular components, how great is each component?

5. What part of a force is useful in producing motion in a given direction, which acts at an angle of 45° with that direction?

Ans. 0.707 of it.

6. What part of a force is useful in a given direction, that acts at an angle of 60° with that direction?

7. John and James are carrying a pail of sand weighing 135 lb. on a bar 9 ft. long. The pail is 4 ft. from John's end. How much of the weight does each boy support?

8. Where must the pail be placed so that James shall support 100 lb.?

9. A man in rolling a barrel up a plank pushed with a force of 200 lb., but exerted his force in such a direction that the useless component of it was 100 lb. How much useful force was exerted on the barrel?

10. Three forces act outward from a point at an angle of 120° with one another. One of them, *A*, equals 1200 dynes, and the other two, *B* and *C*, equal 800 dynes each. What is the direction and magnitude of the resultant of the three forces?

11. A boy is pulling a sled in such a way that he exerts a force of 40 Kg. usefully in moving the sled horizontally and a force of 30 Kg. vertically. How great a force is he actually using?

12. Resolve a force of 80 dynes into two rectangular components one of which shall be $\frac{3}{4}$ as great as the other. What is the magnitude of each component?

13. A team of horses was moving a car by a rope attached to one side of it. They pulled at such an angle that the useful component of their pull was 400 Kg., and the useless component was 300 Kg. How great a pull were they exerting?

14. The sail of a boat was set at an angle of 45° with its keel, and the wind struck the sail at an angle of 45°. How much pressure would the wind exert on the sail which exerts a pressure of 10 Kg. on a surface perpendicular to it? How much force would this wind exert in the direction of the keel? *Ans.* 5 Kg. in the direction of its keel.

15. A steamer is sailing east with a velocity of 240 m. per minute and a man runs north on the deck at the rate of 180 m. per minute. Show by a diagram his actual direction and calculate his actual velocity in centimeters per second.

16. Assuming the radius of the earth to be 4000 miles, determine what the weight of a body would be 4000 mi. from the surface of the earth that weighs 32 lb. on the surface. *Ans.* 8 lb.

17. At what height would a body weigh 50 Kg. which weighs 100 Kg. on the earth's surface?

18. At what distance below the earth's surface will the 100 Kg mass weigh 50 Kg.? How much would it weigh 1000 mi. from the earth's center?

19. The moon is about 240,000 mi. from the center of the earth. How many times less is the attraction of the earth for a small piece of the moon than it would be if that piece were on the earth's surface?

VI. CENTRIFUGAL FORCE

71. Centripetal force. — Curvilinear motion is motion whose direction is changing at every instant of time, that is, continuously. According to the first law of motion a body in motion continues to move in a straight line except when a force acts to change its direction. It follows, therefore, that a force must act continuously upon a body to cause it to change its direction continuously. Moreover, this continuous force must not act in the direction of the motion, but at an angle to its direction, for a force acting in the direction in which the body is moving could change only the magnitude of the motion, not its direction.

Motion in a circle, or circular motion, is a familiar form of curvilinear motion. In such motion the continuous force which acts at right angles to the motion and toward the center of the circle is called the *centripetal* (center seeking) *force.*

The pull we exert on the string when whirling an object around the hand is a familiar illustration of centripetal force, and the fact that the object moves off in a straight line, tangent to the circle, the instant the string is broken, shows that the force must be continuous to produce circular motion.

72. Centrifugal force. — We have learned (§ 25) that a body can exert force only when it encounters resistance and that the force cannot be greater than the resistance

encountered. The resistance which the centripetal force encounters is due to the inertia of the body (§ 23). This resistance is called *centrifugal force.* By the third law of motion the centripetal and the centrifugal forces are equal. The two together constitute a stress.

The magnitude of either one of these forces is calculated by the formula

$$f = \frac{mv^2}{r} \text{ dynamic units, or } f = \frac{mv^2}{gr} \text{ gravitational units.}$$

73. Development of formula for centrifugal force. — The term acceleration is applied to change of direction as well as to change of magnitude of motion. We have seen that a continuous force is necessary to cause either one. If the force is constant, the acceleration is constant.

A body moving with uniform velocity in a circle has a constant acceleration toward the center of the circle and may be said to be falling toward the center with uniformly accelerated motion.

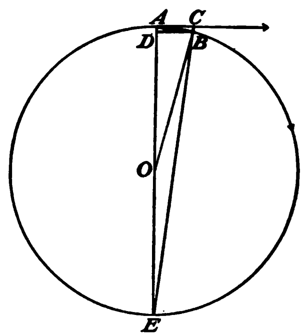

FIG. 30.—Diagram for demonstrating the formula for centrifugal force.

Let r = the radius of the circle in cm.

Let a = the acceleration toward the center in cm. per sec^2.

Let v = the uniform velocity in cm. per sec. of a body moving in a circle.

Let t = the number of seconds required for the body to traverse the arc AE.

Let m = the mass of the body in grams.

AC, the tangent at the point A, represents the path the body would take if the force acting toward the center should cease at that point. DB is drawn parallel to AC. When t is exceedingly small, AB is very short coinciding with the chord AB. CB, or its equal AD, is the distance the body is turned from a straight line, or it is the distance the body moves toward the center with uniformly accelerated

motion in the time t. Hence, $AD = \frac{1}{2}at^2$; and since the body moves with uniform velocity in the circle, $AB = vt$.

By geometry $AD : AB = AB : AE$. Substituting $\frac{1}{2}at^2$ for AD, vt for AB, and $2r$ for AE, we have $\frac{1}{2}at^2 : vt = vt : 2r$ from which we obtain $a = \frac{v^2}{r}$, the value of the *centripetal acceleration* toward the center.

Multiplying both terms of this equation by m, we obtain $ma = \frac{mv^2}{r}$, but $f = ma$; therefore, $f = \frac{mv^2}{r}$.

This formula gives the value of f in dynamic units, in dynes in the C.G.S. system; but if it is divided by g, the value of f is given in gravitational units.

Problems

1. A mass of 49 g. was whirled in a circle by a string 50 cm. long with a velocity of 100 cm. per second. What was the pull on the string?

$$f = \frac{49 \times 100^2}{50} = 9800 \text{ dynes; or, } f = \frac{49 \times 100^2}{50 \times 980} = 10 \text{ g.}$$

2. A fly wheel 16 ft. in diameter made 120 revolutions per minute. What centrifugal force did a portion of the rim of the wheel weighing 100 lb. exert? *Ans.* $400\pi^2$ or 3948 lb.

3. A mass of 500 g. revolved in a circle 120 cm. in diameter with a velocity of 200 cm. per second. What was the centrifugal force?

4. Calculate the number of times the force in the last problem would be increased by doubling the number of revolutions per second.

5. How many times is the centrifugal force increased by increasing the number of revolutions per second 5 times? 8 times? 20 times?

6. How is the centrifugal force affected by doubling the diameter of a revolving wheel, the mass of the wheel and the rate of rotation remaining the same?

74. Illustrations of centrifugal force are familiar to us all. The mud flying from a revolving carriage wheel and water from a grindstone are phenomena due to this force. The inclination of a bicycle rider on turning a corner is necessary to counteract this force. Great damage and

loss of life sometimes result from the bursting of enormous iron fly wheels when by some accident their speed becomes so great that the centrifugal force breaks the wheel in pieces. It should be noted that this force varies as the *square* of the velocity; this means that the centrifugal force is quadrupled when the velocity is doubled and 100 times as great when the velocity is increased 10 times.

There are many useful applications of this force. The centrifugal drier used in laundries is one of them. It consists of a cylinder with perforated sides in which the clothes are placed and revolved very rapidly, the water flying out through the holes in the sides. By this force honey is separated from the comb, sirup from the crystals of sugar, and cream from milk. If oil and water are placed in two inclined tubes (Fig. 31), the oil being the lighter will remain at the top when the tubes are at rest; but when they are revolved, the water and the oil change positions. The denser liquid goes to the outside. This may be illustrated still further by placing some mercury and colored water in a flask and revolving it by a whirling machine. The mercury will form a ring about the center of the flask, the water being on each side of it (Fig. 32).

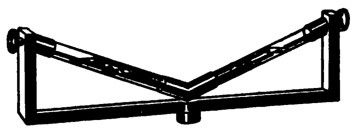

Fig. 31. — Apparatus for showing the effect of centrifugal force on liquids of different densities.

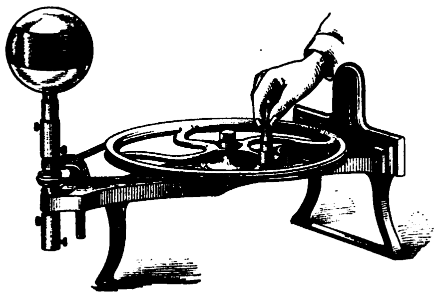

Fig. 32. — Apparatus for applying centrifugal force to mercury and water.

A body always tends to rotate about its center of gravity and upon its shortest axis. This may be illustrated by suspending such an object as a ring, a disk, a chain bracelet, or two balls of unequal size joined by a wire, from a whirling machine by a string and revolving it rapidly. Although the string may not be attached to the object at its center of gravity, yet the body will revolve about that point. The moon, for example, does not revolve about the earth, but the earth and the moon together revolve about their common center of gravity, which is about 2880 miles from the earth's center.

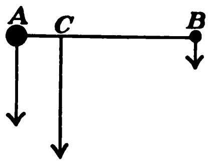

FIG. 33. — Diagram of two bodies revolving about their common center of gravity.

75. The fact that a body tends to rotate about its center of gravity may be used to verify the formula for centrifugal force. Let A (Fig. 33) be a mass of 6 Kg. connected by a rod 120 cm. long to B, a mass of 2 Kg., the mass of the rod being disregarded. By the law of parallel forces the resultant of their weight will be at C, 90 cm. from B and 30 cm. from A, hence C is the center of gravity of the system about which it will revolve. Since the radius of B's path is 3 times that of A's, its velocity will be three times as great. Let $v =$ the velocity of A and $3v$ that of B; then the centrifugal force for $A = \frac{6000 \times v^2}{30} = 200 v^2$ dynes, and that for $B = \frac{2000 \times (3v)^2}{90} = 200 v^2$ dynes. The centrifugal forces of the two bodies therefore balance each other when they revolve about their common center of gravity.

VII. THE PENDULUM

76. Simple pendulum. — A body suspended from a point or axis so as to vibrate to and fro under the influence of gravity is a pendulum. A *simple pendulum consists of a single heavy particle supported by a thread without weight.* Such a pendulum has, of course, no more actual existence than a geometric line; but it is conceived for the purpose

of arriving at the laws of the pendulum. A small dense ball such as a lead bullet suspended by a fine silk thread is an approximation to a simple pendulum and is often used as such in experimental work. Every actual pendulum is a *compound* or *physical pendulum* because the supporting thread or rod must have weight and the bob must have size; it may be conceived to be composed of as many simple pendulums as there are particles in it.

77. Motion of a pendulum. — Let O (Fig. 34) be the center of suspension of a pendulum which is drawn aside to the point A, and let AB represent the weight of the bob. This force AB, which of course acts vertically, may be resolved into two rectangular components, one AC, in line with the cord OA, which has no effect in causing motion but is entirely used in producing tension on the cord, and the other AD, tangent to the arc of motion, which is all used in causing motion toward P. As it approaches the point P the component AD becomes less and less until at P it is zero, while the other component AC increases until it equals AB at P. The inertia of the pendulum carries it past P, but now the component EH begins to retard it and finally brings it to rest at E, when the motion is

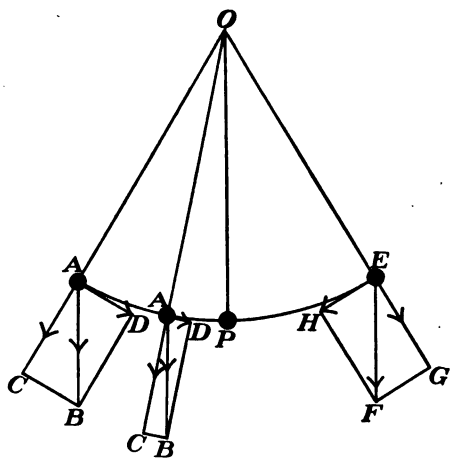

FIG. 34. — Diagram of the motion of a simple pendulum.

repeated in the opposite direction. Were it not for the resistance of the air and the friction of the point of support it would continue to vibrate to and fro indefinitely.

78. Definitions.— The motion of the pendulum to or fro, as from A to E, or E to A, is called a *simple vibration;* while a motion to and fro, as from A to E and back again, is called a *complete vibration.* The point or axis about which the pendulum oscillates is the *center of suspension.* The distance from the position of rest P to the extremity of the arc of vibration A is called the *amplitude* of vibration. It is measured by the angle POA. The *time* or *period* of a pendulum is the time required for a simple vibration. A pendulum whose time or period is a second is called a *seconds pendulum.*

79. Center of oscillation. — We are all familiar with the fact that a short pendulum vibrates faster than a long one. It is evident then that particles in the upper part of a compound pendulum tend to make it vibrate faster than it actually vibrates, and that those in the lower part of it tend to make it vibrate slower than it actually vibrates. There must be therefore an intermediate point or particle between these two sets of particles, which tends to vibrate neither faster nor slower than the pendulum actually vibrates.

The point or particle in a compound pendulum that tends to vibrate neither faster nor slower than the compound pendulum actually vibrates is called its center of oscillation.

If all the other particles of the pendulum but this one could be removed, it would still vibrate at the same rate as the compound pendulum does. It follows therefore that every compound pendulum is equivalent to a simple pendulum whose length is equal to the distance from the center of suspension to the center of oscillation. This distance is the *true length of the compound pendulum.*

Experiment. — Cut a notch in the end of a uniform bar such as a meter stick and cover the top of the notch by a piece of tin or thin brass (Fig. 35). Drive a three-cornered file, the edge of which has been ground sharp, or a knife blade into an upright support with the sharp edge up, and suspend the bar from it by resting the brass strip upon the knife-edge. This will form a compound pendulum. Suspend a simple pendulum consisting of a round bullet and a fine silk thread in front of the bar or at its side, having their centers of suspension at the same level.

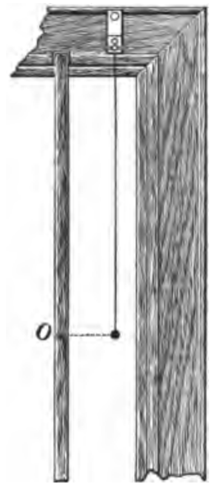

FIG. 35. — A compound pendulum and a simple pendulum compared.

First, make the simple pendulum about half as long as the bar and compare their rates of vibration. The simple pendulum will vibrate the faster.

Second, make the simple pendulum of the same length as the bar and set them both in vibration. The bar will now vibrate the faster.

Third, shorten the simple pendulum and find by trial such a length for it that the rates of the two pendulums are the same. A point O in the bar on the same level as the center of the bullet will be the center of oscillation of the bar pendulum. Bore a hole through the bar at this place so that the lower edge of it shall be exactly at the center of oscillation.

80. Interchangeable points. — The centers of suspension and oscillation are interchangeable; that is, if the pendulum is reversed and the center of oscillation is made the center of suspension, the first center of suspension becomes the center of oscillation, and the pendulum vibrates at the same rate as before. This makes it possible to find the true length of a compound pendulum with great accuracy by finding two points on opposite sides of its center of gravity about which it will vibrate in the same time.

If the bar pendulum (Fig. 35) is reversed and suspended on the knife-edge at its center of oscillation O, it will still

vibrate at the same rate as the simple pendulum. In a bar of uniform thickness, when the center of suspension is at one end, the center of oscillation is at a point two thirds of the length of the bar from the center of suspension.

81. Laws of the pendulum.—The laws of the simple pendulum apply to the compound pendulum when its true length and not its apparent length is taken.

I. *The time of a pendulum is independent of its amplitude.* This law holds true only when the amplitude is small. If, for example, the arc through which the pendulum swings is at first about 5° or 6° and gradually becomes less, its time of vibration does not appreciably change; but if a pendulum is made to swing through a large arc, its time of vibration becomes shorter as the length of the arc diminishes. The law is sometimes stated as follows: *The vibrations of a pendulum are isochronous* (Galileo, 1583).

II. *The time of a pendulum is independent of its mass.*

III. *The time of a pendulum is directly proportional to the square root of its length.* Let t and l = respectively the number of units of time and of length of one pendulum and t' and l' = the number of units of time and of length of another pendulum. Then $t : t' = \sqrt{l} : \sqrt{l'}$.

IV. *The time of a pendulum is inversely proportional to the square root of the acceleration due to gravity, g.*

82. Experimental verification of the laws of the pendulum. — Galileo was the first to discover and verify experimentally these laws. His attention was drawn to the subject in 1583 by a swinging chandelier which still hangs in the cathedral at Pisa. These laws may be roughly illustrated by experiments in the class room, but accurate experiments are suited only to the laboratory.

Experiment 1. — Suspend two simple pendulums (a bullet supported by a fine silk thread, the center of the bullet being taken for the center of oscillation) of the same length side by side. Pull them

aside through small but unequal arcs and let go of them simultaneously. The two pendulums will swing in unison for a long time if their lengths are accurately equal.

In like manner start them vibrating again, making one of them swing through a very long arc. The one vibrating through the long arc will soon fall behind the other in its vibration.

Experiment 2. — Repeat the first part of the preceding experiment, using a wooden or a hollow ball of the same size as the bullet in place of the bullet. This will illustrate the second law.

Experiment 3. — Make one pendulum exactly 100 cm. long, and another 81 cm. long, and start them vibrating at the same time. The shorter one will make exactly 10 vibrations while the longer makes 9. Whatever the time of the longer one, the time of the shorter one is $\frac{9}{10}$ as great; for, suppose the time of the longer one is to be 1 second, then it will take it 9 seconds to make 9 vibrations, and accordingly 9 seconds for the other to make 10 vibrations. Hence the time of the shorter one is $\frac{1}{10}$ of 9 seconds, which is 0.9 sec. The times of the two pendulums are therefore 1 sec. and 0.9 sec., but $1 : 0.9 :: \sqrt{100} : \sqrt{81}$. In the same way the times of two pendulums 81 and 64 cm. long may be compared, or two pendulums one of which is four times as long as the other.

Experiment 4. — Support two pendulums of the same length side by side, using an iron ball for the bob of one of them. If some magnets are placed under the iron ball pendulum, it will vibrate faster than any other pendulum, that is, its time will be shorter. The attraction of the magnets for the iron ball is an equivalent to an increase in the force of gravity.

83. Theory of the pendulum. — The mathematical theory of the pendulum was first worked out by Huygens (1673), who discovered the interchangeability of the centers of suspension and oscillation. The laws of the pendulum are expressed mathematically by the formula $t = \pi\sqrt{\frac{l}{g}}$ which can easily be reduced to the form $g = \frac{\pi^2 l}{t^2}$. By the use of this formula the value of g for any place can be obtained when the time and length of a pendulum at that place have been experimentally determined; or the length

of a seconds pendulum can be calculated for any place when g is known for that place. For the northern latitudes of the United States the seconds pendulum is about 99.3 cm. or 39.1 in.

The United States government has determined the value of g at many places by means of the pendulum. The following are some of the results obtained:

Washington, D.C.	980.10 cm.	Cleveland, O.	980.23 cm.
Boston, Mass.	980.38	Chicago, Ill.	980.26
Ithaca, N.Y.	980.29	Cincinnati, O.	979.99
Charlottesville, Va.	979.92	Denver, Col.	979.60
Philadelphia, Pa.	980.18	Pikes Peak, Col.	978.94

84. Uses of the pendulum. — The chief use for the pendulum is in the measurement of time (Fig. 36). The first and third laws of the pendulum make its use possible in regulating the motion of the hands of a clock. The motive power is supplied by weight or a spring, an impulse being given to the pendulum by the escapement wheel at each swing sufficient to overcome the resistance of the air and the friction at the point of support. If the clock gains time, the pendulum is lengthened by lowering the bob; and if it loses time, it is shortened by raising the bob. In his early experiments with falling bodies before he discovered the laws of the pendulum, Galileo measured time by weighing the amount of water flowing from an orifice.

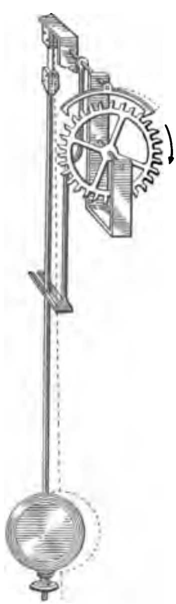

Fig. 36. — A clock pendulum.

Problems

1. How long is a seconds pendulum at Washington, D.C.?

2. If a seconds pendulum were a meter long,

how long would a pendulum have to be to make a simple vibration in 2 seconds? 3 seconds? 4 seconds?

3. At what value of g would a seconds pendulum be a meter long?

4. If a seconds pendulum is 99.3 cm. long, what is the length of a pendulum whose time is 1.5 seconds?

5. If the time of a pendulum 88 in. long is 1.5 seconds, how long is a pendulum whose time is 1.2 seconds?

6. How does the time of one pendulum compare with that of another twice as long? three times as long? five times as long? n times as long?

7. A 50-centimeter pendulum was found to make a simple vibration at a certain place in 0.7101 second. What was the value of g at that place?

8. A clock having a seconds pendulum 39.1 in. long keeps correct time in winter. How much will it lose a day in summer if it becomes 39.2 in. long by expansion?

VIII. WORK[1]

85. Work *is force acting through space.* For example, when a horse pulls steadily on a load moving it 100 feet, the horse exerts force through a space of 100 feet and does work. If the horse should pull with the same force for any length of time and yet not move the load at all, he would do no work upon it according to the meaning given to that word in physics. Force without motion is not work, neither is motion without force. When a bullet is shot from a gun, work is done on the bullet because the gases resulting from the explosion of the powder exert force upon the bullet through the length of the gun barrel; if the bullet penetrates an embankment, it does work, since it exerts force through the space it penetrates. The

[1] It often happens that a familiar word is taken from ordinary language and given a special and peculiar meaning. When a word is thus appropriated and given a limited and precise meaning, it becomes a technical term. The student must learn to use such a word with precision and exactness. The word *work* is used in this way in physics.

body exerting the force is said to do the work, and the body upon which the force is exerted and which is moved is said to have work done upon it.

86. Measurement of work. — Work, therefore, consists of two elements or factors, *force* and *space*. It is obvious that if the force acting through a given space is doubled, the work done is also doubled, or if the space is doubled, the work done is doubled; hence when both the force and the space are doubled the work done is four times as great. From such considerations it follows that the amount of work done is measured by the product of the force and the space, or

$$w = fs.$$

When the force and the motion do not have the same direction, the work is not computed by multiplying all the force by all the space, but by multiplying all the force by that component of the motion lying in the direction in which the force acts, or by multiplying all the space by that component of the force acting in the direction of the motion. For example, let AC (Fig. 37) represent a force acting upon a body at C pushing it from C to D. In this case the work done is not equal to $AC \times CD$, but to $EC \times CD$, EC being the rectangular component of the force AC acting in the direction CD; or the work done equals $AC \times CF$, CF being the rectangular component of the motion CD lying in the direction of the force AC.

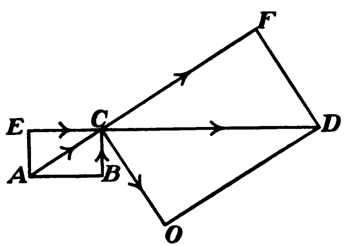

FIG. 37.—Diagram of work done by a body acting with a force AC through a space CD.

87. Units of work. — A unit of work = a unit of force acting through a unit of space. There are two kinds of units of work in use, the **dynamic** and the **gravitational.** A dynamic unit of force acting through a unit of space does a dynamic unit of work. A gravitational unit of

force acting through a unit of space does a gravitational unit of work.

The *erg*, the *joule*, the *kilogrammeter*, and the *foot pound* are the units of work in use.

1. The erg is the work done by a force of one dyne acting through a space of one centimeter.

The number of dynes of force × the number of centimeters of space = the number of ergs of work. The erg is sometimes called the *dyne centimeter* of work. The erg is a unit so exceedingly small that a multiple of it called a *joule* is often used in scientific calculations.

2. A joule = 10,000,000 ergs, or 10^7 ergs of work.

3. The kilogrammeter is the work done by a force of one kilogram acting through a space of one meter.

The number of kilograms of force × the number of meters of space = the number of kilogrammeters of work.

4. The foot pound is the work done by a force of one pound acting through a space of one foot.

The number of pounds of force × the number of feet of space = the number of foot pounds of work.

88. Classification of units of work.

Units of work	dynamic or absolute	erg (dyne centimeter) joule (10^7 ergs)
	gravitational	kilogrammeter foot pound

89. Illustrative problems. — A team drew a load of logs weighing 3 tons on smooth ice 1295 ft. How much work did the team do if the average pull exerted on the load throughout the distance was 684 lb.?

SOLUTION. — 1. A force of 684 lb. acting through a space of 1295 ft. is equivalent to a force of 1295 × 684 lb., or 885,780 lb. acting through a space of one foot, or 885,780 foot pounds of work.

In solving this problem the mass and the weight of the load are

not considered, since the work done depends solely on the two factors of work, force and space.

2. How much work is done in carrying 30 bricks to the top of a building 60 ft. high, each brick weighing 6 lb.?

SOLUTION. — Weight of bricks = 30 × 6 lb. = 180 lb. In this case the force exerted must be equal to the weight of the bricks. Hence the work done, in foot pounds, is 180 × 60 = 10,800.

3. How many ergs in a kilogrammeter at Washington?

SOLUTION. — At Washington $g = 980.1$ cm. and hence at that place 1 gram = 980.1 dynes and 1 Kg. = 980,100 dynes. 1 meter = 100 cm. Hence 1 Kgm. = 100 × 980,100 = 98,010,000 ergs.

90. Power or activity. — In computing amount of work done *time* does not enter into the problem. Thus, if 1000 bushels of grain are carried from a vessel to the top of an elevator, the amount of work done is the same whether it is done in an hour or a day or a month. In fact, time has nothing to do with the problem.

But in estimating the ability of an agent to do work, time is a factor. For example, if one machine can do a certain amount of work in one third the time required by another to do the same work, the first has three times the power for doing work that the second one has.

The *power* or *activity* of a machine is the rate at which it is able to do work. The *horse power* is the unit of power in common use in this country. Doing work at the rate of 33,000 foot pounds per minute, or 550 foot pounds per second, constitutes a horse power (H.P.). One horse power is equivalent to 4562 Kgm. per minute. A 10 H.P. engine is one that can do 330,000 foot pounds of work per minute.

In the C.G.S. system the unit of power is the *watt*, which means ability to do work at the rate of *one joule per second*. 746 watts = 1 horse power. Electrical machinery is commonly rated in *kilowatts*. A kilowatt, which means 1000 watts, $= 1\frac{1}{3}$ horse power nearly ($746 \times 1\frac{1}{3} = 994\frac{2}{3}$).

A watt hour is a unit of work and equals 3600 joules, and a kilowatt hour equals 3,600,000 joules of work.

IX. ENERGY

91. Energy *is capacity for doing work.* We are all familiar with the fact that a body in motion can overcome resistance and exert force through space. Illustrations of this are occurring about us constantly. The wind may turn a windmill or propel a boat, and a falling hammer will drive a nail. We instinctively fear a swiftly moving body, and we know that such a body, especially if its mass is large, can do great damage when resistance is encountered. Such facts teach us that a body in motion can do work and it therefore possesses energy. *A body in motion always possesses energy because of its motion.* Such energy is called *kinetic energy* (literally motion energy).

92. Measurement of kinetic energy. — Since the energy of a body is its capacity for doing work, the amount of work it can do is a measure of its energy; and hence energy and work are measured by the same units, — ergs, joules, kilogrammeters, or foot pounds.

All experience teaches us that the faster a body is moving or the greater its mass, the greater the force it can exert and the greater the space through which it can exert the force. Hence the amount of kinetic energy a body possesses depends on its mass and its velocity. The exact relation between the kinetic energy, mass, and velocity of a body is expressed by the formula:

$$e = \tfrac{1}{2} mv^2.$$

This formula means that

I. *The kinetic energy of a body is proportional to its mass.*

II. *The kinetic energy of a body is proportional to the square of its velocity.*

93. Development of the formula for kinetic energy. — Suppose a force f moves a mass m with uniformly accelerated motion through a space s, giving it an acceleration of a and a velocity of v. The work done or the energy imparted to the mass is given by the formula, $e = fs$; but $f = ma$ and $s = \frac{1}{2}at^2$. Substituting these values for f and s in the formula, we have $e = ma \times \frac{1}{2}at^2$. Again, $a = \frac{v}{t}$; substituting $\frac{v}{t}$ for a in the last equation, we have

$$e = m\frac{v}{t} \times \frac{1}{2}\frac{v}{t}t^2,$$

which is easily reduced to the form $e = \frac{1}{2}mv^2$.

Since this formula is developed by the use of the formula $f = ma$, which is the formula for dynamic units of force, it gives us dynamic units of energy. Whenever this formula is used, the result is ergs of energy, provided the mass is expressed in grams and the velocity in centimeters per second. The formula

$$e = \frac{mv^2}{2g}$$

gives the kinetic energy in gravitational units; in foot pounds, when the mass is expressed in pounds, the velocity in feet per second, and g in feet per second^2; in kilogram-meters, when the kilogram, meter, and second are used to express these quantities.

Problem. — How much kinetic energy has a mass of 4 Kg. moving with a velocity of 7 m. per second?

4 Kg. = 4000 g. and 7 m. = 700 cm. Substituting in the formula we have

$$e = \frac{1}{2} \times 4000 \times 700 \times 700 = 980{,}000{,}000 \text{ ergs of energy.}$$

Or, using the other formula,

$$e = \frac{4 \times 7 \times 7}{2 \times 9.8} = 10 \text{ Kgm. of energy.}$$

Problem. — A falling mass of 10 lb. strikes the ground with a velocity of 8 ft. per second. What is its kinetic energy just before it strikes? *Ans.* 10 ft. lb.

94. Potential energy. — We have learned that a body in motion possesses energy because of its motion; but a body at rest may also possess energy. A mass of iron such as the weight of a clock or the hammer of a pile driver when lifted has energy because it can do work when allowed to fall. The water in a milldam, although at rest, can do work because it is at a higher level than the stream below the mill. Again, a coiled watch spring or a bent bow can exert force through space and hence possesses energy, and gunpowder because of the relations of its constituent parts has capacity for doing work and hence possesses energy even though it be at rest. The energy in all these cases is due to the relative positions of the bodies or to the relative positions of the parts of the body.

The energy of a body due to its position or the relative positions of its parts is called potential energy.

In all cases of potential energy the bodies, or parts of a body, hold their positions in opposition to a stress, hence potential energy is often called *energy of stress.* For example a lifted weight is under gravitational stress and the earth and the weight have been separated from each other in opposition to this stress. A coiled watch spring is under an elastic stress. The parts of the gunpowder are under atomic stress or stress of chemical affinity.

Potential energy is measured or calculated by the product of the stress or force by the strain or displacement, or, $e = fs$.

If, for example, a mass of 8 Kg. is suspended 4 m. above the ground, its potential energy is 32 Kgm. (8×4), because the force acting upon it is 8 Kg. and the displace-

ment is 4 m. and 32 Kgm. of work would have to be done upon it to raise it from the ground to its position.

95. Forms of energy. — Every one deals with energy continually, and it presents itself to us in various forms or aspects; but in whatever form it may appear, it is either energy of motion, that is, kinetic, or energy of position, that is, potential. The chief forms of kinetic energy are:

1. Energy of mechanical motion, *i.e.* motion of any mass of matter of sensible size, called mechanical energy.
2. Energy of molecular motion, called heat.
3. Energy of electricity in motion, called the electric current.
4. Energy of wave motion in the ether, called radiant energy. Light is radiant energy.

The chief forms of potential energy are:

1. Energy of gravitational stress or gravitational separation, as a lifted weight.
2. Energy of molecular stress due to extension, compression, or distortion, as a coiled spring.
3. Energy of atomic stress or chemical separation, energy possessed by gunpowder, coal, etc.
4. Energy of ether stress or electrical separation, as electrification.

96. Transformation of energy. — The changing of one form of energy to another form of energy is called *transformation of energy.* Each form of energy can be changed either directly or indirectly into any other form of energy. Indeed these transformations are going on all about us continually. As a pendulum swings to and fro its energy alternately changes from kinetic to potential and from potential to kinetic; at the extremity of its arc its energy is all potential, at the center all kinetic, and between

these two points part kinetic and part potential. A steam engine transforms the kinetic energy of heat into the kinetic energy of mechanical motion. In our bodies the potential energy possessed by the food we eat is transformed into kinetic energy of heat and also into kinetic energy of mechanical motion.

Energy in the potential form always tends to become kinetic, or the potential energy of a system tends to become as small in amount as possible.

The whole science of physics is very largely a study of the transference and transformation of energy.

97. Conservation of energy. — The amount of energy in the universe is never increased or decreased by any changes through which it passes. This fact is known as the principle of the *conservation of energy.* It is to energy what indestructibility is to matter. The doctrine of the conservation of energy has been said to be the greatest principle discovered in the realm of science during the nineteenth century.

98. Transference of Energy. — Energy may be transferred from one body to another. It follows from the principle of conservation of energy that when one body gains energy it gets it at the expense of some other body. *Doing work always consists in the transfer of energy from the body doing the work to the body upon which the work is done.* Hence a body can gain energy only by having work done upon it. Very often, but not always, when energy is transferred from one body to another, it is at the same time transformed.

99. Energy valuable. — Energy has a commercial value in the markets of the world and men buy and sell it every day. A steamship or railway company, or a householder, buys coal because the coal has energy. As soon as the coal is stripped of its energy, it has no value, and

the material of the coal, the ashes and the gases, is thrown away. The energy of the coal is used to propel the ships and the trains of cars and to warm our houses.

Mill owners often buy water from canal companies with which to run the machinery of their mills. In reality they buy, not the water, but the energy which the water possesses; and when the water has parted with its energy to the machinery of the mill, it goes on its way, of no further use to the mill owner.

It is no exaggeration to say that we eat food for the sake of its energy. The heart expends many foot pounds of energy every day in propelling the blood through the body, and many foot pounds of energy are also required to keep our bodies warm. We walk and run and climb stairs, rise up and sit down, in innumerable ways expend large amounts of energy every day, and all of it comes from the food we eat. Our bodies are machines for transforming and transferring energy.

Problems

1. How much work must be done to cause a displacement of 180 cm. of a 10 Kg. mass against a resistance of 200 g.?

2. What force in dynes will lift a mass of 5 Kg.? How many ergs of work are done in lifting a mass of 5 Kg. 20 cm.?

3. How much work is done in lifting a 10 Kg. mass vertically 180 cm.? Give the answer in kilogrammeters, in ergs, and in joules.

4. Suppose a carpenter pushes a saw downward with a force of 10 lb. and draws it back with a force of 6 lb. If he makes 30 complete strokes 15 in. long in sawing off a board, how much work does he do?

5. A constant force of 100 Kg. was applied to a body and moved it 800 m., but the direction of the force made an angle of 45° with the direction in which the body moved. How much work was done?

Ans. 56,568 Kgm.

6. What is the power of an agent which could do the work of the last problem in 1.5 min.?

7. How many joules in a kilogrammeter where g equals 980 cm. per second^2? How many joules in a kilogrammeter at the equator?

8. An inclined plane is 12 ft. long and 4 ft. high. How much work is done in moving a mass of 300 lb. up the plane, the force being applied parallel to the plane and friction being neglected?

9. An inclined plane is 16 ft. long and 4 ft. high. How much work is done in moving a mass of 300 lb. up the plane, as in problem 8?

10. In problems 8 and 9 calculate the work done when the force acts parallel to the base of the plane, the lengths of the bases being respectively 11.31 ft. and 15.49 ft. *Ans.* 1200 ft. lb.

11. How much work is done in moving a mass of 200 lb. up an inclined plane 10 ft. long and 3 ft. high?

12. What is the kinetic energy of a mass of 20 lb. which has a velocity of 32 ft. per second? 64 ft. per second?

13. What is the kinetic energy of a mass of 5 Kg. which is moving with a speed of 4.9 m. per second? (Give three answers, that is, answers in three different units.)

14. What is the kinetic energy of a ball weighing 10 lb. after it has been falling 5 sec.?

15. It took a ball weighing 100 Kg. 4 sec. to fall from the top to the bottom of a canyon. How much potential energy did it have at starting? (Find the depth of the canyon.) How much kinetic and how much potential energy did it have when it had been falling 2 sec.? How much and what kind of energy did it possess just as it struck the bottom?

16. How much kinetic energy has a mass of 20.58 Kg. which has a velocity of 2 Km. per second?

17. What is the kinetic energy of a cannon ball weighing 100 lb. and having a velocity of 2000 ft. per second?

18. What is the horse power of a steam engine which is able to lift 180 tons of ore per hour from the bottom of a mine shaft 1100 ft. deep?

19. About 198,000 joules of energy are required per hour for a 16 candle power incandescent electric lamp. How many such lamps can be kept burning by a 60 horse power steam engine?

20. What must be the horse power of a steam engine which is to run a 250 kilowatt dynamo?

21. What must be the horse power of a motor to operate an elevator which with its maximum load weighs 4000 lb., if the speed of the elevator is to be 90 ft. per min. and friction adds two thirds as much more to the work to be done?

X. MACHINES. PRINCIPLE OF MOMENTS

100. A Machine is a contrivance for the transference of energy, or both the transference and the transformation of energy at the same time; it is therefore an instrument for doing work. A steam engine is a machine because it transfers energy from the steam to the moving parts of the engine, at the same time transforming the kinetic energy of heat into kinetic energy of mechanical motion. A dynamo is a machine because it transforms the kinetic energy of mechanical motion into the kinetic energy of the electric current. Our bodies may be considered as machines since they transfer and transform energy.

It must be remembered, however, as one of the fundamental facts of physics, that a machine does not and cannot create energy; it can only give out the same amount of energy as has been given to it, being merely the agent for transferring it from one body to another. Many attempts have been made to construct machines which, when once started, will run themselves, giving out more energy than they receive. Such an attempt, called the search for perpetual motion, always results in failure, and must do so because of the principle of the conservation of energy.

101. Law of machines. — Since transferring energy to a body is doing work upon it, work is done upon a machine when energy is given to it, and work is done by the machine upon the body to which it transfers the energy. There are therefore always *two quantities of work* in the operation of a machine, and by the principle of the

conservation of energy these two must be equal. This is the fundamental *law of machines*. It may be stated as follows:

The work done upon a machine = the work done by the machine; or the energy applied to a machine = the energy given out by it. Each of these quantities of work must of course be composed of the two factors of work, the force factor and the space factor. Hence in the operation of a machine there are always two forces and two spaces to be considered.

102. Power and weight. — The force factor of the work done upon the machine is called the *power* or *effort*. It is the *force applied* to the machine. The force factor of the work done by the machine is called the *weight* or *resistance*. It is the *force exerted* by the machine in overcoming resistance and equals the resistance overcome.

Representing the power or effort by p and the space through which it acts by d, and the weight or resistance by w, and the space through which the weight acts by d', the law of machines may be expressed as follows:

$$p \times d = w \times d'.$$

The power × the distance the power acts = the weight × the distance the weight acts.

When the product of two quantities equals the product of two others, two of them may be made the extremes and two the means of a proportion. Hence, the law of machines may be expressed as follows:

$$w : p = d : d'.$$

The weight is to the power as the distance through which the power acts is to the distance through which the weight acts.

Thus the law of machines may be stated in different ways; but the student should bear in mind that these are

equivalent expressions, — not different laws, but different methods of stating one law.

103. The mechanical advantage of a machine. — If we consider the equation $p \times d = w \times d'$, we shall see that the factor p may be made very small provided the other factor d is made correspondingly larger; also that w may be very great, if the other factor d' is very small. In other words, by a machine a small force acting through a large distance can overcome an enormous resistance through a small distance. For example, a machine can be so constructed that when the power works through a distance of 50 ft. the weight will move only 0.001 ft. In that case, if p is 10 lb., w will be 500,000 lb. ($10 \times 50 = w \times 0.001$, $w = 500{,}000$ lb.); but p will move 50,000 times as fast and 50,000 times as far as w.

On the other hand, d' may be made very large, provided w is made correspondingly small, and w will act rapidly over a large distance. In this case the resistance overcome will be small, but speed and distance will be great. We can thus by a machine either gain force at the expense of speed, or speed at the expense of force. A machine can increase the force, but not the energy applied to it; there is no such thing as the conservation of force.

The ratio of the weight or resistance overcome to the power or force applied to a machine, $\frac{w}{p}$, is called the *mechanical advantage* of a machine. In the illustration given above, the mechanical advantage is 50,000, because w is 50,000 times as great as p.

In a sewing machine the power applied by the feet is much greater than the resistance overcome by the needle; the mechanical advantage is very small, being less than 1, but what is lost in power is gained in speed. Again, machines are in use which cut in two great rods of iron

as easily as you would break a pipe stem; here the mechanical advantage is very large, but the distance through which the resistance is overcome is very small compared with that through which the power acts.

104. Efficiency of machines. — Some of the energy given out by a machine is always used in doing work upon the machine itself, overcoming the friction of the moving parts, resistance of the air, etc. The energy so used is usually transformed directly into heat and is wasted. Since it is impossible to construct a machine in which there are no internal losses due to friction, etc., the *useful* work done by a machine is always less than the work done upon the machine; and the law of machines may be stated as follows:

The work done upon a machine = the useful + the wasted work done by the machine.

The *efficiency* of a machine is the ratio of the useful work done by the machine to the total work done upon the machine. If there were no wasted work, that is, if the machine were perfect, this ratio would be unity or 100 %. The efficiency of a machine is usually expressed as a per cent. For example, suppose 480 ft. lb. of work is done upon a machine and that 360 ft. lb. of useful work is done by the machine; its efficiency is $360 \div 480 = .75$, or 75 %. When a machine is said to have an efficiency of 96 %, the useful work done by the machine is 96 % of the total work done upon the machine, 4 % being wasted.

105. The simple machines. — The *lever*, *wheel and axle*, *pulley*, *inclined plane*, *screw*, and *wedge* are known as the *six simple machines* or *mechanical powers*. They simply transfer energy, not transform it. All other machines are mechanically only modifications or combinations of these simple machines.

In studying the laws of a few simple machines, we shall

treat them as if there were no wasted work, regarding them as having an efficiency of 100 %.

106. The lever is a simple machine which consists of a rigid bar capable of turning about a fixed axis, called the *fulcrum*. There are three classes of levers, the class to which a lever belongs being determined by the relative positions of the fulcrum, power, and weight.

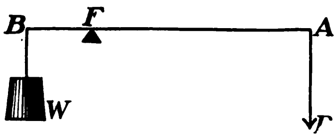

FIG. 38. — Diagram of a lever of the first class.

(1) In the *lever of the first class* (Fig. 38) the fulcrum is between the power and the weight. One half of a pair of scissors, the pedal of a piano, and a hammer when used to pull a nail are examples of this class. The distance *FA* is called the *power arm* and *FB* the *weight arm* of the lever.

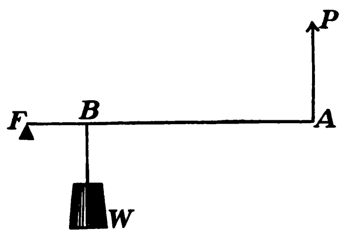

FIG. 39. — Diagram of a lever of the second class.

(2) In the *lever of the second class* (Fig. 39) the weight is between the power and the fulcrum. The parts *AF* and *BF* are respectively the power arm and the weight arm. A bar thrust under a body in order to lift it illustrates this class of levers. The point where the end of the bar rests on the ground constitutes the fulcrum.

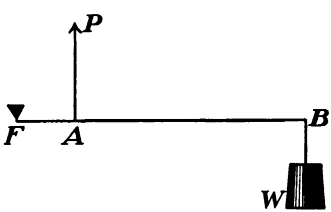

FIG. 40. — Diagram of a lever of the third class.

(3) In the *lever of the third class* (Fig. 40) the power is between the fulcrum and the weight. *FA* is the power arm and *FB* is the weight

arm. With this lever the power is always greater than the weight and is sacrificed for speed or quickness of movement. The forearm is a lever of the third class; the elbow joint is the fulcrum. The muscle which exerts the power is in the upper arm and is joined to the forearm by a ligament very near the elbow. The hand constitutes the weight.

107. The law of the lever. — Let AFB (Fig. 41) be a lever and suppose that the power P turns it to the position $A'FB'$. The work done by the power is $P \times A'c$ and the work done upon the weight is $W \times B'e$. By the general law of machines these two expressions are equal to each other, $P \times A'c = W \times B'e$. From this equation we may form the proportion, $W : P = A'c : B'e$. It can be shown by geometry that $A'c : B'e = AF : BF$. Hence, $W : P = AF : BF$ and $W \times BF = P \times AF$. Therefore, (1) *The weight is to the power as the power arm is to the weight arm.* Or, (2) *The power times the power arm equals the weight times the weight arm.*

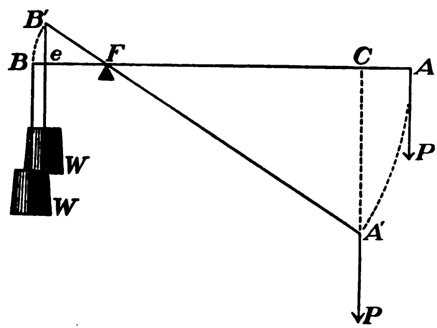

Fig. 41. — Diagram to illustrate the law of the lever.

Since by definition the mechanical advantage of a machine is the ratio of the weight to the power, the *mechanical advantage of the lever is equal to the inverse ratio of its arms.*

108. The moment of a force is its importance in producing rotation about an axis. For example, in closing a

door, we know that a push near the hinges does not close it so readily as one near its outer edge. This shows that the moment or importance of a force in swinging the door depends upon the distance of its point of application from the hinges or axis of rotation.

Experiment. — Let a small hole be bored through a meter stick at its 50 cm. mark, and let the meter stick be balanced in a horizontal position on a nail passing through this hole. A small piece of tin bent so as to clasp the stick will assist in balancing it. Suspend by a loop of thread a 50 g. weight A (Fig. 42) 40 cm. from the nail on one

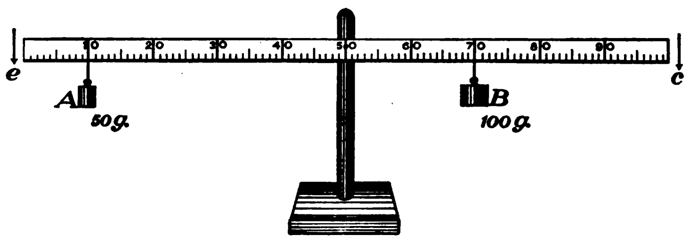

FIG. 42. — Apparatus to illustrate the principle that the moment of a force depends on its distance from the axis of rotation.

side and a 100 g. weight B 20 cm. from the nail on the opposite side. They will exactly balance each other. The weight B evidently has a tendency to rotate the ruler about the nail in a clockwise direction, as indicated by the arrowhead c; the weight A tends to turn it in a counter-clockwise direction, as shown by the arrowhead e. The importance, however, of the 100 g. weight at 20 cm. from the nail in producing rotation is the same as that of the 50 g. weight at 40 cm. from the nail. Hence, the moment of a force depends upon its distance from the axis of rotation as well as upon its magnitude.

109. Measure of the moment of a force. — The moment of a force is measured by the product of the force into the perpendicular distance between its line of direction and the axis of rotation. For example, the moment of the force A is $50 \times 40 = 2000$, and of B, $100 \times 20 = 2000$.

The moment of a force tending to produce rotation in

a clockwise direction is called positive, while that tending to produce rotation in the opposite direction is called negative. The moment of *B* is positive and of *A* negative.

110. Law of moments. — *When a body is at rest or in equilibrium, the algebraic sum of all the moments acting upon it is zero;* or the sum of the positive moments is equal to the sum of the negative moments. For example, if 10 g., 20 g., and 100 g. are placed respectively 15, 25, and 40 cm. from the nail on the right, and 50 g. and 200 g. respectively 13 cm. and 20 cm. from the nail on the left, the ruler will be in equilibrium.

Positive Moments	Negative Moments
$10 \times 15 = 150$	$50 \times 13 = 650$
$20 \times 25 = 500$	$200 \times 20 = 4000$
$100 \times 40 = 4000$	Sum $= 4650$
Sum $= 4650$	

The law of the lever, the power × the power arm = the weight × the weight arm, is simply the law of moments applied to the lever; for the power and the weight always tend to turn the lever in opposite directions about the fulcrum, the moment of one being positive and of the other negative.

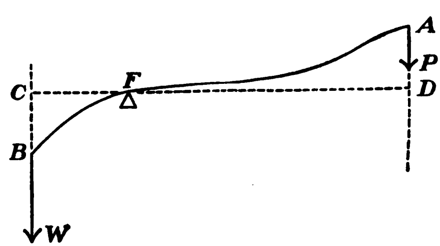

FIG. 43.—Diagram of a bent lever.

If the direction of the force is not at right angles to the bar or if the bar be bent, then the moment of the force is not the product of the force times the distance from the axis to the point of application of the force, and in the case of the lever the apparent length of the arm is not its real length. Thus (Fig. 43) *FD*,

not FA, is to be considered the power arm of the lever AFB, and $P \times FD$ is the moment of P about F as an axis. Likewise CF is the weight arm and $W \times CF$ is the moment of W about the axis F.

111. Problem. — Let AB (Fig. 44) be a bar 24 m. long with a mass of 1200 Kg. suspended 3 m. from A and one of 1800 Kg. 5 m. from the end B. What is the pressure on the supports at each end? This problem may be solved by the principle of parallel forces (§ 55), but the solution is much more easily accomplished by the law of moments. Let p be the pressure at A and p' at B. First, consider A as an axis of rotation. The two weights will have positive moments, and the reaction of the support at B will form a negative moment about A. Since the bar is at rest, the positive and negative moments are equal. Therefore $3 \times 1200 + 19 \times 1800 = p' \times 24$. (The reaction at B is equal to the pressure upon it.) Again, take B as an axis of rotation. The two weights will have negative moments, and the reaction of A a positive moment about B. Hence, $5 \times 1800 + 21 \times 1200 = p \times 24$.

Ans. $p = 1425$ Kg., $p' = 1575$ Kg.

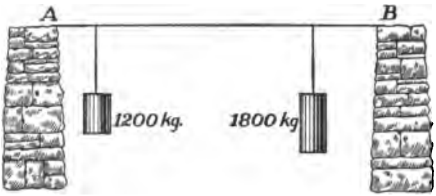

Fig. 44. — Diagram of a problem in moments of forces.

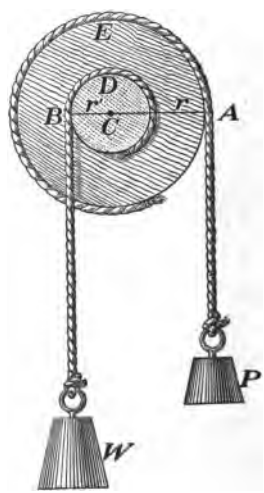

Fig. 45. — Diagram of a wheel and axle.

112. The wheel and axle (Fig. 45) consists of a wheel E and a cylinder D, united so as to turn together on a common axis. Often the wheel is replaced by a crank as in the windlass (Fig. 46) or by bars as in the capstan (Fig. 47) which is used on board ships for raising the anchor or

for bringing the ship to the dock. A similar arrangement is used for moving buildings, the power being supplied by a horse hitched to a bar and traveling in a circle around the machine, the rope which is attached to the building being wound around a vertical cylinder or axle. The cranks and large sprocket wheel of a

FIG. 46. — A windlass.

FIG. 47. — A capstan.

bicycle constitute a wheel and axle, the cranks corresponding to the wheel and the sprocket to the axle.

113. Law of wheel and axle. — It is clear (Fig. 45) that when the wheel and axle revolves once around, the power acts through a space equal to the circumference of the wheel, while at the same time the weight moves through a distance equal to the circumference of the axle. Hence, by the general law of machines, power × circumference of wheel = weight × circumference of axle, or $w : p =$ circumference of wheel : circumference of axle. By geometry, circumferences of circles are to each other as their radii. Hence, denoting the radius of the wheel Ac by r, and the radius of the axle Bc by r', $w : p = r : r'$. *The mechanical advantage of the wheel and axle equals the ratio of the radius of the wheel to the radius of the axle.*

It follows from the last proportion that $p \times r = w \times r'$.

Hence the wheel and axle furnishes another illustration of the law of moments.

Figure 45 shows that the wheel and axle may be regarded as a modified form of the lever of the first class, the axis being the fulcrum, the radius Ac the power arm, and the radius Bc the weight arm.

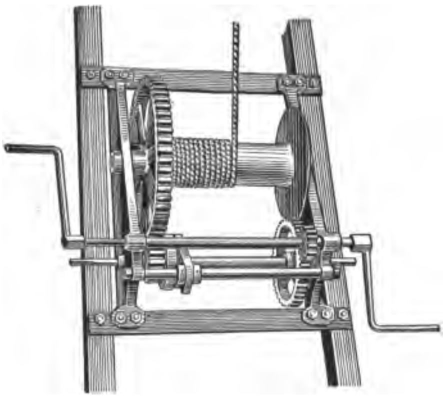

FIG. 48. — A train of wheel work used on a derrick.

114. Trains of wheel work. — Several wheel and axles are often combined (Fig. 48,) the energy being communicated from one to the other by belts or by cogwheels or gears. In such cases the weight or resistance of the first constitutes the power of the second, the weight of the second the power of the third, and so on. The ratio of the weight to the power may be determined by the continued application of the law for the simple wheel and axle, or from the general law of machines the following law may be derived: *The weight is to the power as the continued product of the radii of the*

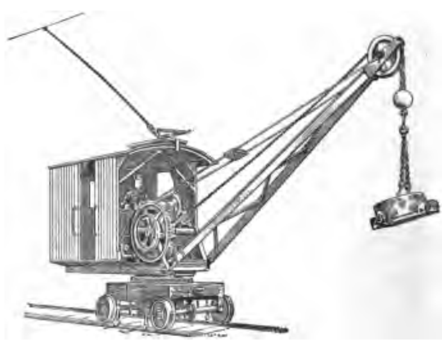

FIG. 49. — A traveling crane.

wheels is to the continued product of the radii of the axles. A windlass is generally one of the essential parts of derricks (Fig. 48) and cranes (Fig. 49) which are used for lifting and moving very heavy weights.

In some combinations the radius of the part to which the power is applied, or the wheel, is often much smaller than that of the axle. This is always the case when great speed is required. The force factor or power is sacrificed for the space factor or speed. The buzz saw and the bicycle are good illustrations. When, however, it is desired to sacrifice speed for power the radii of the axles are made small in comparison with those of the wheels.

115. The pulley (Fig. 50) consists of a wheel called **a** ***sheave***, turning freely on an axis fixed in a frame called a ***block***. The sheave has a grooved rim to hold a cord or rope. A pulley serves the purpose of *changing the direction* of the pull on the rope; and, friction aside, the tension is the same at every point in the rope, no matter how many pulleys it passes over, just as it would be if it were straight. This is sometimes known as *the principle of transmitted tension*. For example (Fig. 52), if a pull of

Fig. 50. — Pulleys.

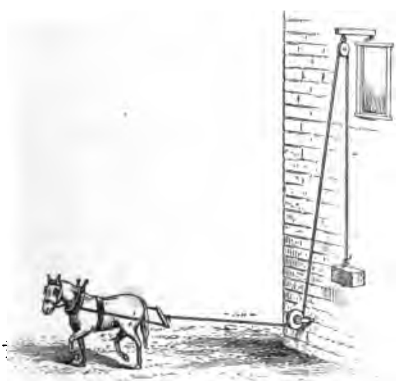

Fig. 51. — Diagram of two fixed pulleys.

10 Kg. is exerted at *P*, there will be a stress of 10 Kg. at *B*, *C*, *D*, and at every other point throughout the length of the rope.

A pulley is either *fixed*, as at *A* (Fig. 52), where the block is fastened to some immovable body, or *movable*, as at *B*, where it rises and falls with the weight.

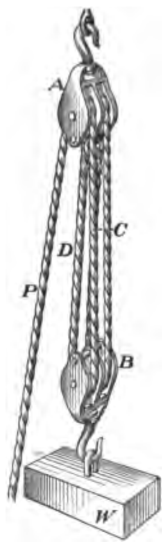

FIG. 52. — Block and tackle.

If only fixed pulleys are used, as in Figure 51, nothing is gained except a change of direction of the force employed. Thus a horse pulling horizontally may be used to raise objects vertically; but the weight and the power, except for friction, will be equal, and the distances through which they are exerted will also be equal. To gain a mechanical advantage, combinations of fixed and movable pulleys are used as shown in Figure 52. Such an arrangement is called *a block and tackle.*

116. The law of the pulley. — In Figure 52 when the weight *W* is raised one foot, each of the four parts of the rope supporting *W* must be shortened one foot, and consequently the part to which the power *P* is applied must be lengthened four feet; that is, *P* will move four times as far as *W* does. In general, if there are *n* parts of rope supporting the movable pulley and the weight, the power must move *n* times as far as the weight. Hence, by the general law of machines, *W* is *n* times as great as *P*, or $W = n \times P$ and $\frac{W}{P} = n$. Therefore the mechanical advantage of the pulley equals the number of parts of the rope supporting the weight.

117. The differential pulley (Fig. 53) consists of two sheaves or pulleys A and a fastened rigidly together, the diameter of one being slightly less than that of the other. L is a movable pulley to which the weight is attached. A continuous chain passes around the pulleys upon the circumference of which are spurs fitting the links of the chain to keep it from slipping. If the power is applied to the chain at c, pulling it down until A makes a whole revolution, the part e is wound up on A a distance equal to $2\pi R$, and the part d is unwound from a a distance of $2\pi r$, R and r being the radii of A and a, respectively. The parts d and e are therefore shortened by an amount equal to $2\pi R - 2\pi r$, which is equally divided between them. The weight is therefore lifted half that amount or $\frac{2\pi R - 2\pi r}{2}$. This is the distance the weight moves when the power moves $2\pi R$. Applying the law of machines, we have

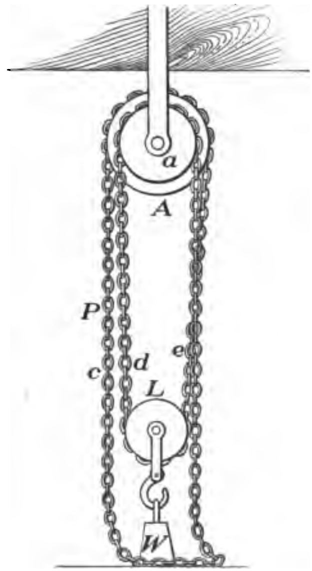

FIG. 53. — Diagram illustrating the differential pulley.

$$P \times 2\pi R = W \times \frac{2\pi R - 2\pi r}{2}, \text{ or } \frac{W}{P} = \frac{2R}{R-r}.$$

118. The inclined plane as a machine. — It has been shown (§ 58) that the weight on an inclined plane is to the force holding it in place as the length of the plane is to its height. By applying the law of machines to the plane, the same result is obtained.

The work done in lifting the weight W (Fig. 54) against gravity from the bottom to the top of the plane is $W \times h$, and the work done by the power P in pulling the weight from A to C is $P \times l$. By the

law of machines these two amounts of work are equal, therefore

$$W \times h = P \times l, \text{ or } W : P = l : h. \quad \text{Hence,}$$

the mechanical advantage of the inclined plane equals the ratio of the length of the plane to its height.

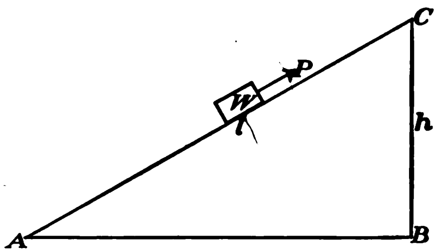

FIG. 54.—Diagram to illustrate the use of an inclined plane as a machine.

119. The screw is a cylinder A (Fig. 55) around whose circumference winds a spiral groove. The ridge between the adjacent parts of the groove constitutes the *thread* of the screw. The screw turns in a block B called a *nut* within which there are a spiral thread and groove exactly corresponding to those of the screw. If the thread or groove winds around the cylinder twenty times in one inch of its length, the screw is said to have "twenty threads to the inch." In that case the *pitch* of the screw, which is the distance from one thread to the next measured along a line parallel to the axis of the cylinder, is one twentieth of an inch. When the screw is turned once around it is moved lengthwise in the nut a distance equal to its pitch. A screw having twenty threads to the inch

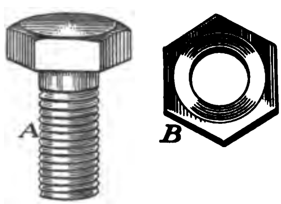

FIG. 55.—Screw and nut.

must be turned through twenty complete revolutions to advance it one inch.

120. The law of the screw. — The power is usually applied to the screw at the circumference of a wheel or at the end of a lever, and hence travels in a circle, and the weight is overcome by the movement of the screw lengthwise in the nut. When the screw is turned once around, the power acts through a space equal to the circumference of the wheel, while at the same time the weight is moved through a distance equal to the pitch of the screw.

Let r denote the radius of the wheel or the length of the lever, and d denote the pitch of the screw.

In one revolution of the screw the work done by the power $= P \times 2\pi r$, and the work done upon the weight $= W \times d$. Hence by the general law of machines,

$$W \times d = P \times 2\pi r, \text{ or } W : P = 2\pi r : d.$$

Therefore,

The weight is to the power as the circumference through which the power acts is to the pitch of the screw. That is, the mechanical advantage of the screw equals $\frac{2\pi r}{d}$.

121. The uses of the screw. — Since the distance through which the resistance is overcome by a screw is so very small in comparison with that through which the power acts, there is a very great gain of force at the expense of space, although the efficiency of the screw is small because of the large amount of friction. The screw is often used to overcome great resistances very slowly and through small distances. It is used for raising great masses, such as buildings (Fig. 56), and compressing cotton and other substances.

FIG. 56. — Jack screw, used for raising buildings.

A very important use of the screw, especially for scientific purposes, is in the measurement of exceedingly small distances with great accuracy. The dividing engine, micrometer screw (Fig. 57), and the spherometer make use of the screw for this purpose. In such instruments a graduated circle is attached to the screw so that

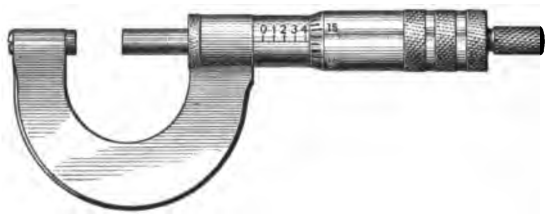

FIG. 57. — Micrometer screw.

it can be turned through a known fraction of one revolution. For example, if the pitch of the screw is 0.5 mm. and the circle is divided into 50 equal parts, turning the screw through one of the divisions of the graduated circle will measure $\frac{1}{50}$ of 0.5 mm., which is 0.01 mm., because one whole revolution of the screw measures 0.5 mm. More than 25,000 parallel lines have been ruled on glass in the space of an inch by a dividing engine.

XI. FRICTION

122. Friction. — Resistance to the sliding of one body over another is called *friction.* It is due to the roughness of the two surfaces and may be lessened, but never entirely destroyed, by making them smoother or by filling the depressions with oil or some other lubricant.

Friction is greater just as the body begins to move than it is after the motion is once established, hence two kinds of friction have been recognized: (1) *friction of rest or static friction*, and (2) *friction of motion or kinetic friction.*

Resistance to motion is very greatly diminished by having one of the surfaces roll upon the other instead of slide. The ball bearings of a bicycle illustrate this. There is still, however, resistance to the motion which is called *rolling friction* to distinguish it from sliding friction, but strictly speaking it is not properly called friction.

123. Value of friction. — Friction always opposes motion, and when motion occurs, the energy used in overcoming it is transformed into heat and is wasted by being diffused among surrounding objects. Friction is therefore the great obstacle to perfect efficiency in machines.

And yet we could not dispense with friction. The difficulty of walking on very smooth ice shows us that we could neither walk nor run if there were no such thing as friction; we could hold nothing in our hands; nails, bolts, and screws would hold nothing together; machines could not be run by belts; locomotives, electric cars, automobiles, and bicycles would be useless, because they depend on the friction between their wheels and the roadway for their propulsion.

124. Laws of friction. — Coulomb (1821) established by experiment the following laws of friction:

1. *Friction is directly proportional to the pressure between the two surfaces.* If, for instance, a brick is drawn along on a surface, the friction is doubled by placing another brick upon the first one because this doubles the pressure.

2. *Friction is independent of the extent of the two surfaces in contact.* According to this law the friction between a brick and the surface on which it slides is the same whether the brick rest upon its broad side, its narrow side, or upon its end, because the pressure, the weight of the brick, is the same in the three cases; only the extent of surface is changed.

3. *Kinetic friction is independent of the speed of the moving parts.*

125. Coefficient of friction. — The ratio between the friction and the pressure is called the *coefficient of friction*, *that is*,

$$\textit{coefficient of friction} = \frac{\textit{friction}}{\textit{pressure}}.$$

Since the friction is proportional to the pressure or varies as the pressure, it follows that the coefficient of friction for any two surfaces is a constant quantity; because if the pressure is multiplied by any quantity, the friction is also multiplied by the same amount, and the quotient of one by the other is not changed. It depends on the nature and roughness of the surfaces in contact.

The laws of friction may be verified by means of the inclined plane; but very accurate and concordant results are not to be expected. The third law is only approximately true, and in fact large variations from all the laws are generally met with in practice.

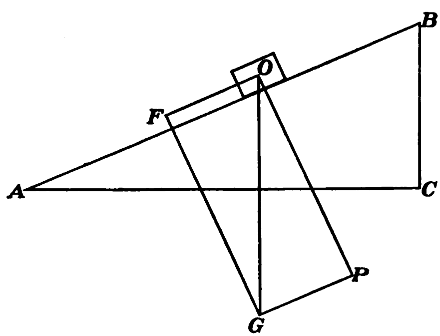

Fig. 58. — Diagram of the friction of a block sliding on an inclined plane.

Let O (Fig. 58) be a block of pine resting on a pine board AB. The end B is raised until the block slides down the plane with uniform motion after being started. The weight of the block represented by OG may be resolved into two components, OP, representing the pressure on the board, and OF, which equals the friction because it just overcomes it. Hence,

$$\frac{OF}{OP} = \text{the coefficient of friction.} \quad \frac{OF}{OP} = \frac{BC}{AC},$$

therefore *the coefficient of friction equals the height of the plane divided by its base when the body slides with uniform motion down the plane.*

The angle BAC is called the *critical angle* for sliding friction. To verify the laws for statical friction, the angle is increased until the block just begins to move. The angle is then called the *limiting angle* of friction or the *angle of repose.*

The first law is tested by placing different weights on the block O. The angle does not change with different pressures and hence the ratio $BC \div AC$ is constant and the law is verified.

The second law is illustrated by turning the block on edge so as to present less surface to the plane. The angle still remains the same as before.

Problems

1. If the power or effort applied to a machine is 250 lb. and it acts through a distance of 50 ft. while the weight moves through a distance of half an inch, how great is the resistance overcome?

2. What is the gain in a machine in which the force applied is 100 times as great as the resistance overcome?

3. A meter stick whose center of gravity is at the 50 cm. mark weighs 250 g. It is balanced on a knife edge at its 75 cm. mark by a weight suspended from its 90 cm. mark. How great is the weight? What is its moment? (See § 65.)

4. A meter stick is balanced on a nail through its center of gravity at its 50 cm. mark. A 50 g. mass is suspended at its 70 cm. mark, a 100 g. mass at its 85 cm. mark, and a 200 g. mass at the 92 cm. mark. If a 500 g. mass is placed at the 20 cm. mark, where must a mass of 70 g. be placed to keep the bar in equilibrium?

5. Show by a diagram how to rig a block and tackle so that its mechanical advantage shall be 5.

6. How many fixed and how many movable pulleys in a block and tackle in which the mechanical advantage is 3? 4? 5? 6? 7?

7. The weight arm of a lever of the first class is attached to the power arm of another of the same kind so that the weight of the first lever becomes the power of the second. The power arm of the first is 3 ft. and its weight arm 6 in. long, and the power arm of the second is 4 ft. and its weight arm 3 in. What resistance can be overcome by the second lever by applying a power of 1 lb. to the first?

8. The handles of a wheelbarrow measured from the axis of the wheel are 5 ft. long. A load of 280 lb. is placed 16 in. from the axis of the wheel. How much must a man lift to wheel the load? To what class of levers does the wheelbarrow belong?

9. A weight of 10 lb. is 6 in. from the fulcrum of a lever of the second class. If the power is 2 lb., how long is the lever?

10. The crank of a derrick (Fig. 48) is 20 in. long, and the wheel on the axle turned by the crank has 16 teeth. The larger wheel on the second axle has 48 teeth and the smaller one, 10 teeth. The large wheel on the third shaft has 120 teeth and the axle about which the rope winds is 6 in. in diameter. If a force of 100 lb. is applied to the crank, what force is exerted by the rope?

11. Four men are raising an anchor weighing 1 ton by a capstan the barrel of which is 10 in. in diameter. The circle described by the handspikes is 8 ft. in diameter. How much force must each man exert?

12. A power of 60 Kg. applied to a wheel 80 cm. in diameter exactly balances a weight on the axle of 800 Kg. What is the diameter of the axle?

13. The pilot wheel of a boat is 4 ft. in diameter and the axle is 8 in. The rudder on being moved offers a resistance of 150 lb. What power must be applied to the wheel to turn the rudder?

14. In a differential pulley the diameter of the larger sheave is 12 in. and that of the smaller one is $11\frac{3}{4}$ in. How much force must be applied to the chain to lift a mass of 9600 lb.?

15. How long must an inclined plane be which is 4 m. high to enable a car weighing 400 Kg. to be pushed up it by a force of 20 Kg.?

16. The screw of a letter press has 6 threads to the inch and the diameter of the wheel is 12 in. If a force of 30 lb. is applied to the wheel, what pressure is exerted by the plate?

17. A jackscrew has 4 threads to the inch and the lever used to operate is 3 ft. long. If the efficiency of the screw is 50 %, how much force must be applied to the lever to raise a mass of 3 tons?

18. In the last problem how far must the power travel to raise the weight 6 in.?

19. If it requires a force of 9 lb. to draw a block of wood weighing 36 lb. along a horizontal board with uniform velocity, what is the coefficient of friction?

20. If the coefficient of friction of wood sliding on wood is 0.30, how high must one end of a board be placed to allow a block of wood to slide down it with uniform velocity, the base of the plane being 10 ft.?

XII. STATES OF MATTER

126. The molecular structure of matter. — It is universally accepted that any piece of matter is not one continuous whole, but is a collection of a vast number of exceedingly small particles called *molecules*. It is also believed that these particles are in ceaseless motion and not in permanent contact with one another. It may seem incredible to a beginner in science that a piece of solid iron, for instance, is composed of separate moving particles; but a great number of facts, many of which we shall study, support this theory; many of them, if not all, are fully and satisfactorily explained by this theory and are not explainable if this theory is discarded.

It is further believed that molecules are composed of still smaller particles called *atoms*, and it has been assumed for many years that these atoms are indivisible and indestructible — being the ultimate divisions of matter. Recent investigations and discoveries, however, have lead to the abandonment of the idea that atoms are indivisible and indestructible, and we now have the electronic hypothesis of the composition of matter.

According to this hypothesis all matter is composed of

molecules, molecules of atoms, of which some seventy or eighty different kinds are known to chemistry, and atoms of *electrons*. All electrons are supposed to be alike, but there are different kinds of atoms because the numbers of electrons composing them differ. If, as seems to be the case, each electron consists of an electric charge, we may come to believe that all matter is composed of electricity.

127. Size of molecules.—Molecules are almost inconceivably small, many times too small to be seen by the most powerful microscope; yet their size has been investigated, and the limits to their probable size can be stated with a considerable degree of certainty. Lord Kelvin's famous comparison in regard to the size of molecules is this: If a drop of water were magnified to the size of the earth, the molecules in it would be smaller than cricket balls but larger than fine shot.

128. Porosity.—If the molecules of a body are not in contact with one another, it follows that there must be spaces between them. These spaces are called *pores* and give to matter the property of *porosity*. Pores in this physical sense are not visible channels such as extend through a sponge or a piece of charcoal, but spaces of insensible size existing between the molecules of all substances.

The porosity of metals has been shown by attempts to compress water by squeezing a closed leaden shell full of water. The water came through the pores of the lead, appearing on its surface as perspiration appears on the skin. The same result followed when a silver shell was used and also when the silver was heavily plated with gold.

The rapid passage of gases through sheet rubber, hot iron, and steel also illustrates the porosity of matter.

129. States of matter. — Matter exists in three physical states, or forms, — the *solid*, the *liquid*, and the *gaseous*. Wood is an example of the solid, water of the liquid, and air of the gaseous state of matter.

A solid is a body that has a shape and a volume of its own, and it offers resistance to a change of its shape and volume. The characteristic property of a solid is *rigidity*, or resistance to change of shape.

A liquid is a body that has a volume, but not a shape of its own; it takes the shape of the vessel containing it. A liquid offers resistance to a change of volume, but no permanent resistance to a change of shape.

A gas has neither shape nor volume of its own, but it takes the shape and volume of the vessel containing it. Gases expand indefinitely as the pressure upon them is diminished, always filling the vessels containing them. Thus we may have a vessel partly full of a liquid with a definite surface separating it from the space above it, but this is impossible with a gas.

The term *fluid* includes both liquids and gases. The characteristic property of a fluid is *mobility*, a term having reference to the ease with which a fluid changes its shape and its parts slide over one another. No fluid offers a permanent resistance to a change of shape, and the slightest force will cause a movement among its particles. It is upon this property of mobility that many of the phenomena which we are to study depend.

The term *vapor* is used to designate the gaseous portion of a substance that is part liquid and part gaseous at the same time. Thus, the space about us at all times contains some water in an invisible form called water vapor, and the space above any liquid always contains some vapor of that liquid. A solid or liquid when passing into the gaseous state is said to *vaporize*. All true gases or vapors except a few, which like iodine have color, are invisible. A fog or mist or a so-called cloud of steam is not a gas or vapor. True steam is invisible.

130. Liquids incompressible. — Although liquids and gases are alike in some respects, they differ from each other very greatly in one important particular, that is, in compressibility. Gases are easily compressible, but liquids are not. It is an easy matter to force a large quantity of air into a small space, as is done in inflating a pneumatic tire and in filling tanks with compressed air for operating the brakes of electric cars. On the other hand, it requires enormous pressures to change the volume of a liquid even a little; the amount of the compression is so small that liquids are commonly said to be incompressible. Of course, if a liquid is not compressed by pressure, it does not expand as a gas does when the pressure on it is removed.

131. The same substance in different states. — It is a familiar fact that water can exist in any one of the three states, the solid, the liquid, or the gaseous, according to the conditions of temperature and pressure. What is true of water, however, is true of many other substances; they may exist in any one of the three states, if the proper conditions of temperature and pressure are attained. Liquid and even solid air can be produced. It is not possible, however, to liquefy or vaporize some substances, simply because a proper temperature for the purpose cannot be reached without causing a chemical change in them. Wood, for example, cannot exist in the liquid form, because the heat necessary to liquefy it would entirely change its character.

132. Molecular forces. — Forces acting between the molecules of a body at distances too small to be perceived are known as molecular forces. When sufficiently near together, molecules attract one another with very great force. This molecular attraction is called *cohesion* when it exists between molecules of the same kind, and *adhesion*

when it acts between unlike molecules. When a piece of iron or glass is broken, it is the force of cohesion that is overcome. The clinging of chalk to the blackboard, mud to the shoes, and flour to the hands are illustrations of adhesion.

The distance through which molecular attraction acts is so extremely small that generally when a hard body is broken, the parts cannot again be brought close enough to restore cohesion; yet many powdered substances, such as graphite of which the leads of lead pencils are made, may be welded into solid masses by great pressure. Cohesion may be reëstablished between the parts of soft bodies like wax, putty, and gold with comparative ease. In filling a tooth, thin leaves of gold are packed so firmly that they are made to cohere in one solid mass.

XIII. MOLECULAR FORCES AND MOTIONS IN GASES

133. Molecular motions in gases. — In gases the molecules are believed to be so far apart and move with so great rapidity that each one is independent of the attraction of the neighboring molecules, so that it moves in a straight line until it collides with another molecule or strikes the wall of the containing vessel; it then bounds off in another direction. Under ordinary conditions the *free path*, as the course from one encounter to another is called, is very short; although in comparison with the size of the molecule itself it is very long. The encounters or collisions of a single molecule with other molecules in a second are numbered by millions. If a gas is compressed and made more dense, the molecules are of course crowded more closely together, and their average free path becomes shorter and the collisions more frequent.

That the spaces between the molecules of a gas are large

in comparison with the diameter of the molecules is shown by the great change of volume in a gas when it is condensed into a liquid; steam, for instance, when changed to water, occupies less than $\frac{1}{1600}$ of its original volume.

The velocities of gas molecules are very great indeed. It has been calculated that the molecules of hydrogen gas under standard conditions have an average velocity of about a mile per second. Oxygen molecules under the same conditions have a velocity only one fourth as great. Since an oxygen molecule weighs sixteen times as much as a hydrogen molecule, but moves one fourth as fast, its kinetic energy is the same as that of the hydrogen molecule. Prove it by using the formula $e = \frac{1}{2} mv^2$.

134. Elastic tension of gases. — Pressure must be exerted at all times on a gas to keep it from expanding. The force with which the gas tends to expand is called its *elastic tension*, or, in the case of a vapor, *vapor tension*.

The elastic tension of a gas at rest must always be equal to the pressure upon it; for, if the tension were less than the pressure, the gas would be compressed; or, if the tension were greater, the gas would expand until pressure and tension became equal.

Fig. 59. — Experiment illustrating the elastic tension of air.

Experiment 1. — To illustrate the elastic tension of air, place an air-tight rubber bag or a toy balloon containing only a small quantity of air under the receiver of an air pump and exhaust the air from the receiver. As the air about the bag is taken away and the pressure on the outside of it diminishes, the air within the bag expands, inflating it to its full capacity (Fig. 59). Upon admitting the air again to the receiver, the bag shrinks to its original size. Thin glass cubes are sometimes burst by the elastic

tension of the air within them when the pressure on the outside of them is reduced by taking the air surrounding them away.

Experiment 2. — Fit a bottle about half full of water with a stopper through which one arm of a U-shaped glass tube extends (*A*, Fig. 60). Place the other arm of the tube in a bottle (*B*) about one fourth full of water. The arms of the tube should extend nearly to the bottom of both bottles. Place the apparatus under the receiver of an air pump and exhaust the air from it. As the pressure in the receiver lessens, the elastic tension of the air in *A* causes it to expand and forces the water from the bottle through the tube into *B*. When the air is admitted again into the receiver, the water is forced back into *A*. Thus the elastic tension of the air in *A* and the outside pressure balance each other and hold each other in equilibrium.

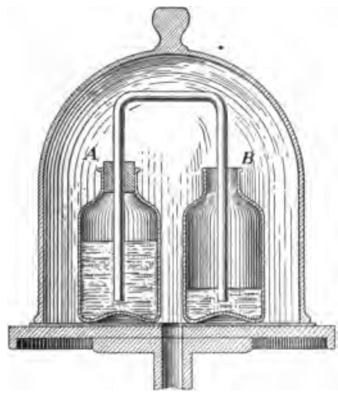

FIG. 60. — Experiment illustrating the elastic tension of air.

The elastic tension or outward pressure of a gas is easily explained by the motion of its molecules. Since even a cubic centimeter of a gas contains many billions of molecules, and since these molecules are moving with very great velocities, it is clear that the blows or impacts of the molecules against the walls of the containing vessel must be exceedingly numerous — so numerous and frequent in fact as to be practically continuous. This bombardment against the sides of the vessel by the molecules causes the pressure, and if the side of the vessel gives way, the molecules because of their motion follow it up and the gas expands.

Consider for a moment how rapidly the gases in a cannon must expand in order to propel the shot from the gun with great velocity. Do you imagine a gas could

expand so quickly and with such great force if its molecules were moving sluggishly or not moving at all? Or can you conceive how a gas can expand in such a way, unless it is composed of small particles with spaces between them?

135. Air pump. — The air pump, as its name implies, is an instrument for removing air from a closed vessel. Its most essential part is a hollow cylinder or barrel *CC* (Fig. 61), the interior of which is very smooth, so that a piston, *P*, a sort of close-fitting plug, can be easily moved to and fro within it. This piston is usually covered or packed with leather and fits the cylinder so closely that no air can pass between it and the sides of the cylinder. *T* is a tube leading to the vessel from which the air is to be exhausted, and *o* is a small hole through the piston. The opening *o* and the entrance to the tube *T* are closed by valves *v* and *s*.

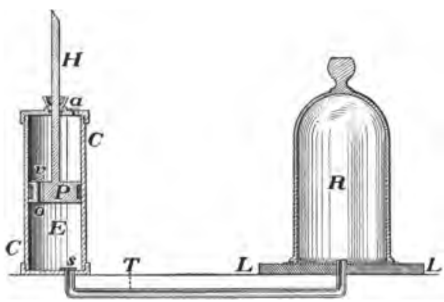

Fig. 61. — Diagram of an air pump.

An air pump for school purposes is usually fitted with a plate, *LL*, to the center of which the tube *T* extends. This plate is a flat metallic disk perhaps a foot in diameter, so smooth and true that when a glass bell jar *R* is placed upon it, mouth downward, an air-tight vessel is formed from which the air can be taken by the pump. This bell jar is called a *receiver*.

136. Action of the air pump. — When the piston is raised by the rod *H*, the pressure in *E* decreases and the downward pressure of the atmosphere closes the valve *v*.

The air in the receiver, because of its elastic tension, expands and lifting the valve *s* fills the space *E*.

When the piston begins to descend, it compresses the air in *E* slightly and the valve *s* closes and *v* opens. This allows the air in *E* to pass to the upper part of the cylinder and at the next upward stroke of the piston out into the atmosphere. When the cover of the cylinder is air-tight, another valve is placed at *a* which opens when the piston rises but closes when it descends. Observe that all valves open outward, thus allowing air to flow out but not in.

With such an air pump as this a very high degree of exhaustion cannot be attained because the elastic tension of the air finally becomes too weak to lift the valves. To remedy this the more expensive pumps are so constructed that the valves are opened and closed automatically by the movement of the piston and not by the air. But even then a perfect vacuum cannot be made, though the valves and piston fit so perfectly that there is no leakage; for if the receiver has, for example, a volume of three liters and the barrel of the pump a volume of one liter, the first double stroke of the piston will remove one fourth of the air and leave three fourths of it. By the second double stroke one fourth of the remainder is taken and three fourths of the remainder is left, that is, three fourths of three fourths of the air remains after two double strokes of the piston, or $(\frac{3}{4})^2$. After ten double strokes the amount remaining would be $(\frac{3}{4})^{10}$. Thus it is evident that, no matter how many strokes are made, a portion of the air still remains in the receiver.

FIG. 62. — Vertical section of a condensing pump.

137. The condensing pump. — If the valves of an air pump all opened in the opposite direction, the air would be forced into the receiver instead of being withdrawn from it. Such a pump is a condensing pump. A bicycle pump (Fig. 62) is an illustration of a condenser in which the

piston itself acts as a valve. It is packed with a cup-shaped piece of leather, *aa*, which allows the air to pass readily on the up stroke; but on the down stroke the air under compression enlarges the rim of the cup so that it fits the cylinder closely and the air cannot pass by it. Compressed air, or air which has been forced into large receivers by condensing pumps, is very extensively used. The brakes of electric cars and railroad trains and a large variety of pneumatic tools, such as drills and hammers, are operated by compressed air.

138. Diffusion. — When a room or a vessel is filled with several gases of different densities, the gases do not arrange themselves permanently in layers according to their densities, the densest at the bottom and the lightest at the top; but they intermingle so that in time each gas is distributed throughout the whole space as uniformly as it would be if it were the only gas present.

This intermingling of different gases is called *diffusion*. The diffusion of gases is readily explained by the relatively large spaces between the gas molecules and their great rapidity and freedom of motion.

We should expect a dense gas to diffuse more slowly than a rare one because the molecules of a dense gas are either closer together or in less rapid motion, and experiment shows this to be true, the rates of diffusion of gases being inversely proportional to the square roots of their densities.

The composition of air affords a striking illustration of the diffusion of gases, as the proportions of its different constituents are practically the same everywhere.

Experiment. — Fill two small bell jars, one with hydrogen, the other with oxygen, and cover the mouth of each with a glass plate. Then place them together mouth to mouth with the hydrogen jar above the other and withdraw the glass plates. Since oxygen is 16 times as

dense as hydrogen, we might expect it to remain in the lower jar and the hydrogen in the upper one; but after standing 15 or 20 minutes a test will show that the two gases are thoroughly mixed. A mixture of these two gases is explosive. Insert the plates between the jars and take them apart. Apply a lighted taper to the gas in each. A slight explosion in each case will show that the gases have diffused into each other.

The greater rate of diffusion which a light gas possesses over that of one more dense is easily shown when the two gases are separated by a porous partition.

Experiment. — Close a battery cup made of unglazed earthenware with a firm, thick cork which has been rendered impervious to air by being soaked in hot paraffin, and connect this cup with a bottle by a glass tube as shown in Figure 63. The bottle should be nearly full of water and be provided with a jet tube extending nearly to the bottom. Care must be taken that the stoppers and tubes all fit air-tight.

Place over the porous cup a battery jar or a bell jar full of hydrogen or illuminating gas. A jet of water will soon issue from the jet tube which is due to the increased pressure in the porous cup. This pressure is transmitted through the tube connecting the cup to the bottle, thus forcing the water out of the jet tube, and it is due to the fact that the hydrogen diffuses into the cup through its pores faster than the denser air diffuses outward from the cup into the bell jar.

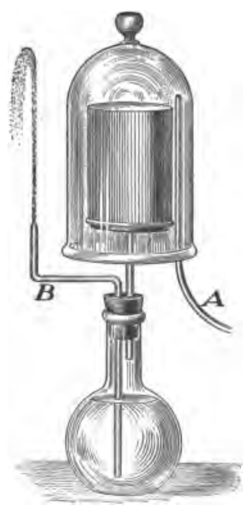

Fig. 63. — Apparatus illustrating the diffusion of hydrogen gas through a porous cup.

If now the bell jar is removed, the reverse will happen. The hydrogen will diffuse out of the porous cup faster than the air about it can pass in, and as a consequence a diminished pressure in the cup will be the result. This will be made evident by the water rising in the tube from the bottle to the porous cup, if that tube is long enough to extend into the water.

XIV. MOLECULAR FORCES AND MOTIONS IN SOLIDS

139. Molecular forces and motions in solids. — In solids the force of cohesion has its greatest intensity, and the molecules, although in constant motion, do not move from point to point within a body with great freedom as they do in gases and liquids. Yet it has been shown that when a body of lead is placed upon a mass of gold, some of the latter finds its way slowly up through and into the upper part of the former. Thus we know that there is diffusion to same extent among solids at ordinary temperatures. At higher temperatures, about 500° C., it has been proved that all metals diffuse into each other to a considerable degree. Such facts as these seem to make it certain that the molecules of solids as well as those of gases and liquids are in constant motion.

Different kinds of solids show great diversity in their characteristics and properties. This fact is doubtless due to differences in their cohesive forces acting within them, as well as to variations in their molecular motions.

140. Strain. — When a solid is bent or twisted or changed in shape or size in any way, it is said to be *strained*, and a change of shape or a change of size of a body is called a *strain*. We thus see that there are two kinds of strain, one consisting of a change of shape, and the other of a change of size.

141. Elasticity is the property which matter has of exerting force to recover from a strain. Just as soon as a force begins to strain an elastic body, that is, to change its size or shape, an elastic force begins to resist the straining force and tends to restore the body to its original size or shape. Elasticity has been called *power of recovery*. A body is not elastic because it can be

stretched, bent, twisted, or compressed; but because it exerts force to resist and recover from such strains.

The restraining force and the restoring or elastic force together constitute a *stress*. The student should be careful not to call a strain a force; the stress is a force, and the strain results from the action of the stress.

As there are two kinds of strain, so there are two corresponding kinds of elasticity, *i.e.* elasticity of *form* or *shape*, and elasticity of *size* or *volume*. Solids possess both kinds of elasticity, but fluids have only that of volume.

Illustrations of this property of matter are very common. The bent bow and a stretched rubber band strive to recover their shape, and compressed air in an air gun strives to resume its volume. Steel is made into springs of many forms which well illustrate this property. Glass and ivory are very elastic.

Experiment. — Let a smooth marble block having a plain surface be coated with a thin layer of printer's ink. If a glass or ivory ball is now touched to the block, only a small black spot will be made upon it, since the plane of the block is tangent to the ball; but if the ball is dropped upon the block from the height of a meter or so, a much larger black spot is made upon it. This shows that the ball is slightly flattened upon striking the marble. The rebound of the ball is caused by the elastic force exerted by the ball in recovering its shape.

142. Limit of elasticity. — If the restoring force of a strained body continues constant and causes the body to resume its former size or shape immediately, no matter how long the strain has lasted, the body is *perfectly elastic*. All fluids are perfectly elastic. Glass and steel are practically so. A piece of glass that had been bent slightly for twenty-five years was observed to recover its shape perfectly as soon as the distorting force was removed.

When a body is strained beyond a certain limit, it suffers a permanent change of shape, or a *set*. The point at

which this is about to take place is termed the *limit of elasticity*.

143. Hooke's law. — *Whenever a body is strained within the limits of its elasticity, the force with which it reacts is proportional to the amount of the strain.* This is known as Hooke's law.

The archer in bending his bow illustrates this law. If he bends the bow a little, only a small force is required; but if he bends it twice as much, then he must exert twice as much force as before, and the bow will exert twice as much force on the arrow. A spring balance and the butcher's scale also act on the same principle. A three-pound weight distorts the spring just three times as much as a one-pound weight, and the pointer attached to the spring moves three times as far.

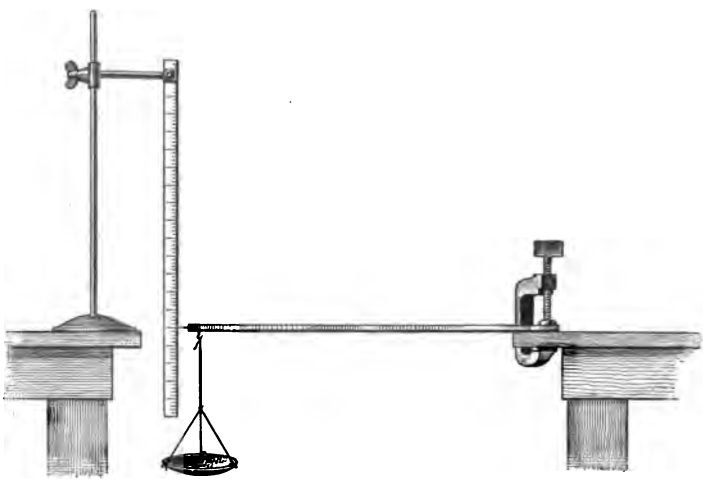

FIG. 64. — Apparatus to illustrate Hooke's law.

Experiment. — Fasten a meter stick in a horizontal position (Fig. 64) by clamping it at one end to a firm support, and suspend a scale pan from the other end of the stick. Then insert a needle in

the free end of the meter stick for an index, and support another meter stick just back of the needle in a vertical position. Place in succession 100 g., 200 g., etc., in the scale pan, and measure by means of the needle and the upright meter stick the amount the horizontal one is bent by each weight. The amount of the bending will be found proportional to the weight in the pan. This experiment illustrates the truth of Hooke's law.

144. Plasticity. — A solid which exerts no elastic force to recover from a strain is said to be perfectly plastic, or inelastic; putty, moist clay, and dough are practically so. Such bodies offer resistance to change of shape because of their viscosity, not because of lack of plasticity.

A plastic body that may be beaten into thin leaves is called *malleable.* Gold, the most malleable substance, can be beaten into leaves so thin that 300,000 of them are required to make a pile an inch high.

A plastic body that may be drawn out into fine wire is termed *ductile;* gold, silver, and platinum are the most ductile substances.

A body that breaks when the strain exceeds a very small limit is called *brittle.*

XV. MOLECULAR FORCES AND MOTIONS IN LIQUIDS

145. Molecular motions in liquids. — The spaces between the molecules in a liquid are so much smaller than in gases that the molecules are never entirely free from one another's attraction, and the cohesive force in liquids is considerable. The molecules of liquids have, however, great freedom of motion, so that they wander from point to point throughout the whole mass of the liquid. This freedom which liquids as well as gases possess renders the explanation of many phenomena relating to them comparatively simple and easy to understand.

146. Viscosity. — Although there is great freedom of motion among the molecules of fluids, there is some resistance to the gliding of one part of a fluid over another part of it. This resistance offered to the motion of the parts of a body when gliding over one another is called ***viscosity.*** It it a sort of internal friction. All fluids are more or less viscous. Ether and alcohol are liquids of small viscosity, being termed *mobile* or *limpid.* Honey, molasses, and balsam are liquids of large viscosity. The viscosity of water is shown by the fact that it soon comes to rest after being given a whirling motion. Gases are also viscous. This is evident in air, from the fact that a stream of air always drags some of the surrounding air along with it.

A perfect fluid, if there were such a thing, would have no viscosity and no rigidity; it would be perfectly mobile.

147. Diffusion of liquids. — Liquids as well as gases when placed in contact intermingle to a greater or less extent, but the diffusion is not unlimited as with gases. Some liquids, such as oil and water, diffuse into each other scarcely at all or to a limited extent; while other liquids, such as water and alcohol, mingle without limit.

FIG. 65. — Experiment illustrating the diffusion of one liquid into another.

This diffusion occurs even when the liquids are left wholly undisturbed; it must be, therefore, that the molecules of the liquids are in motion.

Experiment. — Place some water colored with blue litmus in a small, deep jar (Fig. 65) and then pour a small quantity of sulphuric acid very carefully into the bottom of the jar by means of a thistle tube. The acid, which is nearly twice as dense as the water, will remain for a time at the

bottom of the jar but will gradually diffuse upward, making itself evident by changing the color of the liquid from blue to pink.

148. Solution. — When a lump of sugar is placed at the bottom of a deep dish of water, it gradually disappears from sight and in time the whole mass of water is sweetened by it. It must be that the sugar is composed of small particles, or molecules, too small to be seen, and that they are in motion. When a solid or a gas diffuses in this way through a liquid, it is said to be *dissolved* in it. The molecules of the solid are distributed uniformly throughout the liquid, and the solid as such disappears. The liquid in which the solution takes place is called the solvent. The mobility of the molecules of a dissolved solid approaches the mobility of a gas, and recent investigation has shown that some of the laws relating to solids in solution are similar to some of the most important laws of gases.

149. Osmosis and osmotic pressure. — Two different liquids or solutions when separated by a porous partition or membrane often pass through the membrane and intermingle at different rates, that is, one liquid passes through the partition in one direction faster than the other liquid passes through in the opposite direction. This diffusion of two liquids at different rates through a porous membrane is called *osmosis*.

Many animal membranes, such as the walls of an intestine and also the walls of many plant cells, permit a solvent, such as water, to pass through while not allowing the dissolved substance to pass in the opposite direction. Such membranes are termed *semipermeable*.

The water or the solvent passes through the membrane into the presence of the dissolved solid, and there is, on that side of the membrane, an increased pressure called *osmotic pressure*. This pressure is due to the presence of

the molecules of the dissolved solid. It resembles in some respects gas pressure, and some of the most important laws of gas pressure apply also to osmotic pressure. Osmotic pressures are often very great, and osmosis and osmotic pressure play a very important part in the circulation of liquids in plants and animals.

FIG. 66. — Vertical section of an apparatus to illustrate osmotic pressure.

Experiment. — Bore a hole about 2.5 cm. in diameter and 8 or 10 cm. deep in a carrot, using a carpenter's bit for the purpose. Then fill the hole almost full with dry sugar and add water to within a centimeter of the top. The hole should now be closed with a rubber stopper in which are a glass plug and a long piece of barometer tubing. Be sure that the stopper is pushed firmly into the carrot. Support the carrot (Fig. 66) in a tumbler or wide-mouthed bottle full of water so that the water shall cover the greater part of its surface. The cell walls of the carrot constitute a semipermeable membrane which permits the water to diffuse through into the cavity containing the sugar, but the sugar molecules cannot pass outward. The osmotic pressure due to the dissolved sugar forces the water up the tube. It has been known to rise with such an arrangement more than 3 meters in a few hours, but this is by no means a measure of the full amount of the pressure.

XVI. SURFACE TENSION AND CAPILLARITY

150. Adhesion between gases, liquids, and solids. Absorption. — Whenever a gas is in contact with the surface of a solid or liquid, the force of adhesion acts between the gas and that surface. The result is that the molecules of the gas near the

surface are drawn closer together and thus a film of condensed gas covers the solid or liquid surface. All surfaces about us are thus covered with a film of air which is denser than ordinary air. For this reason it is very difficult to remove the last traces of air from a glass vessel when a vacuum is being formed in it.

A small lump of charcoal because it is filled with small holes has a very large amount of surface with which a gas can come in contact, and hence it is able to absorb a volume of gas many times its own volume. Spongy platinum will absorb or condense a jet of hydrogen so rapidly that the heat generated will ignite the hydrogen.

Experiment. — Collect over mercury a large test tube full of dry ammonia gas and insert under the mouth of the test tube a lump of charcoal which has shortly before been heated. The ammonia gas will at once begin to disappear, the mercury rising in the tube to take its place (Fig. 67). This absorption of the gas by charcoal is due to the adhesion between the gas and the charcoal surface.

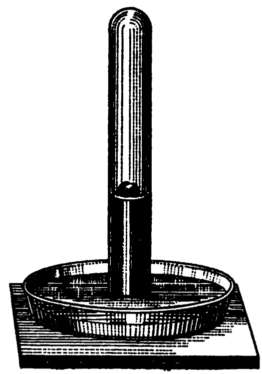

FIG. 67. — Apparatus to illustrate the absorption of ammonia by charcoal.

151. Adhesion between liquids and solids. — Whenever the adhesion between a solid and a liquid exceeds the cohesion within the liquid itself, as is the case with water and clean glass, the solid is *wet* by the liquid ; but when the cohesion exceeds the adhesion, as in the case of mercury and glass, the liquid does not wet the solid.

Experiment. — Attach three cords to a disk of glass about 8 cm. in diameter and suspend it in place of one of the pans of a beam balance in such a way that the disk shall be as nearly horizontal as possible. The under side of the disk should be perfectly clean. First counterpoise the disk by some shot or weights and then place a dish

of water beneath it at such a height that the balance beam shall be horizontal when the disk rests on the surface of the water (Fig. 68). Add carefully more weights to the opposite pan. At first the water beneath the disk will be lifted above the level of the water in the dish, and finally the disk will be lifted from the water. The under side of the disk, however, will be found to be wet. This shows that the water itself was pulled apart, not the water separated from the glass, and that the adhesion of water to glass is greater than the cohesion between the water molecules themselves.

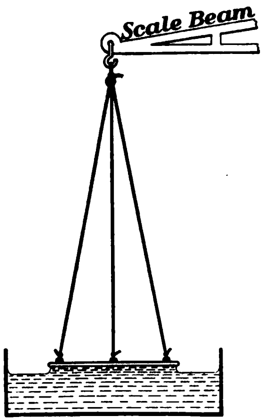

FIG. 68.—Experiment illustrating adhesion between a liquid and a solid.

Repeat the experiment, using mercury instead of water. If the mercury is clean and dry, more force will be necessary to lift the glass disk than before; but in this case the glass breaks away from the mercury, the cohesion between the mercury molecules being greater than the adhesion between the mercury and the glass.

152. The surface of a liquid differs in some respects from its interior and has peculiar properties. Any molecule in the interior of a liquid, as at *A* (Fig. 69), is surrounded on all sides by other molecules and is attracted by them equally in all directions, each molecular attraction on one side being balanced by an equal attraction on the opposite side; hence such a molecule is in equilibrium with respect to the molecular attractions acting upon it. These attractions are due to the force of cohesion.

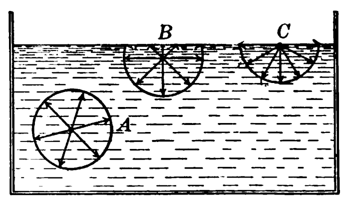

FIG. 69.—Diagram of forces acting on a particle near the surface of a liquid.

A molecule, however, at or near the surface of a liquid, as at *B* and *C* (Fig. 69), is more or less free from molecular attractions in directions exterior to the liquid, and hence the attractions toward the interior are not balanced by a molecular attraction outward. The result is that the surface molecules exert a pressure on the interior of the liquid and cause the surface to act as if it were an elastic stretched membrane.

153. Surface tension. — The tendency of the surface of a liquid to contract and to act as an elastic membrane is called *surface tension.*

It follows as a result of surface tensions that the free surface of a liquid must always be trying to become as small as possible. The form of raindrops, dewdrops, globules of mercury, and soap bubbles proves this to be true, because geometry teaches that any given bulk has the smallest possible surface when it is in the form of a sphere. All liquids when free from distorting influences take the spherical form, the form whose surface has the least possible area. Shot is manufactured on this principle. The molten metal is poured through sieves at the top of a tower, and the bits of metal take the spherical form while falling. Large drops of a liquid are usually distorted by their own weight, but in the following experiment the distorting effect of gravity is overcome.

Experiment 1. — Make a mixture of alcohol and water, using nearly twice as much alcohol as water. This mixture can easily be adjusted by the addition of small quantities of water or alcohol at a time so that a large globule of olive oil will remain suspended in it at any depth. The fact that the oil takes the spherical form affords an interesting proof of surface tension.

Experiment 2. — By means of a clay pipe blow a large soap bubble, then remove the pipe from the mouth and observe the contraction of the bubble. To show that the air is being driven from the bubble point the stem of the pipe toward a candle flame. Faraday was able

in this way to extinguish the candle. This experiment illustrates surface tension because the bubble contracts and forces out the air.

Experiment 3. — Form from a piece of iron wire a ring 10 or 12 cm. in diameter, allowing the end of the wire to project for a handle; tie to the ring a loop of thread. Dip this ring into a soap solution, thus forming a film over it with the loop lying in the film (*a*, Fig. 70). If now the film within the loop be broken by being touched with a hot wire or a piece of blotting paper, the loop will be made to take immediately the form of a circle as shown at *b*.

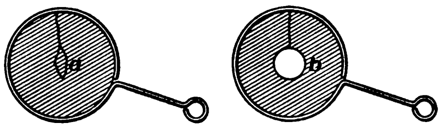

Fig. 70. — Experiment to illustrate surface tension.

Geometry teaches that the opening *b* has the largest possible area when it is circular. Since it is as large as possible, the area of the film outside of the loop covering the remainder of the ring must be as small as possible. That it becomes as small as possible is proof of surface tension.

154. Surface of a liquid in contact with a solid. — If you examine the surface of water where it is in contact with the side of a clean glass dish, you will find it turned upward as shown at *a*, Fig. 71. Sometimes, when the glass is not clean, the surface of the water next to the glass takes the form shown at *b*, or if the glass is greasy, the surface is turned downward as shown at *c*. Pure

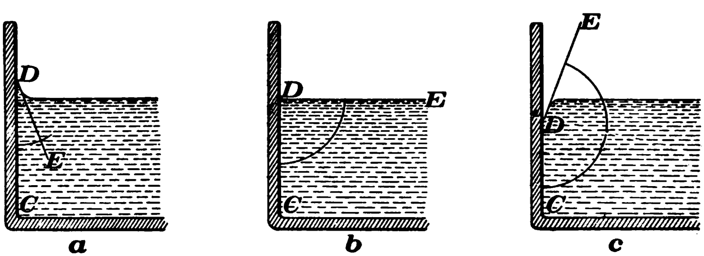

Fig. 71. — Diagram of the surfaces of liquids in contact with a solid.

mercury in contact with clean glass takes the form shown at *c*.

The angle *CDE* which the liquid makes with the solid at the point of contact is called the *angle of contact.*

If the adhesion between the solid and the liquid is greater than the cohesive attraction of the liquid molecules for each other, the liquid wets the solid and the angle of contact is acute, as at *a*. If the attraction between the solid and liquid equals the attraction of the liquid molecules for each other, then the angle of contact is 90°, as at *b*; but when the cohesion is greater than the adhesion, then the liquid does not wet the solid, and the angle of contact is obtuse, as at *c*.

155. Capillarity. — When a clean glass tube of fine bore is placed vertically in the water, the water rises in the tube, standing at a higher level within the tube than in the vessel around it. The angle of contact causes the surface of the water in the tube to be concave upward (*a*, Fig. 72), and the surface tension, tending to contract this surface, trying to straighten it out as it were, raises the water in the tube.

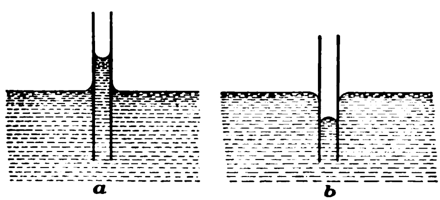

FIG. 72.—Diagram to illustrate the elevation and the depression of liquids by capillarity.

When the tube is placed in mercury, the angle of contact is obtuse, causing the surface to be convex upward (*b*, Fig. 72), and the surface tension forces the mercury in the tube to a lower level than in the dish around it. This elevation or depression of a liquid by surface tension is called capillarity.

Capillary action is most noticeable in the fine hairlike tubes, hence the name (Latin, *capillus*, a hair). Oil rising

in the wick of a lamp and the absorption of ink by a blotter are familiar illustrations of capillary action.

156. Laws of capillarity.

I. *Liquids are elevated in tubes when they wet them or when the angle of contact is acute.*

II. *Liquids are depressed in tubes when they do not wet them or when the angle of contact is obtuse.*

III. *The elevation or depression of the liquid is inversely proportional to the diameter of the tube.*

The third law is easily illustrated by dipping several clean glass tubes having different bores in water. The water will stand highest in the tube having the smallest bore, and lowest in the tube of largest bore.

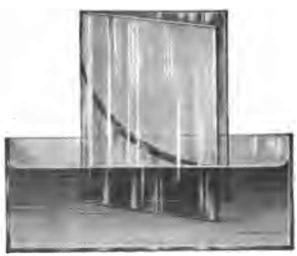

FIG. 73. — Experiment illustrating capillarity between two glass plates.

The same law may be illustrated by wetting two glass plates and bringing them together along a vertical edge, as in Figure 73. The water will rise highest where the plates are nearest together.

XVII. TRANSMISSION OF PRESSURE BY FLUIDS

157. Pressure in fluids at rest is due to two causes, (1) pressure exerted upon the fluid from without by the walls of the containing vessel, and (2) pressure due to gravity or the weight of the fluid itself.

The wall of an inflated bicycle tire exerts pressure on the air within it, and the piston of a bicycle pump exerts pressure on the air within the pump.

It is evident that the lower part of a fluid must support the weight of the fluid that rests upon it, and hence it is

easy to understand how gravity or the weight of the fluid must cause pressure in a fluid.

The word *pressure* when used with exactness in physics does not denote the *total force* exerted on the *whole* of a surface, but the *force per unit area.* Thus, if a force of 100 g. acts on a surface of 5 sq. cm., the pressure is 20 g. *The pressure on any surface is found by dividing the total force on the surface by the area of that surface;* and conversely, *the total force exerted on any surface is found by multiplying its area by the pressure upon it.*

158. Pressure at any point within a fluid at rest is equal in all directions. This is true whether the pressure is due to the walls of the containing vessel, or to gravity, or to both combined. Imagine a minute cube of water about the point *a* in a vessel full of water (Fig. 74), and suppose the pressure on this cube to be greater in one direction than in any other. The cube would then be moved in the direction of the greater force because of the mobility of fluids; but the fluid is supposed to be at rest, and hence there can be no motion and hence no force at any point within a fluid greater in one direction than another. The same reasoning holds for air as well as water.

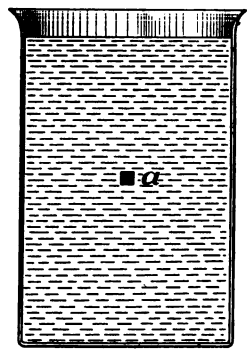

FIG. 74. — Diagram of a minute cube of water in a jar of water.

159. Fluid pressure perpendicular to the surface. — *The pressure of a fluid at rest is always perpendicular to any surface upon which it is exerted.* Suppose it were not perpendicular and let *pr* represent the pressure of a fluid on the surface *AB* (Fig. 75). This force could be resolved into two rectangular forces

cr and *er*, one perpendicular and one parallel to the surface. Because of the mobility of the fluid it is evident that the component *cr* would cause motion along the surface. But by hypothesis the fluid is at rest. Hence there can be no component along the surface producing motion, and this is true only when the pressure *pr* is perpendicular to *AB*.

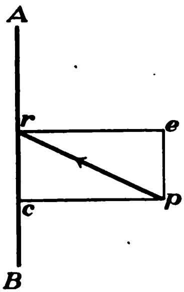

Fig. 75. — Diagram to illustrate the direction of fluid pressure.

160. Transmission of pressure. — Pascal's law is as follows: *Pressure exerted at any place upon a fluid inclosed in a vessel is transmitted undiminished in all directions to every part of the interior of the vessel.*

Imagine a box (Fig. 76) to be filled with wheat or with smooth bright bicycle balls. Because the kernels of wheat or the balls slide over one another easily the contents of the box will exert pressure on its sides as well as on the bottom; and if additional pressure is exerted on the contents through an opening in the top by means of the block *B*, the wheat will tend to overflow or to lift the cover, exerting pressure upwards as well as on the bottom and sides of the box. If the box were filled with rusty balls or rough irregular bodies that would not slide over one another, little or no extra pressure would be exerted on its sides and top, but the pressure would be transmitted only to the bottom directly beneath the block *B*. The action of the wheat or the balls illustrates very imperfectly how fluids transmit pressure, imperfectly because

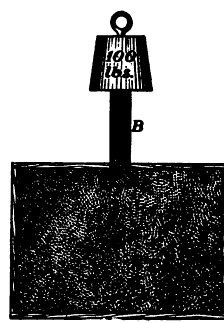

Fig. 76. — Diagram to illustrate the transmission of pressure.

there is much friction when the kernels of wheat or the balls are moved about, whereas the mobility of fluids is practically perfect.

161. Illustrations of transmitted pressure. — If a bottle fitted with a perforated rubber stopper is filled with water, it can be easily broken by forcing a smooth brass rod just large enough to fill the hole in the stopper down into the water. In making this experiment it is necessary to fasten the stopper in place (Fig. 77), and no air bubbles should be left under the stopper. In forcing the rod into the water the pressure is exerted on the water at the end of the rod and this pressure is transmitted to the whole interior surface of the bottle.

FIG. 77.— Experiment to illustrate the transmission of pressure by water.

Suppose, for example, the diameter of the rod to be 0.5 cm. and the force with which it is pushed into the bottle is 5 Kg. The area of the end of the rod would be 0.2 sq. cm. (nearly). Hence every square centimeter of the interior surface of the bottle would be subjected to a pressure of 25 Kg., or more than 350 pounds to the square inch. The bottle would probably burst long before any such pressure was reached. If you study this example carefully, you will understand that the smaller the rod the greater will be the pressure per unit area and the greater the total force on the interior of the bottle. If this bottle were connected by a tube of any length whatever to another bottle full of water, the same pressure would be transmitted to that also.

The water mains of a city afford an illustration of the transmission of pressure by a liquid while the air brake in use on railroad trains illustrates the transmission of pressure by gases.

162. The hydraulic press, which has a very extended use in industrial operations, affords, perhaps, the best illustration of Pascal's law. It consists of two watertight cylinders, C and D (Fig. 78), connected by a pipe E. In each cylinder there is a plunger or piston, A and B.

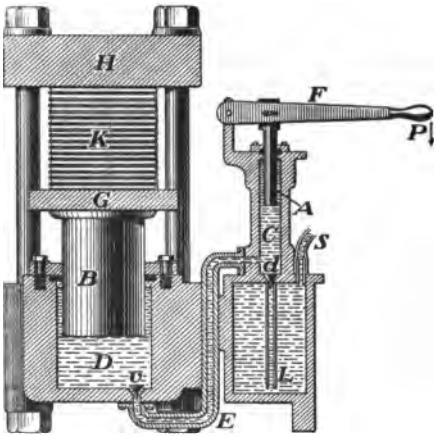

FIG. 78. — Diagram of the hydraulic press.

These pistons work in the same way as the rod described in the preceding paragraph. They are free to move up and down and yet fit so perfectly where they enter the cylinders that little or no water leaks out. The valve v allows water to flow from C to D, but not in the opposite direction, and the valve d allows water to enter C from the supply tank L, which is kept full by the pipe S, but not to flow back. When the plunger A, which may be operated by a lever or by steam, is lifted, water enters C through d; when A is forced down the water flows from C to D, and the pressure exerted on A is transmitted from the end of A to the end of B, as well as to all other parts within the cylinders. The pressure on B exactly equals that on A, but the total force on B is as many times greater than on A as the area of B is greater than that of A.

When used as a press, the substance to be compressed, K, is placed between a platform G, on the top of B, and a rigid structure H, above it. Many of the passenger elevators in high buildings are operated on the

same principle. Generally the car of the elevator is moved by wire cables and pulleys attached to the piston *B* (although sometimes the car is placed immediately on top of *B*, the cylinder *D* being a deep one extending down into the ground beneath the elevator).

The hydraulic press illustrates the general law of machines. The student should be able to show that the force on *A* times the distance *A* moves down, equals the force on *B* times the distance it moves up.

XVIII. PRESSURE IN FLUIDS DUE TO GRAVITY

163. Weight of a fluid causes pressure. — The reason why there is pressure in a fluid, due to gravity or the weight of the fluid itself, is easily understood from the fact that the lower portions must bear the weight of the upper portions. Because gravity acts downward, we can readily see why there should be a downward pressure; but it has been shown (§ 158) that at any point in a fluid at rest there is an equal pressure in all directions, — upwards and laterally, as well as downwards. It is because fluids transmit pressure in all directions that the downward pressure due to gravity causes pressure in other directions as well.

164. Pressure in air due to gravity. — We live at the bottom of a great ocean of air. The height of the air above the surface of the earth has been estimated by some to be about 45 miles and by others at about 200 miles, but the density of the air decreases so gradually that it is impossible to place any exact limit to its height. Air has weight; and the air about us, the surface of the earth, and our bodies must bear the weight of the air above us.

Although this weight is considerable, we are unconscious of it because the pressure due to it is the same in all directions, and also

because it is balanced by the elastic tension of the air and gases within the tissues and liquids of the body. This elastic tension of the air within the body is shown by the process of "cupping" used by physicians. When a cup is placed upon any portion of the body and the air is exhausted therefrom, the skin bulges up into the cup because of this elastic tension. The pressure on the outside of the body and the elastic tension of the air within the body are equalized by the outside air being taken into the body by the lungs and distributed throughout the tissues of the body by the blood.

One liter of air at 0° C. and under standard pressure weighs 1.2932 g., or the density of air is 0.0012932 g. per cubic centimeter.

Experiment to show weight of air. — Fit a large bottle with a perforated stopper through which a glass tube extends, and place a thick-walled rubber tube over the end of the glass tube, the rubber tube being provided with a clamp. Make sure that the stopper and clamp close the bottle air-tight. Weigh the bottle, tube and all, carefully, and then, connecting it to the air pump by the rubber tube, exhaust the air from it as thoroughly as possible. Close the tube by the clamp and weigh again. The difference in the two weights will be the weight of the air removed. Dip the end of the tube into a jar of cold water, and open the clamp to allow the water to flow into the bottle. Hold the bottle so that the water in it and in the jar are at the same level. (Can you tell why the water flows into the bottle?) Weigh again as before and find the weight of water in the bottle. The weight of the water in grams equals the number of cubic centimeters of air removed. (Why?) Calculate the density of the air removed.

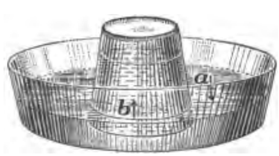

FIG. 79. — Experiment to illustrate the downward pressure of air.

Experiment 1. — Place a tumbler in a basin of water so that it will be filled, then invert it and lift it nearly out of the water. The water will not fall out of the tumbler while the edge of it remains below the surface of the water in the basin. The downward pressure of the air on the surface of the water in the basin is transmitted by the water in all directions, so that at the point *b* (Fig. 79) there is an upward pressure equal to the downward pressure at *a*.

Experiment 2. — The fountain in vacuo shows the pressure of the air. A tall glass vessel (Fig. 80) provided with a stopcock at the bottom has a jet tube extending up into it. The air is first removed from the vessel by the air pump. The lower end of the vessel is then placed in water and the stopcock is opened. Since the outer end of the jet tube is under water, the air cannot enter; but the pressure of the air on the water forces it up into the vessel in a strong jet. A large bottle closed with a rubber stopper through which a glass tube extends may be used for the experiment.

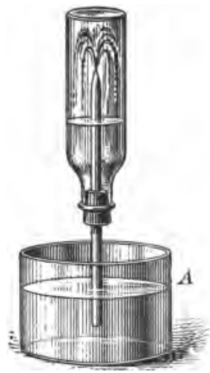

FIG. 80. — Simple form of the fountain in vacuo.

Experiment 3. — Fill a large test tube with an even rim to the top with water, and cover its mouth with a card which has been soaked in water a few moments. Being sure that the card touches the rim all round, hold it in place and invert the test tube. The hand may now be removed from the card and the water will not fall out (Fig. 81). It is the upward pressure of the air that prevents it. The test tube may be held horizontally, or obliquely, — in fact, turned through a complete circle, — and the water will be held in the test tube by the pressure of the air.

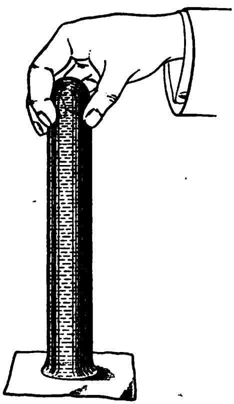

FIG. 81. — Water supported in a test tube by the upward pressure of the air.

Experiment 4. — The Magdeburg hemispheres (Fig. 82) are two hemispherical cups of iron or brass usually about 10 cm. in diameter. The edges of the cups are turned so true and smooth that when they are placed together they form an air-tight sphere. One of the cups has a stopcock ending in a screw, by which it can be

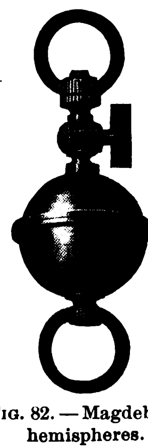

FIG. 82. — Magdeburg hemispheres.

attached to an air pump or to which a handle may be screwed. The other cup is also provided with a handle. Place the cups together and attach the sphere to the air pump. Remove the air from the sphere and close the stopcock so that the air cannot enter again. Remove the sphere from the pump and attach the handle to it. If the exhaustion is nearly complete, it is quite possible that no two boys of the class can pull the hemispheres apart. It does not matter in what direction the pull may be, — whether horizontal, oblique, or vertical, — they are still held together by atmospheric pressure.

This experiment also illustrates the elastic tension of the air. The more air there is within the sphere, the more easily are the parts separated. When the stopcock is open, the hemispheres fall apart easily, because the elastic tension of the air within equals the pressure of the air without.

These experiments show us that there is pressure in air in all directions, but they do not show that it is equal in all directions at any point. One more experiment, however, will make this clear.

Experiment. — Fasten a piece of thin sheet rubber over the large end of a student-lamp chimney (a rubber band will hold it in place better than a string), and close the other end of the chimney with a stopper through which a glass tube about 1 cm. in diameter extends. Pass a rubber tube over this glass tube. Then partially exhaust the air from the chimney by means of the air pump or by the lungs. It may be necessary to close the tube by a clamp to prevent the air reëntering the chimney, or possibly you can do this by pinching the tube between the thumb and finger. The air will press the sheet rubber into the chimney, as shown by Figure 83. *The distance it is pressed in will be the same whatever the position in which the chimney is held, thus showing that the pressure is equal in all directions.*

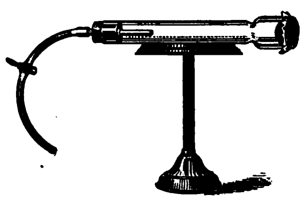

FIG. 83. — Sheet rubber stretched by the pressure of the air.

"But," some one will say, "the rubber is drawn in by suction." Another experiment will show that this is not true.

Experiment. — Cover the sheet rubber of the last experiment with a plate of glass that fits so that no air can get under it, and repeat the experiment. The rubber will not be drawn in by "suction," but as soon as the plate is slipped off it will be pushed in by atmospheric pressure. Moisten the glass to make it slide easily on the rubber. Really there is no such thing as suction.

165. Pressure in liquids due to gravity. — We have shown that at any point in air there is pressure which is equal in all directions, and that this pressure is due to the weight of the air itself.

Likewise at any point within a liquid, there is pressure which is exerted equally in all directions and which is due to the weight of the liquid itself. In liquids pressure due to gravity is proportional to depth.

Experiment. — *A*, *B*, and *C* (Fig. 84) represent three glass tubes open at both ends, each containing mercury in the lower bend. When these tubes are lowered into a deep jar of water, the water at the opening of the tubes exerts pressure which is transmitted by the inclosed air to the mercury, causing it to rise in the long arm of the tube. The difference in the height of the mercury in the two arms is a measure of the pressure exerted by the water at the mouth of the tube. The tube *A* measures the downward pressure, *B* the lateral, and *C* the upward pressure. If the three tubes are placed in succession in water so as to measure the pressure at the same point, it will be found equal in all directions.

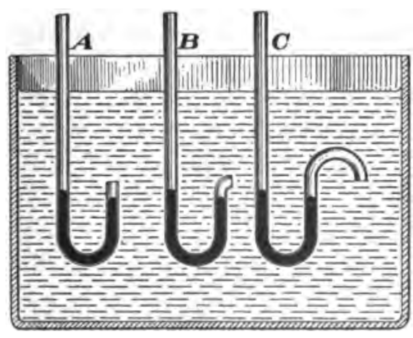

FIG. 84. — Diagram of apparatus for measuring the downward, lateral, and upward pressure in a liquid.

Experiment. — Figure 85 illustrates an apparatus for measuring upward pressures in water. It consists of a thistle tube over the mouth of which at *a* is stretched thin sheet rubber. A disk of wood

e rests upon this rubber diaphragm and is fastened to a metal rod extending up through the tube and supporting the pan *A* at the top. Any motion of the pan up or down is indicated by the lever *B*. The whole is supported by a platform attached to a tall glass jar. The position of the lever is marked at *o* when there is no water in the jar, and then a 100 g. weight is placed in the scale pan and water poured into the jar carefully until the lever *B* is brought back to *o*. The upward pressure at *a* then balances the downward pressure of the 100 g. weight. Let *b* be the surface of the water at this adjustment. Substitute now a 200 g. weight for the first weight and again add water until the lever is brought to the mark at *o*. It will be found that the water is at *c*, the depth from *c* to *a* being just twice the depth from *b* to *a*. Other weights may in the same manner be tried, and it will be found that the depth of *a* below the surface of the water is proportional to the weights in the pan *A*. This will prove that *upward pressure in a liquid is proportional to the depth;* but since pressure at any point is equal in all directions, the same law applies to other directions as well. *D* is a siphon for removing water from the jar, and *E* is a thistle tube for filling the jar.

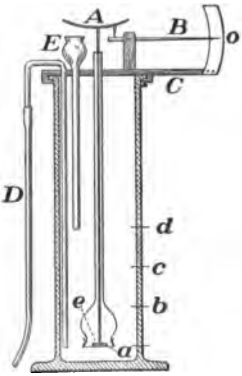

FIG. 85. — Diagram of apparatus for measuring upward pressures in water.

166. The surface of a liquid at rest.— *Under the action of gravity alone the free surface of a liquid at rest is always level or horizontal.*

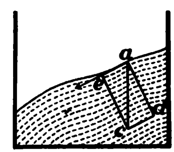

FIG. 86. — Diagram of the effect of gravity on a particle in the surface of a liquid.

Let *a* (Fig. 86) be a particle in the surface of a liquid which is supposed to be at rest while the surface is not level. Let *ac* represent the force of gravity acting on the particle. This force *ac* may be resolved into two rectangular components, *ad* perpendicular and *ae* tangent to the surface at *a*. The component *ae* would be unopposed

and hence would produce motion; therefore the liquid cannot be at rest as supposed. It can be at rest only when the component *ae* is zero, and this can occur only when *ad* coincides with *ac* and the surface is perpendicular to *ac*, that is, when the surface is level.

167. Pressure in a horizontal plane. — Pressure in fluids due to gravity is equal at all points in the same horizontal plane. This is true no matter what the shape of the vessel may be and follows from the fact that pressure varies with the depth. Since the surface of a liquid at rest is horizontal, all planes through the liquid parallel to the surface are also horizontal; hence all points in any such plane have equal depth and consequently equal pressures.

Observe that pressure due to gravity differs from pressure exerted externally on a fluid in this respect. While the first is the same at all points in the same horizontal plane, it is different in different planes, but the latter is the same at all points throughout the fluid, whether in the same horizontal plane or not.

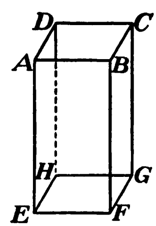

FIG. 87. — Diagram to illustrate the pressure on the bottom of a vessel filled with a liquid.

168. Pressure at any depth below the surface of a liquid. — The total force on the bottom of a vessel with vertical sides (Fig. 87) equals the weight of the liquid in the vessel. If the vessel is full, the volume of the liquid equals the area *EFGH* times its depth *AE*, and the weight of this liquid equals its volume times its density. Therefore the *total force* on the bottom of the vessel equals its area × the depth × the density of the liquid, but the *pressure* on the bottom equals the total force divided by its area (§ 157), or *depth* × *density*.

This is true not only of the pressure on the bottom but at any other depth and in any other direction. In general then, *the pressure in a liquid at any depth and in any direction is equal to the product of that depth by the density of the liquid.*

For example, the downward pressure at any place in the surface PG (Fig. 88) is equal to the depth AB times the density of the liquid filling the vessel, and the upward pressure on the surface LH is equal to the depth SL × the density of the liquid. If the surface is not horizontal, as FH, the pressure is not uniform, but the *average pressure* upon it is equal to the *average depth* of the surface × the density of the liquid. The average depth of a surface is the depth to its center of area; for example, ac is the average depth of the surface FH.

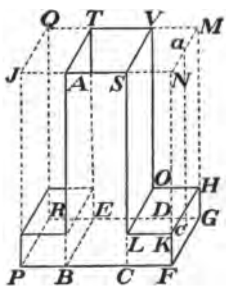

FIG. 88.—Diagram to illustrate the pressures on different surfaces in a vessel filled with a liquid.

The total force on any surface below the surface of a liquid may be computed by first finding the pressure upon it and then multiplying this pressure by the area of the surface; or since pressure equals depth × density, the *total force = area × depth × density.*

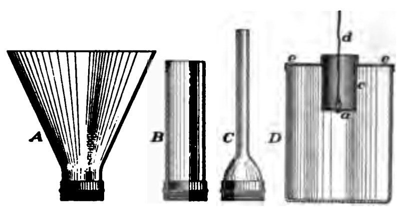

FIG. 89.—Diagram of Pascal's vases for showing that pressure on the bottom of a vessel does not depend on the quantity of liquid in it.

Experiment.—*Pascal's vases.* It has been shown that the total force on the bottom of a vessel depends on the area of the bottom, and the depth and the density of the liquid. It does not depend on the quantity of

the liquid or the shape of the vessel. This can be shown by an apparatus known as Pascal's vases. One form of this apparatus consists of a tube c (Fig. 89) fastened to a large circular disk ee which serves to support the vases on a battery jar D. The upper end of the tube c is threaded so that any one of the three vessels, A, B, or C can be attached as shown in Figure 90. The bottom of the tube is closed by the disk a, held in place by the string d, which is attached to one end of a scale beam.

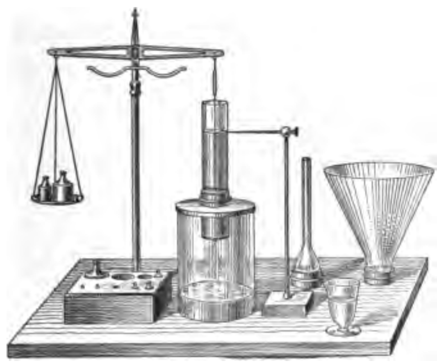

FIG. 90. — Apparatus for experiments with Pascal's vases.

Arrange the apparatus as shown in Figure 90, and counterpoise the disk a by placing some weights in the pan. Then add a 200 g. weight to the pan and pour water carefully into the vase until the pressure on the disk a is just sufficient to loosen it from the bottom so that the water begins to escape. Note the height of the water in the vase when this happens. Repeat the experiment with 400 g. in the pan, and then substitute the other vases for the first one used and repeat the operations again. According to the laws stated above, the height above a should be the same in all three vases with the 200 g. weight and twice as high with the 400 g. weight.

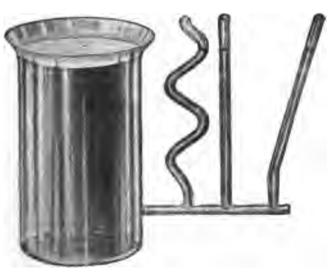

FIG. 91. — Apparatus to illustrate the principle that "water seeks its level."

169. Liquids in connected vessels. — When several communicating vessels of any size or shape whatever contain a liquid, the surfaces of the liquid in all the vessels will have the same level. The saying is, "Water always seeks

its level." This follows because the pressure is proportional to the depth and is the same at all points in any horizontal plane. Figure 91 shows an apparatus for illustrating this principle. Cities are supplied with water on this principle. If the supply of water comes from a reservoir upon a hill, the water will rise to the same height as that of the water in the reservoir in the buildings in the various parts of the city, although the pipes connecting them with the reservoir pass down into valleys and over elevations. The pipe, however, must not pass over an elevation higher than the reservoir itself. This principle holds for liquids at rest. When the water is flowing from an opening, the pressure and the level decrease as the opening is approached.

Problems

Density of water = 1 g. per cubic centimeter, or 62.4 lb. per cubic foot, or 0.578 oz. per cubic inch. Density of mercury = 13.6 g. per cubic centimeter.

Suppose the dimensions of the vessel represented in Figure 88 to be as follows: $SL = 40$ cm., $SC = 48$ cm., $SV = 15$ cm., $KL = 12$ cm., $FP = 36$ cm., and $AS = 12$ cm.

1. Compute the average pressure and the total force on the lower half of the side *SVLO* when the vessel is full of water. Solution: Its area $= 20 \times 15 = 300$ sq. cm. Average depth $= \frac{1}{2}(20 + 40) = 30$ cm. Density of water = 1 g. per cubic centimeter.

Total force $= 300 \times 30 = 9000$ g. *Ans.*

2. Compute the pressure and the total force on the surface *CG* when the vessel is full of mercury. *Ans.* 117,504 g.

3. Compute the pressure and the total force on the surface *LH* when the vessel is full of water. *Ans.* 7,056,000 dynes.

4. Compute the pressure and the total force on the surface *AC* when the vessel is full of water. *Ans.* 13,824 g.

5. Compute the pressure and the total force on the side *SO* when the vessel is full of water.

6. Compute the pressure and the total force on the surface *KG* when the vessel is full of mercury.

7. What is the weight of the water in the box when it is full, and what is the total force on the bottom of the box?

Ans. 11,520 g. and 25,920 g.

8. A cubical box 4 ft. on each edge is connected with a reservoir on a hill 4 miles distant. The surface of the water in the reservoir is 498 ft. above the top of the box. Compute the pressure and the total force on each of the six sides of the box.

9. A cube 18 cm. on each edge was placed in an iron tank which was then filled with water and closed. A hole 1.5 cm. square was made through the top of the tank and a rod of the same size was pressed into the hole with a force of 52 g. Compute the pressure and the total force produced on the surface of the cube.

10. What change would there be in the pressure on the cube if the rod were half as large as in the last problem?

11. What is the average pressure per square inch on the interior of the water pipes of a city which has a standpipe 150 ft. high, the surface of the water in this pipe being on the average 145 ft. above the pipes of the city?

12. When the gas of the city mains will support a column of water 2 in. high, what is the pressure in the mains per square inch?

XIX. THE BAROMETER

170. Historical. — If a tube fitted with an air-tight piston (Fig. 92) is placed in water and the piston drawn upward, the water will follow it. We say commonly that it is drawn up by "suction"; in reality it is pushed up by atmospheric pressure acting on the water in the dish. Until 1643, however, the true explanation was not known, and men were greatly puzzled because, try as they might, water could not be raised in this way to a height greater than 34 ft. This was one of the problems left by Galileo for his pupil Torricelli to solve.

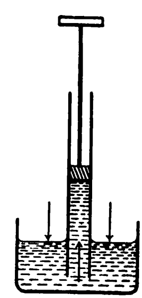

FIG. 92. — Diagram to illustrate raising water by the pressure of the atmosphere.

Torricelli, suspecting the true cause,

reasoned that if the weight of the air was sufficient to support a column of water 34 ft., but no higher, it would support a column of mercury as high as 13.6 is contained in times 34 ft., because mercury is 13.6 times as dense as water. $34 \div 13.6 = 2.5$ ft., or 30 in. Hence he made his famous experiment.

171. Torricelli's experiment. — For this experiment a glass tube closed at one end and somewhat more than 30 in., or 76 cm. long, is needed. The tube is filled with mercury and the open end closed with the finger. It is then inverted and the finger removed after the open end is placed beneath the surface of mercury in an open dish. As soon as the tube is placed in an upright position the mercury falls in the tube to a height about 30 in. above that in the dish, leaving a vacuum at the top (Fig. 93). A vacuum so formed is called a "Torricellian vacuum."

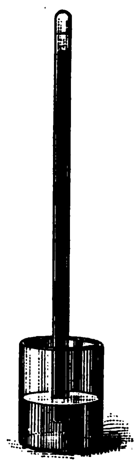

FIG. 93. — Torricelli's experiment.

Experiment. — To show that it is the air that supports the mercury in the tube, place Torricelli's apparatus under a tall receiver of an air pump and exhaust the air from the receiver. As the air is removed, the mercury falls in the tube, but when it is admitted again to the receiver, the mercury rises to its original height. In fact, the height of the mercury affords a measure of the amount of air pumped from the receiver. Instruments based on this principle are sometimes attached to air pumps for this purpose and are called *manometers*.

Pascal thought that if Torricelli's explanation was right, the air at the top of a mountain would not support so high a column of mercury as it would in the valley below. He found this to be true, and thus the truth of Torricelli's explanation was confirmed.

172. The barometer is an instrument for measuring atmospheric pressure. If Torricelli's apparatus were

placed in a frame to support it and a ruler placed by the side of the tube so that the height of the mercurial column could be observed at any time, one would have all the essential parts of a barometer. The height of the column above the surface of the mercury in the cistern is a measure of the atmospheric pressure. If it is observed at regular intervals, the height is found to be varying constantly. At sea level its average height is 76 cm., or 29.92 in. This does not mean necessarily that it often stands exactly at that height, but that this height is the average of a great number of observations taken from day to day and year to year. On the borders of the Great Lakes, Erie, Huron, Michigan, and Superior, which are about 600 ft. above sea level, the average height of the barometer is about 74.3 cm., or 29.3 in.

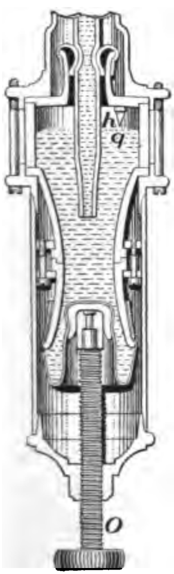

FIG. 95. — Diagram of a section of the cistern of a barometer.

FIG. 94. — Barometer.

173. Fortin's barometer. — Figure 94 represents a common form of the barometer. The glass tube is supported within a brass tube upon which the scale is placed. Figure 95 shows the cistern and the lower end of the tube upon an enlarged scale. It is evi-

dent that as the mercury rises and falls in the tube the amount of mercury in the cistern must vary. Hence the zero of the scale, which must be at the surface of the mercury in the cistern, would need to be changed at each observation for the sake of accuracy.

To avoid this the bottom of the cistern is made of flexible leather which can be raised or lowered by the screw *O*. By this means the surface of the mercury can be adjusted to the zero of the scale, which is at *q*, the end of the ivory point *h*.

174. The amount of the atmospheric pressure. — The height of the barometer does not depend on the diameter of the tube. It is true that if the bore of one tube has twice the area of another, there is twice as much mercury in it to support, but the total force supporting it is also twice as great because it is exerted on twice as large an area (§ 168). Suppose the area of the bore of the barometer tube to be 1 square centimeter and the height of the mercurial column to be 76 cm., then the volume of mercury in the tube would be 76 cc. (76×1), and since 1 cc. of mercury weighs 13.596 g., 76 cc. would weigh $76 \times 13.596 = 1033.3$ g. Hence the average atmospheric pressure at sea level is 1033.3 g. per square centimeter, which is equivalent to 14.7 lb. per square inch. The pressure of *one atmosphere*, as this pressure is called, is usually spoken of as 15 lb. per square inch or 1 Kg. per square centimeter.

It is customary to speak of a pressure of so many centimeters or millimeters; thus, a pressure of 76 cm. or 760 mm. The meaning is that the pressure is equivalent to that of a column of mercury of the height mentioned.

Since the elastic tension of the air always equals the pressure upon it, the barometer measures it as well as the

atmospheric pressure, and it is expressed by the same numbers.

175. Isobaric lines. — A line drawn on a map through places of equal atmospheric pressure is called an *isobaric line* or an *isobar.*

Before the height of the barometer at one place can be compared with that at another, the heights at both places must be reduced to standard conditions. These are sea level and freezing point of water or 0° C. Warm mercury is less dense than cold mercury, and hence the air can support a higher column of it when it is warm than when it is cold. Therefore, when the height of the barometer is taken, the temperature of the mercury in it is also observed. When the mercury is warmer than 0° C., a small amount is subtracted from the actual reading; if it is colder, the correction is added, the amount to be added or subtracted being found in tables printed for the purpose. We have already seen that the barometric height is less the higher the place of observation above the sea. To allow for this variation the height of the place of observation above sea level must be known, and then by the aid of tables the amount to be added to the actual reading is obtained. The actual height of the barometer is thus "reduced" or changed to what it would be if the mercury of the barometer were at 0° C. and the place of observation were at sea level.

The height of the barometer at any one time or place is of no very great value in forecasting the weather, although rapid changes in its height are important. It is by comparing barometric readings made simultaneously at regular intervals at many different places that the barometer becomes most useful for that purpose.

At the same moment of time each day at each Weather Bureau station throughout the United States the height

of the barometer is read and reduced to standard conditions. It is then telegraphed to Washington and to the principal stations, where the isobars are printed on the weather maps. These maps are then distributed by mail to the people.

176. The weather map. — Figure 96 is a copy of a map issued by the United States Weather Bureau. The heavy lines are the isobars drawn for each tenth of an inch between the highest and the lowest barometric reading for the morning of issue. The region of low barometer, marked "Low," is called the *cyclonic area.* The air flows into this area from all directions, forming a sort of whirlpool of air. In the United States the direction of the wind about these areas is counter-clockwise, that is, east of the area the wind is from the south, and west of it the wind is from the north. These cyclones are continually passing over the country with considerable regularity and along pretty well-defined paths. The forecaster depends largely upon his knowledge of the movements of these cyclonic areas in predicting the weather. The student must not confuse these cyclones, which are of common occurrence, with destructive storms such as tornadoes and hurricanes, which are often but improperly called cyclones.

XX. BOYLE'S LAW

177. Boyle's law. — Gases, as we have learned, are compressible by pressure and also exert a pressure outward, which is called elastic tension, this elastic tension always being equal to the pressure on the gas. The relation between the elastic tension of a gas, or the pressure upon it, and its volume was first discovered by Robert Boyle, and is expressed by the following law:

The volume of a given quantity of gas is inversely

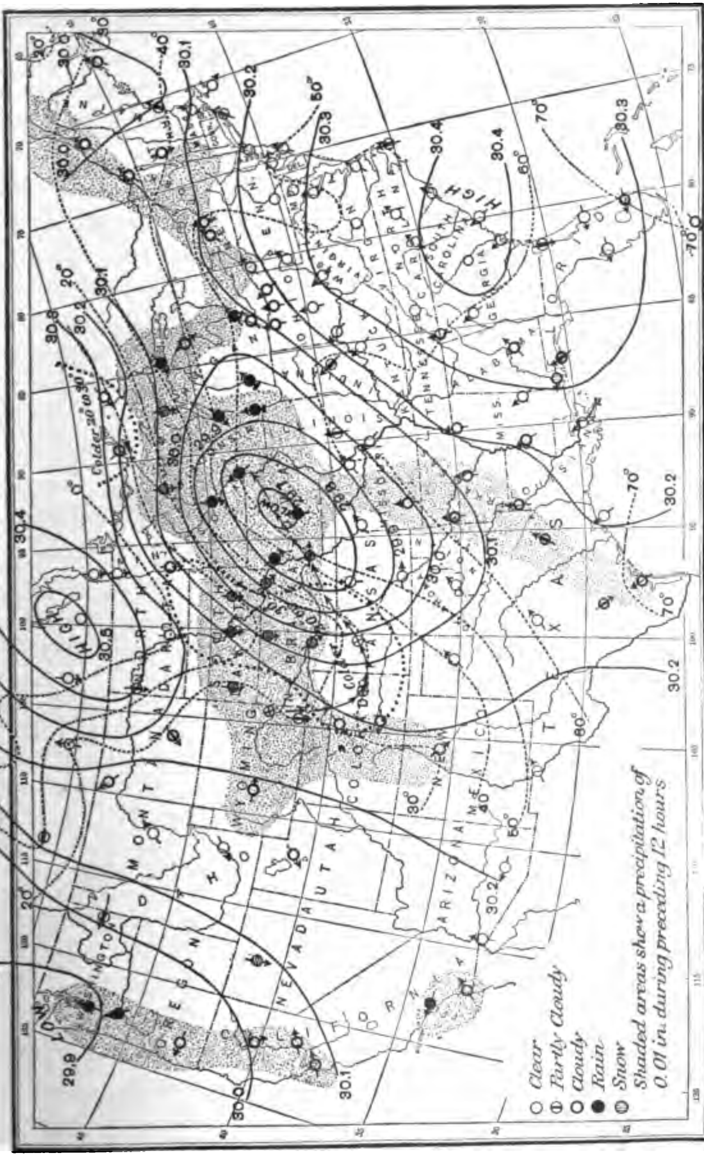

FIG. 96. — United States Weather Map, showing isobars.

proportional to its elastic tension or to the pressure upon it.

This means, for example, that when the pressure on a gas is doubled, its volume becomes half as great; or if the pressure is multiplied by five, its volume becomes one fifth as great, etc. To express this law algebraically, let p_1 represent the pressure upon a gas when its volume is v_1, and p_2 the pressure when the volume becomes v_2; then,

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}, \text{ or } p_1 v_1 = p_2 v_2.$$

This last expression shows that although the pressure and volume have changed, their product is the same as at first; hence, Boyle's law is sometimes stated as follows:

The product of the volume times the pressure of a given mass of gas is constant, if its temperature does not change.

Experiment. — To prove this law Boyle used a long J-shaped tube (Fig. 97) the short arm of which was sealed at the top A. The bend of the tube was then filled with mercury to the same level BC in both arms. This inclosed a quantity of air in the short arm from B to A which was under a pressure of one atmosphere. More mercury was then poured into the tube until it rose to D in the short arm, compressing the air into half its former volume. When this was done, the mercury stood at F in the long arm, and it was found that the column of mercury EF exactly equaled the mercury column of a barometer. This column EF therefore exerted a pressure of one atmosphere on the air in the short arm; but the outside air itself was also pressing on the mercury with a force of one atmosphere, so the whole pressure on the air in the short arm was two atmospheres. In the same way, the pressure was found to be three atmospheres when the volume of air became one third as great.

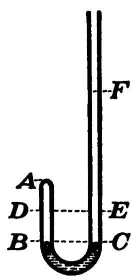

Fig. 97. — Diagram of the apparatus for proving Boyle's law.

Boyle's law should be studied experimentally by the student in the laboratory.

178. Boyle's law explained. — We have shown that the pressure which a gas exerts on the walls of the containing vessel is supposed to be due to the bombardment of those walls by the molecules of the gas. If a gas is compressed into half its volume, its density must be twice as great as before, and there must be in a given volume twice as many molecules as at first. It follows then that the blows against the sides of the containing vessel must be twice as frequent as before and the pressure consequently doubled.

Thus we see that Boyle's law is easily and simply explained by the theory that a gas consists of a vast number of very swiftly moving particles.

Problems

1. A quantity of air has a volume of 80 cc. when the barometer stands at 74 cm. What volume will the air have when the barometer reads 75 cm.? *Ans.* 78.93 cc.

2. What is the volume of a mass of air under a pressure of 32 lb. per square inch which has a volume of 64 cu. ft. under a pressure of 40 lb. per square inch? *Ans.* 80 cu. ft.

3. If the capacity of an inflated automobile tire is 600 cu. in., and the air in it has an elastic tension of 88.2 lb. per square inch, how many cubic inches of ordinary air does it contain?

4. What is the volume of a quantity of hydrogen under standard pressure which occupies a space of 33 cc. when the pressure is 73.5 cm.?

5. A chemist collected some oxygen in a tube inverted in a dish of mercury. The mercury in the tube stood 18 cm. higher than the mercury in the dish and the barometer reading at the time was 75 cm. What was the volume of oxygen under standard pressure, if the volume measured 31.6 cc. as collected?

6. The pressure on a gas was measured in grams per square centimeter and its volume in cc., and the product of these two quantities was found to be 1240. What volume did the gas occupy when the pressure was 40 g. per square centimeter? 62 g. per square centimeter? 8 g. per square centimeter?

7. The average density of the air at sea level is about .0013. At the tops of some of the highest mountains in the United States the barometer stands about half as high as at sea level; what is the density of the air at such heights? What proportion of the atmosphere is above the top of such a mountain and what below?

XXI. THE SIPHON AND PUMPS

179. The Siphon. — The siphon is a tube used to convey a liquid up over an elevation to a lower level than that from which it started. Figure 98 illustrates a siphon in operation. To start the flow the tube must first be filled with the liquid. This may be done by suction after the tube is in place, or it may be filled and then placed in position, the ends of the siphon being closed while this is being done. That part of the tube extending from its highest point *a* into the vessel from which the liquid flows is called the *short arm* and that part extending from the highest point in the direction the liquid flows is the *long arm* of the siphon. The real length of the short arm is the vertical distance from the highest part of the tube to the surface of the liquid, as *be*; and the real length of the long arm is the vertical distance from the highest part of the tube to its end, as *cd* (Fig. 98), or if this arm dips into the liquid, the long arm is *cd* as shown in Figure 99. The flow through the siphon continues while the length of the long arm exceeds that of the short arm.

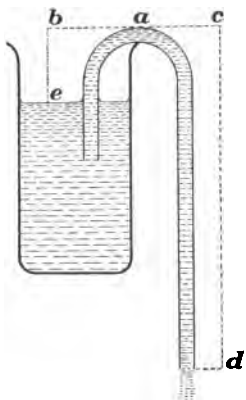

FIG. 98. — Diagram of a siphon in operation.

180. Explanation of the siphon. — The action of the siphon depends on atmospheric pressure. At the point *o*

(Fig. 99) there is an upward pressure of one atmosphere (Why?) and a downward pressure due to the weight of a column of liquid *be* in height; likewise at the point *s* there is an upward pressure of one atmosphere and a downward pressure due to a column of liquid *cd* in height. The resultant pressure at either point, *o* or *s*, is one atmosphere minus the downward pressure at that point. Since *cd* is greater than *be*, the downward pressure at *s* is greater than it is at *o*, and hence the resultant upward pressure at *s* must be less than at *o*. Hence the liquid flows in the direction of the greater upward pressure. It follows from this explanation that if the downward pressure of the column of liquid *be* exceeds one atmosphere, the siphon will not work, and experience shows this to be true.

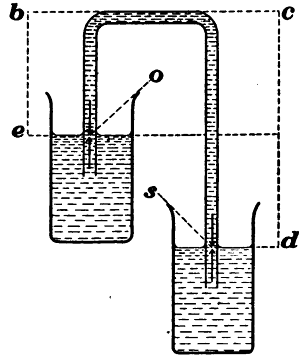

Fig. 99. — Diagram to illustrate the principle of the siphon.

If the short arm of a siphon extends through a two-hole stopper into a large bottle filled with water, the dependence of the siphon on atmospheric pressure is easily shown. When the other hole in the stopper is closed by the finger, the flow through the siphon ceases beginning again when the finger is removed. What would be the theoretical limit to the length of the short arm at sea level for a siphon conveying water? For one conveying mercury?

181. The lifting or suction pump. — In the common cistern pump the piston, barrel, and valves have much the same arrangement as in the air pump (§ 135), and the action is quite similar. The chief difference is that in the air pump it is the elastic tension of the air which causes it to raise the valve *s* and pass into the cylinder, whereas

in the water pump the pressure of the air on the water in the cistern forces the water up the tube T (Fig. 100) into the cylinder when the piston is raised. If the pump has no water in it at first, the air will be pumped out of the cylinder and the tube T by the first few strokes, water taking its place. When the piston descends, the lower valve closes and the upper valve opens, the water passing to the upper side of the piston; when the piston rises

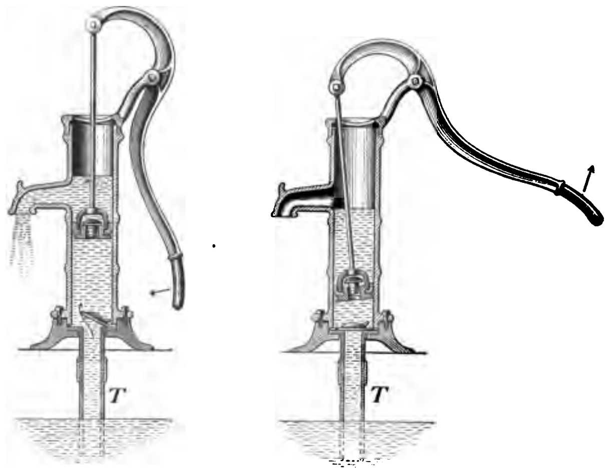

Fig. 100. — Diagrams illustrating the operation of the lifting pump.

again, the water above it is lifted by it and atmospheric pressure forces water up the tube T, filling the cylinder. Of course the water will not follow the piston to a greater height above the water in the cistern than that to which the atmospheric pressure will force it.

182. The force pump. — In the force pump shown in Figure 101, the top of the cylinder, through which the piston passes, is water tight and the water lifted by the piston can flow only through the delivery pipe P. As the pressure above the piston increases, the air in the

dome A is compressed and by its elasticity serves to equalize the varying pressures produced by the alternate upward and downward motions of the piston, and causes a more even flow from P. The valve V is not essential and is often omitted. Another form of force pump is shown in Figure 78.

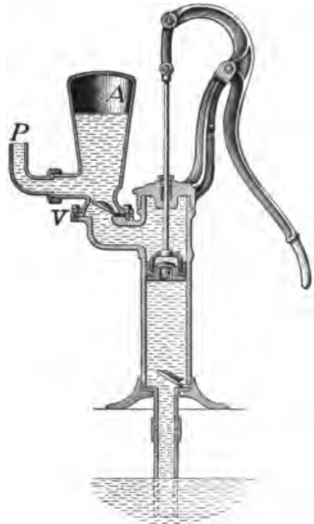

FIG. 101. — Diagram of a force pump.

Problems

1. How much is the atmospheric pressure when the barometric reading is 74 cm.? How much is it when the barometer stands at 28.3 in.?

2. If a barometer were filled with a liquid one eighth as dense as mercury, how would its height compare with that of a mercurial barometer?

3. Compute the height of a barometer filled with water when the mercurial barometer stands at 74 cm.

4. What would be the height of a glycerine barometer (density 1.26 g. per cubic centimeter) when the pressure is 76 cm. by the mercurial barometer?

5. What would be the height of the barometer in the previous example if it were filled with sulphuric acid whose density is 1.84. g. per cc.?

6. When the pressure is 73 cm. the volume of a gas is 560 cc. What is the volume when the pressure is 77 cm.?

7. If the pressure on 120 liters of gas is 4000 g. per square centimeter, at what pressure will it expand to 360 cc.?

8. When natural gas was first discovered in Indiana, it came from the wells at an estimated pressure of 400 lb. per square inch. If it was delivered to the consumer at a pressure of 2 oz. per square inch, how many cubic feet for the consumer did one cubic foot at the well make?

9. A student in the laboratory took the following data in verifying Boyle's law: Barometer, 74 cm.; height of mercury above C (Fig. 97)

in tube F, 38 cm.; in closed tube A, 32 cm.; top of closed tube A, 44 cm. For the second trial the height of mercury in A was 34 cm. Find the height F for the second trial. *Ans.* 56 cm.

10. A given mass of air has a volume of 20 cc. when the barometer stands at 75 cm. What is its volume when the barometer reading is 73.6 cm., the temperature being 16° C. in both cases?

Ans. 20.38 cc.

XXII. BUOYANCY AND SPECIFIC GRAVITY

183. Buoyancy.—Any one who has lifted a large stone under water knows that it is lifted much more easily than when it is out of water. An expert swimmer can keep his body afloat with almost no effort. Evidently the water exerts a lifting force on a body placed in it. Air as well as water exerts a buoyant force on bodies immersed in it. This lifting force which a fluid exerts upon a body immersed in it is called *buoyancy.*

Experiment. — A *baroscope* consists of a hollow, air-tight globe suspended from a scale beam and balanced by a solid counterpoise at the other end of the beam (Fig. 102). Place the baroscope under the receiver of the air pump and exhaust the air from the receiver. When this is done, the equilibrium is destroyed, the counterpoise not being heavy enough to balance the hollow globe. This experiment teaches that the air exerts a buoyant force upon bodies immersed in it. The globe gains in weight when the air about it is removed; the same is true also of the counterpoise, but it does not gain so much as the globe. This experiment, therefore, teaches also that the larger the body, the greater the buoyant force of the fluid in which it is placed.

FIG. 102. — Baroscope.

184. Principle of Archimedes. — *The loss of weight of a body immersed in a fluid equals the weight of the fluid which it displaces.*

A cubic foot of water weighs 62.4 lb. Therefore, according to this principle, any body having a volume of 1 cubic foot loses 62.4 lb. in weight when immersed in water, or a body having a volume of 10 cc. loses 10 g. when immersed in water, because it displaces 10 cc. of water, which weigh 10 g.

185. Theoretical proof of the principle of Archimedes.— Suppose a cube *oc* (Fig. 103) to be immersed in water. The pressures on the opposite sides are evidently equal and hence neutralize each other, but the pressures on the top and bottom of the cube are not equal. The downward force on the top is equal to the weight of the column of water *soa*, the upward force on the bottom *rc* equals the weight of a column of water *rs* in height and having as its base the surface *rc*. Hence the upward force on the cube exceeds the downward force, and the difference between them constitutes the buoyant force. The difference between the two columns which cause the two forces is exactly equal to the volume of the cube itself, hence the force on *rc* exceeds that on *oa* by the weight of a body of water equal to the volume of the cube, or the loss in weight equals the weight of the water displaced.

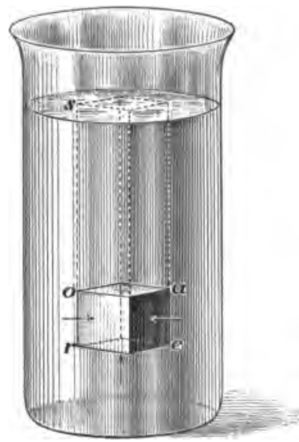

FIG. 103.—Diagram to illustrate the principle of Archimedes.

186. Experimental proof of the principle of Archimedes.— Use for this experiment a metal cylinder about 2 cm. in diameter and 3 or 4 cm. long. Measure its diameter and height accurately and calculate its volume by the formula, $v = \pi r^2 h$. Suspend the cylinder (Fig.

104) from a balance and weigh it. Then place a vessel of water under it and weigh it again while it is immersed in the water. It will weigh less in the water and the loss in grams will be found to equal its volume in cubic centimeters. Since 1 cc. of water weighs 1 g., the cylinder will displace as many grams of water as there are cubic centimeters in its volume.

FIG. 104. — Experiment illustrating the principle of Archimedes.

A cylinder of brass exactly 2 cm. in diameter and 3.5 cm. long has a volume of almost exactly 11 cc. Such a cylinder will be found to weigh 11 grams less in water than in air.

Problems Involving an Application of the Principle of Archimedes

1. A piece of silver sustains a loss in weight of 4.8 g. when weighed in water. What is its volume?

2. A piece of platinum loses 6.8 g. when placed in mercury. What is its volume? What loss in weight would it sustain when immersed in water?

3. A piece of iron loses 31.2 lb. in water. What is its volume?

4. How much buoyant force would be exerted on a block of marble having a volume of 6 cu. ft. when it is placed in water? When placed in a liquid twice as dense as water? In a liquid half as dense as water?

5. How much would a cylinder whose volume is 8.8 cc. lose in weight when immersed in a liquid weighing 1.8 g. per cubic centimeter? In a liquid whose density is 0.8 g. per cubic centimeter?

6. A body having a volume of 5 cc. is buoyed up by a force of 68 g. when immersed in a liquid. What is the density of the liquid?

7. The density of air under standard conditions (0° C. and 76 cm. pressure) is 0.00129 g. per cubic centimeter. What would a body whose volume is 800 cc. and whose weight in air is 1200 g. weigh in a vacuum?

8. A solid whose weight in air is 50 g. has a volume of 50 cc. What is its weight when immersed in water?

9. A piece of glass whose volume was 3 cc. lost 5.4 g. when immersed in a liquid. What was the density of the liquid?

10. A solid loses 15 g. when weighed in water. What is the weight of a body of water having the same volume as the solid?

187. Floating bodies. — It sometimes happens that the buoyant force of a liquid is greater than the weight of the solid submerged in it. In that case the weight of the solid is not sufficient to keep it under the surface of the liquid, and unless held in some way it will rise to the surface and float. A piece of iron, for example, weighing 78 g. would if submerged in mercury displace 136 g. of mercury; hence, according to the principle of Archimedes, the buoyant force would exceed the weight of the iron by 58 g. Therefore iron floats in mercury. When a body floats on a liquid, the buoyant force just equals the weight of the body. Hence, *a floating body displaces its own weight of the liquid in which it floats.*

As a consequence of this law it follows that for every ton of freight placed in a boat the boat sinks enough deeper in the water to displace a ton more of water.

Experiment 1. — Connect the stem of a large funnel by means of a rubber tube to a glass tube about 15 cm. long and support the apparatus in the position shown in Figure 105. Procure a large wooden ball such as a croquet ball and have at hand in a beaker an amount of water exactly equal in weight to the weight of the ball. Place the ball in the funnel with sufficient water to float it and mark the height in the tube at *a* by means of a rubber band. Then remove the ball from the funnel and pour in the water from the beaker. The water will again rise to the same height at *a* as before, thus showing that the ball displaced its own weight of water.

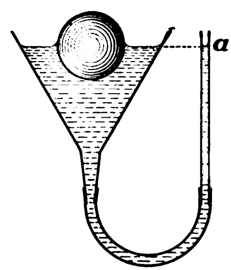

FIG. 105. — Diagram of an experiment to show that a wooden ball displaces its own weight of water.

Experiment 2. — Fill a glass pocket flask with water and place in it a small vial upside down. The vial should be nearly full of water but not quite, a small bubble of air being inclosed by the water (Fig. 106). The amount of air in the vial should be carefully adjusted so that it will barely float in the water. Close the flask with a stopper. If the amount of air in the

vial is properly adjusted, it can be made to rise and sink at pleasure by compressing the sides of the flask between the thumb and fingers.

FIG. 106. — Flask and vial experiment.

If it does not work well, small adjustments may be made by crowding the stopper farther into the flask or by loosening it slightly. There should be a little air in the flask. When the sides of the flask are compressed, the bubble of air in the vial is made smaller by transmitted pressure and hence the buoyant force is lessened; when the pressure is relieved the bubble expands and the vial rises.

This experiment illustrates several things: (1) the elasticity of glass, (2) Pascal's law of transmitted pressure, (3) the compressibility of air, (4) the elastic tension of air, and (5) the buoyant force of liquids.

188. Specific gravity.— *The specific gravity of a substance is a number which expresses how many times denser the substance is than some substance taken as a standard.*

Water at 4° C. is the standard substance for solids and liquids, and air at 0° C. and 76 cm. pressure is the standard for gases.

The density of iron is 486.72 lb. per cubic foot and of water 62.4 lb. per cubic foot. $486.72 \div 62.4 = 7.8$.

Again, in the metric system the density of iron is 7.8 g. per cubic centimeter and that of water is 1 g. per cubic centimeter. $7.8 \div 1 = 7.8$. Hence iron is 7.8 times as dense as water, or its specific gravity is 7.8.

Gold has a density of 11.16 oz. per cubic inch and water a density of 0.578 oz. per cubic inch. $11.16 \div 0.578 = 19.3$.

In metric units the density of gold is 19.3 g. per cubic centimeter and of water 1 g. per cubic centimeter. $19.3 \div 1 = 19.3$. Hence the density of gold is 19.3 times that of water, or the specific gravity of gold is 19.3.

According to the above definition and problems:

$$(1)\quad \textit{Sp. gr. of a substance} = \frac{\textit{density of that substance}}{\textit{density of the standard}},$$

and consequently

(2) *Density of a substance = its specific gravity × density of the standard.*

The specific gravity of a substance is therefore the *ratio* of its density to that of the standard.

It should be observed that the same number expresses both the density and the specific gravity of a substance provided the density is given in metric units. (Why?)

In the above problems the specific gravity has been obtained in each case by comparing the masses of equal volumes, as the mass of a cubic foot of the substance with the mass of a cubic foot of the standard, or the mass of one cubic centimeter of the substance with that of one cubic centimeter of the standard. It is not necessary, however, that the volumes compared shall be unit volumes, but only that the volumes shall be equal.

189. To find specific gravity of solids. — *First, a solid denser than water.* Dividing the weight of the solid by the weight of an equal volume of water gives its specific gravity. The principle of Archimedes affords an easy method for obtaining the weight of an equal volume of water, for by that principle the weight which a body loses in water equals the weight of an equal volume of water. Hence, the specific gravity of a solid is found by dividing its weight in air by its loss of weight in water. This is true whether the body is weighed in pounds, ounces, grams, or any other unit of weight. The specific gravity of the solid may be reduced to density by multiplying it by the density of water.

Second, a solid lighter than water. When a solid is lighter than water it is necessary to tie it to a sinker, such as a piece of lead, in order to weigh it in water. First weigh the sinker in water, then both together in water. Subtract the weight of the sinker in water from the weight of both in water; this will give the weight of the

solid alone in water. Since both together weigh less in water than the sinker alone, the weight of the solid alone in water will be a negative quantity. (What is the significance of the negative sign here?) To find the loss of weight of the solid in water subtract (algebraically) its weight in water from its weight in air. The specific gravity is then found as in the first case by dividing its weight in air by its loss of weight in water.

190. To find specific gravity of liquids.—*First, by use of a sinker.* Find the loss of weight of a piece of glass, such as a glass stopper, first in water and then in the given liquid. By the principle of Archimedes these two losses are the weights of equal volumes of the two liquids. Hence dividing the loss in the given liquid by the loss in water gives the specific gravity of the liquid.

Second, by use of a specific gravity bottle. This bottle has a glass stopper with a capillary bore extending through it so that the bottle can be filled and the stopper inserted without leaving any air bubble under it.

FIG. 107.—Hydrometer.

First weigh the empty bottle, then weigh it full of water, and lastly weigh it full of the given liquid. Subtracting the weight of the empty bottle from each of these weights gives the weights of equal volumes of the water and the given liquid, from which the specific gravity can be calculated as before.

Third, by the use of the hydrometer. This instrument (Fig. 107) consists of a glass tube somewhat enlarged in its lower part and terminating in a bulb which contains shot or mercury to cause the tube to float in a vertical position. Its action is based on the principle that a floating body displaces a weight of liquid equal to its

own weight. Hence the lighter the liquid the greater the depth to which the hydrometer will sink in it. The hollow stem contains a scale which starts at the point to which the instrument sinks in water. The scale is sometimes graduated so that the mark to which it sinks in any liquid indicates the density of that liquid directly, but often the scale is an arbitrary one, the value of its divisions being given in tables. The Beaumé scale, for example, is marked in degrees, and the densities corresponding to these degrees may be found in the *Encyclopædia Britannica* under "Hydrometer." The scale of a hydrometer for liquids lighter than water reads from the bottom of the stem up, and one for heavier liquids reads from the top down. A hydrometer intended for both light and heavy liquids has the beginning of its scale at the middle of the stem. A hydrometer intended for testing milk is called a *lactometer*, one for testing syrup is called a *saccharimeter*, and other names are given to the instrument according to the uses made of it.

Problems

1. A bottle when exactly full holds 23.7 g. of water or 19.9 g. of kerosene. What is the specific gravity of kerosene? Its density in grams per cubic centimeter? In ounces per cubic inch? In pounds per cubic foot?

2. An empty specific gravity bottle weighed 16.002 g. When full of water it weighed 42.900 g. and when full of hydrochloric acid, 46.099 g. What was the specific gravity of the acid?

3. The bottle of example 2 weighed 49.562 g. when full of nitric acid. What was the specific gravity of the acid?

4. A solid in air weighed 4.915 g.; in water, 4.482 g. What was its specific gravity? Its density? Its volume?

5. A piece of wax weighing 12 g. was tied to a piece of lead which weighed 8 g. in water. Both together in water weighed 6.7 g. What was the specific gravity of the wax?

6. A glass stopper which weighed 24 g. in air weighed 17.6 g. in water and 18.8 g. when immersed in alcohol. Find the specific gravity of the alcohol, also its density.

7. A solid weighs 80 g. in air and 72 g. in a liquid whose density is 0.84 g. per cubic centimeter. How many times denser is the solid than the liquid? What is the specific gravity of the solid?

8. A solid whose volume was 2 cu. ft. was supported by a rope in water. The tension on the rope was 499.2 lb. What was the specific gravity and the density of the solid?

9. A body in air weighed 10 oz., which is equivalent to 283.5 g., and in water it weighed 8.8 oz. or 249.48 g. Find its specific gravity, first by the means of English units, and then by the use of the metric units.

10. How is the density of a substance determined from its specific gravity? What is the specific gravity of water?

11. A piece of marble weighing 48.6 g. in air weighed 30.6 g. in water. Find its specific gravity and its density.

12. If the piece of marble mentioned in the last problem were suspended by a string so that one third of its volume is in water, what would be the tension on the string?

13. When a body whose specific gravity is 0.6 floats in a liquid whose specific gravity is 0.8, what fraction of its volume is below the surface of the liquid?

14. A body weighing 72 g. displaces 66 cc. of the liquid in which it floats. What is the density of the liquid?

15. A piece of lead weighing 60 g. in air weighs 54.7 in water and 55.6 g. in alcohol. Find the specific gravity of the lead and of the alcohol.

16. A flask holds 320 g. of a liquid having a specific gravity of 0.8. What is the capacity of the flask?

17. A hollow iron cube 30 cm. on each edge weighs 120 g. How much more ought it to weigh to sink in water?

18. A cube of gold 4 in. on each edge would weigh 44.6 lb. Find its specific gravity and its density. How much would a cube of water of the same size weigh?

19. A cylinder of wood floats in water with one third of its volume above the surface. What is the specific gravity of the wood?

20. A piece of wood whose weight in air is 12 g. and whose specific gravity is 0.64 is tied to a piece of lead which alone in water weighs 19 g. How much do the lead and wood when tied together weigh in water?

21. A piece of aluminum weighs 260 g. in air and 160 g. in water. What is its specific gravity? What is its density in English units? In metric units?

22. What is the weight of a wooden ball which, when placed in a cup full of water, causes an overflow of 36 g.?

23. How much will 8 cu. ft. of a substance weigh whose specific gravity is 11.3?

24. A body whose volume is 50 cc. weighs 525 g. What is its specific gravity?

25. The specific gravity of sulphuric acid is 1.84. A certain volume of it weighs 36.8 g., and an equal volume of another liquid weighs 272 g. What is the density of the other liquid?

26. A solid weighs two thirds as much in water as it does in air and three fourths as much in another liquid as in air. What is the specific gravity of the liquid?

27. What is the displacement in cubic feet of a boat which weighs 5 tons and which carries a load of 1600 lb?

28. If the density of air is 0.00129 g. per cubic centimeter when the barometer is at 76 cm., what will it be when the barometer is at 72 cm.?

29. When air is under a pressure of three atmospheres, what is its specific gravity and its density?

30. Why in the C.G.S. system of units are density and specific gravity numerically equal, and why not in the English units?

31. If the density of air at 76 cm. pressure is 0.00129 g. per cubic centimeter, what is its density at 68 cm. pressure?

32. If 0.094 g. of alcohol vapor would have a volume of 45.6 cc. under standard conditions, what are the density and the specific gravity of the vapor?

33. One liter of hydrogen under standard conditions weighs 0.0896 g. How many times denser is the vapor of alcohol if 42 cc. of it under standard conditions would weigh 0.087 g.?

34. How many times denser than hydrogen is the vapor of chloroform if under standard conditions 0.117 g. of it occupies 21.9 cc.?

CHAPTER II

WAVE MOTION AND SOUND

I. VIBRATORY MOTION

191. Vibratory motion. — When a body traverses any given path repeatedly and in regular intervals of time, its motion is *periodic.* The hands of a watch as well as its balance wheel, the pendulum of a clock, and the earth in its motion around the sun afford illustrations of *periodic motion.*

Vibratory motion is periodic motion in which the moving body is continually reversing its direction, tracing and retracing its path again and again. The motion of a clock pendulum is the most familiar illustration of vibratory motion.

Experiment. — Clamp a slender ruler to a table or in a vise (Fig. 108) and set it in motion by bending and suddenly releasing it. Its motion will be vibratory, because it is periodic and because it repeatedly reverses its direction.

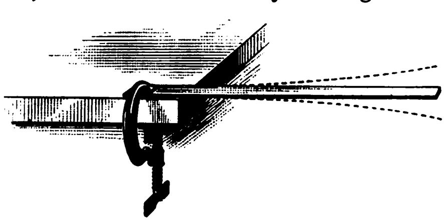

FIG. 108. — Transverse vibrations of a ruler clamped to a table.

Such vibrations are called *transverse* because the direction of the motion is *across* the length of the vibrating body.

Experiment. — Suspend a weight by a spiral spring or an elastic cord and set it in motion by pulling it down and releasing it. Its motion will be vibratory. Why?

Such vibrations are termed *longitudinal* because the direction of the motion is *along* the length of the vibrating body.

Experiment. — Suspend a heavy weight by a long fine piano wire (Fig. 109) and set it in motion by twisting it and releasing it. Such an apparatus is called a *torsion pendulum*. Its motion is vibratory. Why?

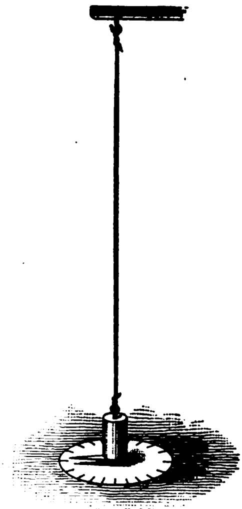

FIG. 109. — Torsion pendulum.

Such vibrations are called *torsional* because the direction of the motion is *around* the length of the vibrating body.

192. Definitions. — A *complete vibration* consists of the motion of a vibrating body from any given stage of its vibration until it reaches that same stage of vibration again. A pendulum, for instance, swinging in the arc *AB* (Fig. 110) makes a complete vibration in moving from *A* to *B* and back to *A* again, or in moving from *C* to *B*, back to *A*, and then to *C* again.

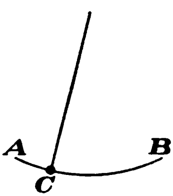

FIG. 110. — Diagram of the swing of a pendulum.

A *vibratory period* is the time required for a complete vibration. The *vibration frequency* of a body is the number of vibrations made by it in a unit of time. If a piano string, for instance, makes 200 vibrations per second, its frequency is 200 per second, and its period is $\frac{1}{200}$ of a second. *Amplitude* of vibration is the distance from the position of a vibrating body when at rest to its extreme point of motion. It is one half the arc through which an ordinary pendulum swings.

193. Simple harmonic motion is the most important form of vibratory motion. If a ship were to move in a small circle with uniform velocity, it would appear to an observer several miles away to be going back and forth in a straight line across his field of view. At one extremity of its apparent path it would seem to be stationary for a time because moving directly toward or away from the observer; then it would seem to increase its speed until at the center of the field of view it would be seen moving with its actual velocity. Its speed would then apparently diminish until at the other extremity of its apparent path it would seem stationary again. The apparent motion of the ship illustrates simple harmonic motion. A body which actually moves as the ship appears to move has simple harmonic motion.

Experiment. — Set the weight of the pendulum (Fig. 109) swinging in a circle. When swinging in this way, it is called a *conical pendulum* because its wire generates the surface of a cone. Place a lamp on a level with the weight of the pendulum and some distance away from it so that the shadow of the weight will be projected on the wall of the room. The shadow of the weight will have simple harmonic motion.

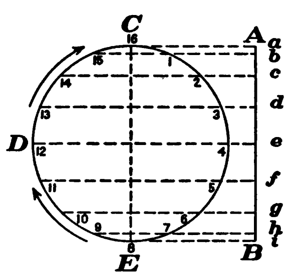

Fig. 111. — A circle of reference for simple harmonic motion.

The observer in viewing the ship projects its motion on the sky beyond. Let the circle *CDE* (Fig. 111) represent the actual path of the ship, and the line *AB* its apparent path, and suppose it to traverse the circle once every sixteen minutes. If the circle is divided into sixteen equal parts, 1, 2, 3, etc., and these points of division are projected on the line *AB* by drawing perpendiculars from them to *AB*, the points 1, 2, 3, etc., will represent the

actual position of the ship, and the points *a*, *b*, *c*, etc., its apparent position at each sixteenth of its period, or at each minute.

On the other hand, if a body is actually vibrating in the straight line *AB* with simple harmonic motion, this same construction will enable one to locate it at each sixteenth of its period. The circle used to locate the vibrating body is called the *circle of reference.* Its radius must equal the amplitude of vibration and its center must be in a line perpendicular to the line representing the path of the vibrating body at its middle point, that is, the center may be at *e* (Fig. 111) or any point in the line *De* or *De* produced. If the vibrating body is to be located at any other fraction of its period, as at each twelfth of its period, the circle must be divided into twelve equal parts instead of sixteen.

194. Isochronous vibrations. — A very important characteristic of simple harmonic motion is that the period of vibration remains the same whether the amplitude is large or small, that is, the period is independent of the amplitude. Vibrations whose period remains constant while the amplitude is changing are termed *isochronous.*

Simple harmonic motion is far from being uncommon or unusual. Air particles in communicating sound have this kind of motion; the particles of piano strings, of the prongs of a tuning fork, and in fact the vibrating parts of all musical instruments, vibrate in this way. Any body vibrating under the action of Hooke's law has simple harmonic motion.

Experiment. — Set the torsion pendulum (Fig. 109) in vibration with an amplitude of a few degrees, and by means of a watch determine its period.

Again, set it vibrating through a large amplitude by twisting once or more times around, and determine its period as before. Although the amplitude may be several times greater than at first, yet the period will be found to be the same. The vibrations are isochronous.

II. WAVES — WAVE MOTION

195. Wave form. — Suppose a series of particles which when at rest lie in the line *WX* (Fig. 112) to be vibrating each with simple harmonic motion, the first in the line AA', the second in the line BB', the third in the line CC',

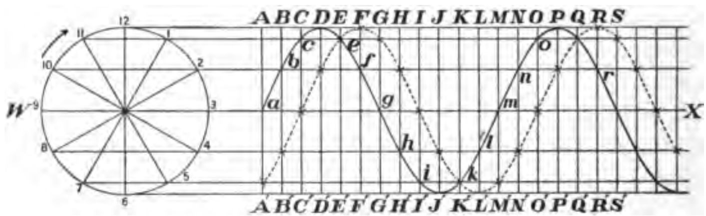

FIG. 112. — Diagram of a wave form.

etc. Suppose also that each is $\frac{1}{12}$ of a period behind the preceding one, the second $\frac{1}{12}$ of a period behind the first, the third $\frac{1}{12}$ of a period behind the second, etc., so that the particles pass consecutively through corresponding positions.

By means of the circle of reference which is divided into twelve equal parts each particle can be located at each twelfth of its period. Suppose the first particle is at *a* going down, being opposite the point 3, then the second will be at *b* opposite the point 2, the third at *c* opposite point 1, the fourth at *D* opposite point 12, the fifth at *e* going up opposite point 11, etc. A smooth curve drawn through these points gives a *wave form* or a *wave* (shown by the heavy line). It is obvious that if the particles all vibrated together instead of consecutively, or if they vibrated in a haphazard manner, such a wave form would not be produced. *A wave form is a configuration of a medium produced by the consecutive vibration of its parts.*

Wave forms similar to this are made when a rope or a carpet is shaken at one end; the particles *a*, *b*, *c*, etc., in

the figure may be supposed to represent the parts of the rope when such waves are formed in it.

Experiment. — Lay a soft cotton rope ten or twelve feet long on a table or on the floor, fastening one end of it to some firm object. Take the free end of it in the hand and without pulling it very taut move the hand up and down once and very quickly. A wave form quite similar to those shown in Figure 112 will pass along the rope.

Tie some pieces of colored twine on the rope at different places and repeat the experiment. It will be observed that each piece of twine makes an up and down motion as the wave form moves along. Notice that this up and down motion starts at the hand and is passed along from point to point in the rope, so that all the parts of it make this motion in succession or consecutively.

Give to the hand a continuous vibratory motion, moving it up and down rapidly and as regularly as possible. A series of wave forms or waves will now pass along the rope one after another. If you watch one of the pieces of twine, you will see that it is in continuous vibration. Each part of the rope evidently possesses energy which came from the hand and which it passes on to the next part of the rope.

196. Wave motion, as the last experiment shows, *is the progression or onward movement of a wave form.* When a wave passes over a field of grain, it is obvious that the grain itself does not move across the field, but that the wave form moves onward while each head of grain merely vibrates in the wind. When we watch water waves, we are apt to think that masses of water are moving onward, but it is no more true of the water than of the grain. One may convince himself of this by observing the motion of some floating object as the waves pass by it.

Wave motion is the result of a vibratory motion which is handed on from particle to particle of the medium. Thus *energy may be transferred by wave motion* from one place to another without the transfer of the medium itself.

197. Wave length. — When waves traverse a medium in succession one after another, they constitute a *train* of

waves, and the distance from any point in one wave of a train to the corresponding point in the next wave is called a *wave length*. Thus, from a to m (Fig. 112), or from D to P, or from f to r is a wave length; while from a to g or f to l is half of a wave length because the particles are in opposite stages of vibration; a, for instance, is moving down while g is moving up.

The points marked $\times$ show the positions that the particles will occupy $\frac{2}{12}$ of a period later than at first, and the wave form at that instant will have passed toward the right, occupying the position shown by the dotted line, the crest which was at D being at F. In $\frac{12}{12}$ of a period the particles will again be back in their original positions, the crest meantime having passed on from D to P. This illustrates a very important principle of wave motion; namely, *A wave always travels a wave length in one vibration period or while each particle is making a complete vibration.*

198. Relation between frequency, wave length, and velocity. — Since a wave travels one wave length in one vibration period, it will travel ten wave lengths while ten vibrations are taking place, or n wave lengths in the time n vibrations occur. Hence, if n equals the number of vibrations per second or the frequency and l the wave length, $n \times l$ will equal the distance passed over by the wave in one second, or the velocity v. Hence, in all wave motion

$$(1) \quad v = n \times l, \quad \text{or (2)} \quad l = \frac{v}{n}, \quad \text{or (3)} \quad n = \frac{v}{l}$$

From these equations it is evident (1) that wave length and frequency are inversely proportional when the velocity is constant, (2) that velocity and frequency are directly proportional when wave length is constant, and (3) that velocity and wave length are directly proportional when the frequency is constant.

Problems

1. What is the length of the sound waves produced in air by a piano string making 300 vibrations per second when sound travels 1140 ft. per second?

2. What must be the frequency of a string when the sound waves are 2.25 ft. long, the velocity being the same as in problem 1? When the waves are 1.9 ft. long? 7.6 ft. long?

3. In problem 1 if the frequency were 600 instead of 300 what would the wave length be? 900? 1200?

4. Waves of light travel through the ether of space with a velocity of about 300,000 Km. per second and some of the waves are 0.00006 cm. long. What is the frequency of the vibrations causing these waves?

5. When a tuning fork having a frequency of 256 produces sound waves 1.36 m. long, what is the velocity of sound?

199. Two kinds of waves.—There are two kinds of waves, *transverse* and *longitudinal.* The waves so far described, such as waves in a rope, are called transverse because each vibrating particle moves to and fro across the direction in which the wave is moving. Each one of such waves is composed of an elevation and a depression, or a *crest* and a *trough.*

The waves of the other kind are termed longitudinal because each particle of the medium vibrates to and fro along the direction in which the wave is moving. Each one of such waves is composed of a *condensation* and a *rarefaction,* not of a crest and a trough.

If you should set up a row of bricks with small spaces between them and tip over the first one, it would tip over the second, and the second would tip over the third, and so on to the end of the row. As each brick strikes its neighbor there would be a crowding together or a sort of a condensation which would be passed along the row in much the same way as the crest traveled along the rope, but

each brick would move longitudinally along the row, and not transversely as the parts of the rope did in our experiment.

Such an experiment with bricks would illustrate a longitudinal wave but very imperfectly, because a medium to transmit such waves must be elastic. If an elastic spiral spring were placed between each brick, the row might then transmit such waves quite well. Air is an elastic medium in which longitudinal waves occur, but as we cannot see them, the row of bricks may help us to form a mental picture of the behavior of the air particles when transmitting waves of condensations and rarefactions.

200. Longitudinal waves in air. — Let the first row of dots (Fig. 113) represent a series of air particles when they are at rest, and suppose the first one a to be driven

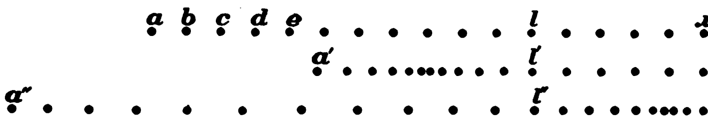

FIG. 113. — Relative positions of air particles in longitudinal waves.

toward x by a blow from some vibrating body, so that it goes as far as a' as shown in the second row.

Because of the elasticity of the air, as soon as a begins to move it will communicate motion to b, driving it toward c, and c will in turn act on d, and d on e, and so on. So that by the time a reaches the position a', the motion will have been transmitted to some distant particle, as l'. All the particles between a' and l' will be closer together than at first, thus forming a condensation which will continue to move on toward x.

But the elasticity of the air will cause a to bound back in the opposite direction and it will be followed in turn by b, b by c, and so on. So that by the time a reaches the

position a'', as shown in the third row, this backward motion will have reached as far as l''. All the particles between a'' and l'' will then be farther apart than at first, and thus a rarefaction will be caused which will follow the condensation toward x.

If blows are given continuously to a, so as to keep it vibrating, a continuous train of waves, each composed of a rarefaction and a condensation, will be sent through the row of particles toward x; and all the particles will be continuously vibrating back and forth in the direction ax. Such waves are very common about us, but we are not familiar with them because we cannot see them.

Experiment. — Support a tin tube in a horizontal position as shown in Fig. 114. The tube should be about 3 m. long and 8 cm. in diameter, the opening at one end being narrowed to about 2.5 cm.

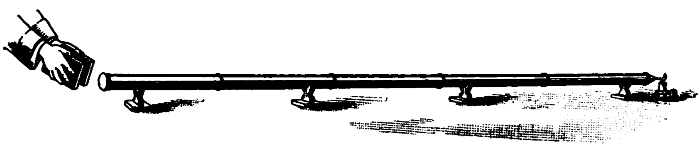

FIG. 114. — Tube for illustrating longitudinal waves in air.

Place a candle so that its flame shall be close to the small opening, and by means of a toy-pistol fire a percussion cap very near the larger opening. A quick forward and backward motion will be given to the candle flame almost instantaneously and possibly it will be extinguished. A condensation is transmitted by the air of the tube from one end to the other, a vibratory motion being handed on from air particle to air particle. That it is a vibratory motion, and not a current of air passing through the tube, is shown by the forward and backward motion of the flame. Again the quickness with which the motion traverses the tube proves that it cannot be a current of air causing it; for it would take about a tenth of a second for the disturbance to travel the length of the tube at 60 miles per hour. This action goes through the tube probably in less than one hundredth of a second or with a speed perhaps ten times greater than that of a tornado.

Tyndall used for this experiment two blocks instead of the pistol, clapping them together at the mouth of the tube; and, to show that it

was not a current of air passing through the tube, he filled it with smoke.

The action of the air in this experiment is analogous to the action of the parts of the rope in the experiment (§ 195); but in this experiment the motion is longitudinal, in that transverse.

201. Wave interference. — It often happens that two or more trains of waves traverse the same medium at the same time. If two trains of water waves, for example,

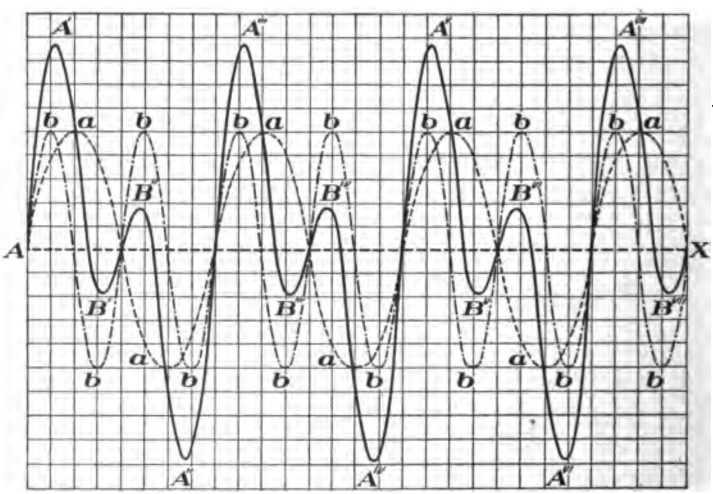

FIG. 115. — Diagram of a resultant train of waves formed from two trains of waves.

should meet so that the crests of one train coincide with the crests of the other train and troughs coincide with troughs, a new train of waves would be formed having higher crests and deeper troughs; but if the crests of one train should coincide with the troughs of the other, then the waves would tend to destroy one another. In like manner waves of condensations and rarefactions may act on each other so as to increase or diminish the amplitude

of vibration. This combining of two or more wave trains in the same medium, whether the result is an increase or a decrease of motion, is called *interference.*

We shall see later how two trains of sound waves may interfere so as to cause silence and waves of light may destroy one another so as to cause darkness. When interference occurs, each vibrating particle of the medium has a motion which is the resultant of the several vibratory motions imposed upon it, and the resulting wave form is often very complex in character.

In Figure 115 the heavy line represents a wave produced by the interference of the two wave trains represented by the dotted lines. Distances above the axis AX being considered positive and distances below it negative, the position for each particle in the resultant wave form is found by taking the algebraic sum of the distances of each particle from the axis in the component waves.

202. Stationary transverse waves.

Experiment.—Fasten one end of a rope 3 or 4 m. long to a stationary object and hold the other end in the hand. Send a trough along it by giving it near the hand a blow from above. Observe that this trough travels to the other end of the rope and is reflected, returning *as a crest.* Send a crest by giving a jerk upward and notice that it is reflected as a trough. The heavier and slacker the rope the more slowly the waves travel and the more readily are they observed.

Keep the hand in rapid vibration so as to send waves along the rope in rapid succession. If the motions of the hand are timed correctly, the waves going one way will meet those going the other way in such a manner as to cause the rope to be at rest at one or more places and to vibrate with considerable amplitude between these places of rest. All motion of the waves along the rope, although still really existing, will be masked, and it will appear merely to vibrate transversely between the points of no vibration.

NOTE.—A long brass spiral is much better than a rope for this experiment.

Such waves are called *stationary waves.* The places of

no vibration are called *nodes* and the places of greatest vibration are called *antinodes.* The part of the rope

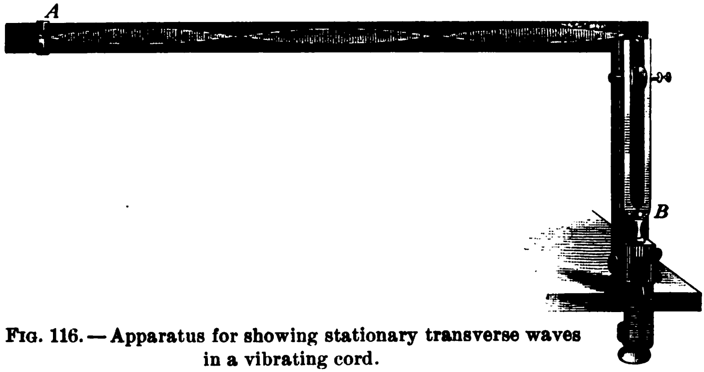

Fig. 116.—Apparatus for showing stationary transverse waves in a vibrating cord.

from one node to another forms a *loop* or *ventral segment.*

If the rope is replaced by a silk cord and the hand by the vibrating prong of a tuning fork or an electric vibrator, the phenomenon may be shown in a more beautiful manner (Fig. 116).

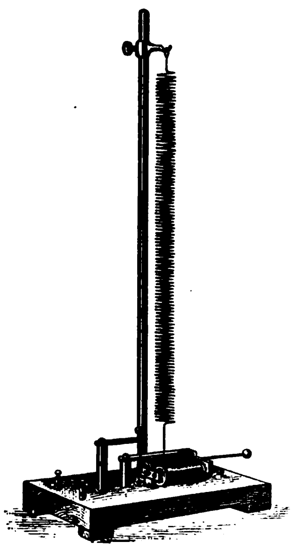

Fig. 117.—Apparatus for showing stationary longitudinal waves in a spiral spring.

203. Stationary longitudinal waves.

Experiment.—Support a light brass spiral spring about 1 m. long and 2.5 cm. in diameter in a vertical position and attach the lower end of it loosely to an electric vibrator (Fig. 117). The hammer of an electric bell which has been weighted to reduce its frequency and provided with stops to control its amplitude, may be used for the purpose. It

may be necessary to adjust the spiral to the frequency of the vibrator in a way analogous to the adjustment of a resonant air column to a fork. This may be done, not by stretching the spring more or less, but by changing the number of turns of wire of the spiral in use.

When adjusted, stationary longitudinal waves will be formed with nodes and antinodes in the spiral. These are caused by condensations and rarefactions traveling up and down the spiral as the troughs and crests traveled along the rope.

III. SOUND. — ITS ORIGIN AND TRANSMISSION

204. Sound *is the form of vibratory motion capable of being perceived by the auditory nerves.*

NOTE. — The definition of sound just given is that used in the science of physics; but sound may also be defined as a *sensation* which is reported to the brain by the auditory nerve. The physiologist and the psychologist who study the phenomena of sense perception would use the latter definition. According to the first definition sound exists independently of the ear, but according to the second one there is no sound when there is no ear to hear; the vibrations which the physicist calls sound may exist, but the sensation which the physiologist calls sound cannot exist under such circumstances.

205. The origin of sound. — *Sound originates in a vibrating body.* It may be caused by the vibrations of a solid, a liquid, or a gas.

By placing a light piece of paper upon a piano string or on that of any stringed instrument, it is easy to see that the string is always vibrating when it is sounding. In wind instruments, such as a flute, a whistle, an organ pipe, or a cornet, the sound is caused by vibrating air.

Experiment 1. — Suspend a small light ball such as a pith ball, or best of all a small hollow glass ball, by a thread so that it just touches the edge of a bell (an inverted bell jar or a glass bowl may be used). Cause the bell to sound by bowing it with a violin bow or by striking it with a rubber mallet made by thrusting a wooden rod through a rubber stopper. The motion of the ball will make the vibrations of the bell evident.

Experiment 2.—Repeat the last experiment, using a tuning fork instead of the bell. If the fork is sufficiently large, suspend the ball by a short thread between its prongs.

Experiment 3.—Clamp a large glass tube about 1.5 m. long exactly at its center (Fig. 118) and suspend a hollow glass ball or a wooden

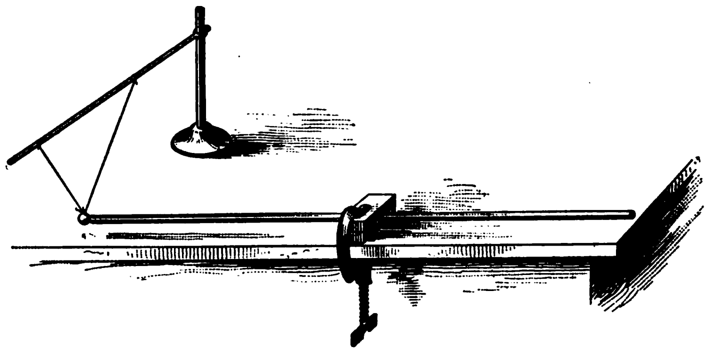

FIG. 118.—Apparatus for showing longitudinal vibrations in a sounding tube.

one so that it rests lightly against the end of the tube. Stroke the tube lengthwise with a damp woolen cloth. A loud tone will be produced and the ball will be thrown violently away from the tube. This shows that the glass of the tube is in vibration while it is sounding. If the tube is wet inside, the water will be gathered into little ridges by the vibrations. (Read Tyndall's *Sound*, page 194.) A metallic rod about a centimeter in diameter and 1 m. long may be used in place of the glass tube for this experiment, but it must be stroked with a cloth covered with resin dust.

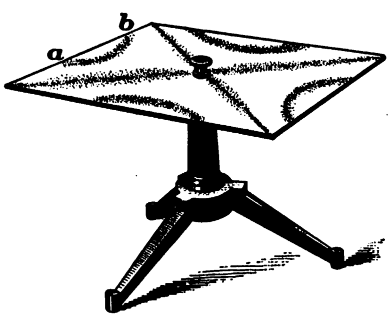

FIG. 119.—Chladni plate.

206. Chladni Plates and Figures.—Experiment.—For this experiment a square brass plate of uniform thickness is best, but a round plate and one of glass may be used. It should be about 2 mm. thick and 30 cm. square and

clamped exactly at its center, or it may be fastened to an upright standard by a screw through its center (Fig. 119). Scatter fine sand over the plate and bow it on one edge. A clear musical tone will be produced, and the dancing motion of the sand will show that the plate is in vibration. The sand will drift away from the parts of it in motion and collect along lines of no vibration, called *nodes*, which divide the plate into an even number of symmetrical parts.

By applying the thumb and finger at the points *a* and *b* and bowing the plate at the middle of an adjacent side, the figure shown here may be produced. A great variety of beautiful figures may be produced by touching the edge of the plate with the finger at different points while the bow is applied at some other point.

207. Vibrations in Gases.—Experiment 1.—Insert a whistle (Fig. 120) in one end of a glass tube containing a small amount of fine dry

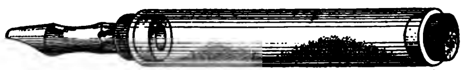

FIG. 120. — Apparatus for showing vibrations in sounding air.

cork dust made by rubbing a baked cork on a piece of sandpaper. When the whistle is held in a horizontal position and blown, the cork dust will rise in gauzy parallel partitions transverse to the length of the tube. When the sound ceases, the dust falls, forming little parallel ridges. The action of the dust shows that the air of the tube is in vibration while it is sounding.

Experiment 2.—*The Singing Flame.* Place some shot or sand in the bottom of a wide-mouthed bottle to give it stability and close it with a two-hole rubber stopper. Through one hole pass a glass tube *a* (Fig. 121) bent at

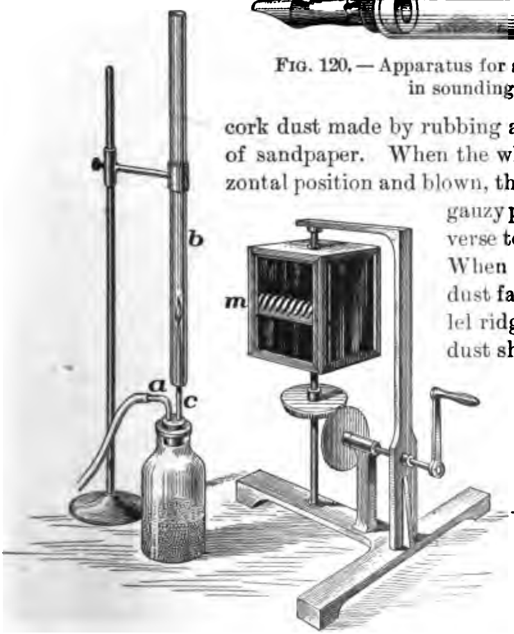

FIG. 121. — Apparatus for showing the vibrations of a singing flame.

right angles, to which a gas tube may be connected, and through the other pass a jet tube *c*. Allow the gas to flow until the air has been expelled from the bottle, and then light the jet at the tip of *c* and support over it a glass tube *b*. When the flame is about $\frac{1}{4}$ of the way up, if the conditions are right, the jet and the air of the tube will be thrown into vibration and a loud musical tone be produced.

The vibrations of the flame are made visible by viewing its image in a revolving mirror *m*. When the flame is not vibrating, its reflection will be drawn out into a smooth band of light; but when it is vibrating, this band will be broken up into separate tongues of flame.

Note. — Were it not that success in this experiment depends on the relative lengths of jet tube and sounding tube the bottle would be unnecessary. The jet tube from its tip to its lower end inside the bottle should be either less than half the length of the sounding tube or longer than the sounding tube. For small sounding tubes, 1 to 1.5 cm. in diameter and 30 to 50 cm. long, the opening of the jet tube should be small enough to produce a flame about 2 cm. long when the gas is turned on with full force. Larger tubes require somewhat longer flames.

208. Transmission of Sound. — Sound is always transmitted from one place to another by waves in some medium. The sounding body by its vibrations sets up in the surrounding medium trains of waves which carry the energy of the sounding body away. These waves are always longitudinal, being composed of condensations and rarefactions, never of troughs and crests.

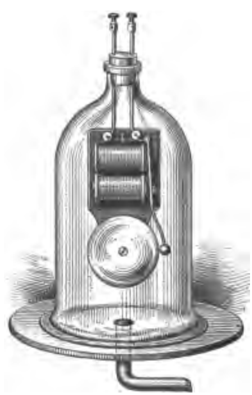

Fig. 122. — Apparatus for showing that sound is not transmitted in a vacuum.

Air is the ordinary medium for the transmission of sound waves, but other gases and solids and liquids also transmit them.

Experiment. — Suspend an electric bell in the receiver of an air pump (Fig. 122), and while it is ringing exhaust the air

from the receiver as thoroughly as possible. The sound will be less and less audible as the exhaustion increases, but it can never become entirely inaudible, partly because a perfect vacuum is unattainable, but mainly because the supports of the bell will transmit some of the sound to the outside air.

After exhausting the air, readmit it by small amounts at a time. At each addition of air the sound will be distinctly louder.

This experiment shows that air or some material medium is necessary to communicate sound.

Experiment. — Insert the stem of a tuning fork in a small wooden disk and rest this upon the surface of water in a tumbler. The sound of the fork, which at first is inaudible at a distance, becomes so when the disk touches the water. The experiment succeeds best when the tumbler rests upon a resonance box of some sort, such as that of a guitar or a violin. The vibrations are transmitted through the water to the box and by the box to the air and thus to the ear.

It is a familiar experience with bathers that water can transmit sound, for when the ear is placed under water, a light tapping under water at a distance can easily be heard. It is said that the approach of a steamer can be heard in this way when it is a mile or two distant.

Experiment. — Place one end of a long pole against the panel of a door and press the stem of a sounding tuning fork against the other end. The vibratory motion will be transmitted by the pole to the door and thence by the air to the ear. By touching the pole and also the panel of the door lightly by the finger tips the vibrations may be felt.

Waves of condensations and rarefactions pass along the rod just as they do through air, but the amplitude of vibration of the wood particles is very small.

209. Form of sound waves in air. — When a pebble is dropped into still water, it starts a wave which travels outward in all directions on the water surface so that it becomes an expanding circle. If pebbles could be dropped at regular intervals at the same spot, a continuous succession

of expanding circular waves would travel outward from the same center, forming a train of circular waves.

In a similar manner, when a body such as a tuning fork vibrates, it causes a train of expanding waves in air. These waves, however, are not confined to a surface as the water waves are, but they expand in all directions so that instead of being circles they are *spheres*, and they are composed of rarefactions and condensations instead of troughs and crests.

Sound waves in air are therefore expanding spherical shells, each shell of condensed air being followed by one of rarefied air. The exterior surface of the shell may be called the *wave front*. All the air particles about the sounding body vibrate longitudinally along the radii of the sphere or in lines perpendicular to the wave front, and the waves enlarge on all sides one wave length during each vibration period.

IV. VELOCITY OF SOUND

210. Velocity of sound in air. — A little observation will teach us that it takes time for sound to travel. Thus, the flash of a gun, the fall of a hammer, or the steam from a whistle at a distance may be seen before the sound is heard ; likewise there is usually an interval of several seconds between a flash of lightning and the thunder which follows it.

Much careful experimentation has given 331.4 *m.*[1] *or* 1087 *ft. per second as the velocity of sound in air at* 0° *C.* This is the velocity of all sounds of ordinary loudness. Very loud sounds travel somewhat faster than this. Roughly speaking, sound may be said to travel at the rate of 1100 ft. per second, or about one mile in 5 seconds.

[1] Violle and Vautier give 331.36 m. per sec.

211. What the velocity of sound depends on. — The velocity of sound in any medium depends on the elasticity and density of that medium.

It varies directly as the square root of the elasticity and inversely as the square root of the density of the medium. It follows from this that sound will travel four times as fast in hydrogen as in oxygen because under the same pressure, or with the same elasticity, oxygen is 16 times as dense as hydrogen. Temperature affects the velocity of sound because it may change either the elasticity or the density of the medium. When uninclosed air is heated, it expands, becoming less dense, while the pressure and consequently the elasticity remains unchanged; hence, as the temperature rises the velocity of sound increases. A rise in the temperature of the air of 1° C. causes an increase in the velocity of sound of 0.6 m. or nearly 2 ft. per second. For example, the velocity of sound at 16° C. is $331.4 + (0.6 \times 16) = 341.0$ m. per second.

Problems

1. What is the velocity of sound in air at 20° C.? At 22° C.? At – 9° C.?

2. What is the velocity of sound in air at 11° C.? At 25° C.?

3. Carbon dioxide is 1.53 times as dense as air at the same temperature and pressure. What is the velocity of sound in it at 0° C.?
Ans. 267.9 m. per sec.

4. What is the velocity of sound in carbon dioxide at 22° C.?

5. What is the velocity of sound in hydrogen at 0° C., the specific gravity of air on a hydrogen basis being 14.43?

6. At what temperature does sound have a velocity in air of 1000 ft. per second?

7. At 18° C. what is the wave length produced by a sounding body whose frequency is 435 per second?

8. What change of frequency occurs in the sound of an organ pipe whose wave length remains constant at 4 ft. when the temperature changes from 5° to 21° C.? *Ans.* Increase of 8 per second.

9. At 18° C. the velocity of sound in a gas is 360 m. per second. What is the specific gravity of the gas, air being the standard?

10. According to Boyle's law, when a gas is compressed, its elasticity and its density are increased at the same rate. What will be the effect on the velocity of a sound if the gas through which it is passing is compressed?

212. Velocity of sound in solids and liquids. — Although the greater density of solids and liquids would tend to make the velocity of sound in them less than in air, it is much greater. In water it is more than 4 times as great as in air, while in some solids, as glass and steel, it is more than 15 times as great. The greater elasticity of the solids and liquids more than counteracts their greater density.

213. Reflection of sound. — When sound waves strike the surface of another medium differing in density from the one in which they are traveling, they are reflected. To reflect sound well, however, the reflecting surface must be large and the irregularities of the surface must be small in comparison with the wave lengths. The wall of a building, a hillside, a bank of forest trees, and even gases and clouds may reflect sound.

An *echo* is a reflected sound separated by an interval of silence from the original sound. The nerves of the ear do not recover from the effect of a sound instantaneously, but the sensation of sound lasts about one tenth of a second after its cause ceases. Consequently if the reflected sound reaches the ear within one tenth of a second after the original sound is heard, it will seem to be a prolongation of that sound. Sound travels about 33 meters or 110 feet in a tenth of a second; hence, the reflecting surface must be more than half that distance away (since it travels the distance twice) to render the reflected sound separate or distinct from the original

sound. The walls of a vacant room or an auditorium often reflect sound repeatedly from one wall to the other, sometimes making it impossible for a speaker to be understood. The presence of furniture or an audience may interfere with the reflection or destroy it. The acoustic properties of a hall are mainly determined by the way the sounds are reflected from its walls and ceiling.

Illustrations of the reflection of sound are seen in the ear trumpet, the megaphone, and the curved surface of the external ear which concentrates the sound waves to the auditory canal by reflection. The laws of reflection, which are the same for sound and light, are much more easily studied under the latter subject.

V. RESONANCE

214. Vibrations communicated to other bodies. — When sound waves strike a body, some of the energy of vibration, instead of being reflected, may be used in setting the body against which the waves strike in vibration. Examples of this are seen in the rattling of windows by the thunder, and often in the vibration of some object of a room whenever some particular note is given by a piano or an organ. The bodies set in vibration by the waves are said to *absorb* the energy of vibration.

Sometimes vibrations are communicated from one body to another more directly than by waves through the air. For example, the vibrations of the strings of a piano or of a violin are transmitted to the sounding-board of the instrument by means of the bridges upon which the strings rest.

Experiment. — Suspend two pendulums having heavy bobs from the same support, making them of equal length, and set one of them in vibration. The supporting frame, if not too rigid, will transmit the vibratory motion from one pendulum to the other and the second

pendulum will absorb nearly all the motion of the other. In a few moments the first pendulum will possess the motion again and so the energy of vibration will pass back and forth from one pendulum to the other.

Next change the length of one of the pendulums and set one of them in vibration. Motion will still be communicated from the first pendulum to the second, but not so well as before. In either case the vibrating pendulum *applies force periodically* to the other through the medium of the supporting frame and sets it in vibration.

215. Forced vibrations *are those set up in a body by the application of a periodic force which is usually if not always communicated to it from another vibrating body.* In all the cases mentioned in the preceding paragraph, force is applied to one body at regular intervals of time, or periodically, by another, to set it in vibration, and each example given illustrates forced vibrations.

To swing a person or to set a large church bell in motion easily, we know that the pushes or pulls must be applied at certain regular intervals of time. If the intervals agree with the natural period of the swing or the bell, small forces may produce wide amplitude of vibration, since the effect is cumulative. It is for this reason that soldiers, when crossing large bridges, break step, lest their step, agreeing with the vibratory period of the bridge, should cause such amplitude of vibration as to destroy it.

216. Resonance consists in the production of forced vibrations in a body which has naturally the same vibratory period as the force causing the vibration, or in the communication of vibrations to one body from another with which it agrees in period and frequency. The experiment given in § 214, when the two pendulums are of the same length, illustrates mechanical resonance. The examples given in the last paragraph also illustrate resonance if the pushes on the swing, the pulls on the bell, or the tramp of the soldiers' feet agree respectively

with the natural period of the swing, the bell, or the bridge. The body which is set in vibration by the periodic force is termed a *resonator*.

Experiment. — Tune two strings of a guitar, bass viol, or sonometer to exact unison and pluck one of them. The other string will be seen to be in vibration, and its sound may be heard if the vibrations of the first one are stopped. The motion of one string is transferred to the other by the air and by the common supports of the strings. The second string is a resonator for the first.

Experiment. — Place two tunning forks mounted on resonance boxes and having the same frequencies some distance apart (Fig. 123). Bow one of them vigorously, allowing it to sound for a few seconds, and then stop its vibrations. The other fork will be found to be vibrating, giving forth sound. Success in this experiment depends upon the exactness with which the two forks agree in frequency. A slight difference, even that caused by warming one of the forks, will prevent resonance.

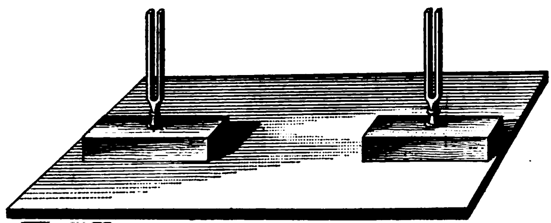

FIG. 123. — Tuning forks arranged for illustrating resonance.

These two experiments illustrate resonance. Forced vibrations in the case of resonance are sometimes termed *sympathetic* vibrations.

The principle of resonance is of very great importance, not only in sound, but also in other branches of physics, especially in electricity and light.

217. Resonance in air. — Bodies of inclosed or partially inclosed air are most excellent resonators. This is shown in the case of the steam whistle, the organ pipe, the flute, and other wind instruments. In all such instruments some small vibrating body, as a reed or a thin jet of air, which of itself produces sound waves too small and indefi-

nite to be heard, sets the air of the resonator in vibration, and the sound is greatly reënforced.

Experiment. — Support a large tin tube, about 1.5 m. long and 8 cm. in diameter, in a vertical position and gradually raise a rose burner which has been mounted on a long tube up into it. The jets of burning gas will set the air of the tube in vibration by resonance, and a very loud sound will result. The tube with its inclosed air is a resonator.

Experiment. — Hold a vibrating tuning fork over a deep cylindrical jar (Fig. 124). At first it is probable that little or no sound will be heard, but if water is poured carefully into the jar, thus changing the length of the air column, the sound will increase in intensity until at a certain depth it will burst forth with considerable power. When the air column is shortened still more, the sound quickly dies away and again becomes inaudible.

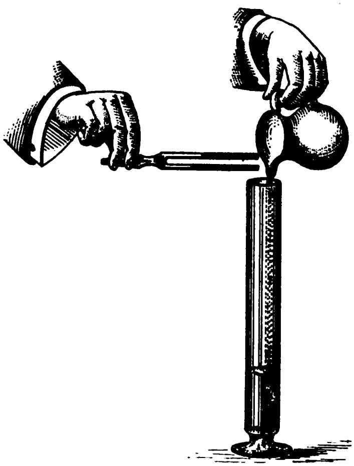

FIG. 124. — Making an air resonator.

In this experiment the volume of inclosed air becomes a resonator. As the column changes in length the rate of vibration of the air particles changes until it finally agrees with that of the fork, and thus resonance is produced.

218. The air resonator explained. — The prong *a* (Fig. 125) of the fork in moving from *b* to *c* produces a condensation below it and a rarefaction above it or between the two prongs. The condensation starts down the tube at the instant the prong starts toward *c* and, if the air col-

umn is of the proper length for resonance, it travels to the water, is reflected, and returns to the mouth of the tube by the time the prong reaches *c*. In the same manner while the prong is moving back to *b* a rarefaction travels down the tube and back again. Thus the prong applies force to the air column at exactly the right intervals to keep it in vigorous vibration and cause resonance.

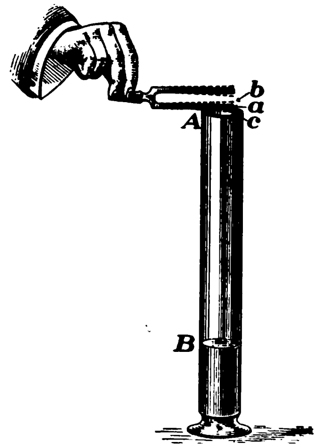

FIG. 125. — Diagram showing how resonance is produced.

Since a wave travels one wave length in one vibration period, and since the condensation travels down the tube and back again while the fork makes half of a vibration, down the tube and back again is equal to half of a wave length. Hence the length *the air column AB is one fourth of a wave length.* In the experiment with the tin tube (§ 217), which is open at both ends, the air column is one half of a wave length.

Resonance also occurs when the length of the air column is three fourths of a wave length. In that case the distance down the tube and back again is one and a half wave lengths and the reflected condensation returns to the fork at the instant the prong reaches *c* a second time.

Similarly there is reënforcement if the column is $\frac{5}{4}$, $\frac{7}{4}$, etc., wave lengths long, but the resonance is best at $\frac{1}{4}$ of a wave length and weakens as the column lengthens.

219. Velocity of sound by resonant air column. — The length of a resonant air column is affected by its diameter, so that it is only approximately equal to one fourth of a wave length. According to Lord Rayleigh about 0.7 of the radius of the tube should be added to the length of the column to obtain the true quarter wave length.

When this and the frequency of the fork are known, the velocity of sound may be determined as follows:

Problem

A fork having a frequency of 256 required at 16° C. an air column 31.8 cm. long for resonance, the diameter of the tube being 4 cm. What was the velocity of sound?

SOLUTION

2 cm. × 0.7 = 1.4 cm., correction for diameter; 31.8 cm. + 1.4 cm. = 33.2 cm. quarter wave length; 33.2 cm. × 4 = 132.8 cm., whole wave length; 132.8 cm. × 256 = 33996.8 cm. = 340 m., the velocity of sound (§ 198).

The velocity of sound at 16° C. (§ 211) is 341 m.

VI. INTERFERENCE OF SOUND

220. Interference of sound waves. — It has been shown (§ 201) how waves may cause interference. The subject of interference is of great importance in the study of sound, which affords many interesting and beautiful illustrations of it.

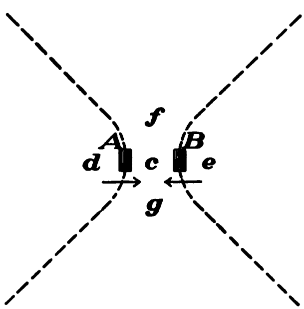

FIG. 126. — Diagram of interference of sound waves around a tuning fork.

The two prongs of a tuning fork always move in opposite directions when vibrating, going toward and then away from each other. Let *A* and *B* (Fig. 126) represent the ends of the prongs of a fork. When they are moving toward each other, a condensation is produced between them at *c* and a rarefaction on the outside at *d* and *e*. Thus a condensation starts at *c*, expanding out into the spaces *f* and *g*, while rarefactions at the same time expand outward

in the spaces *d* and *e*. When the prongs move away from each other, the opposite of this occurs. The dotted lines represent the regions about the fork where condensations and rarefactions border upon and nearly, if not quite, destroy each other. By twirling a sounding fork near the ear these four regions of interference may be easily detected.

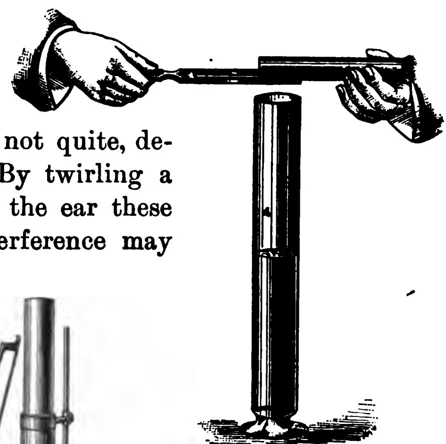

Fig. 127. — Interference of sound waves from the prongs of a tuning fork prevented by a tube held over one of them.

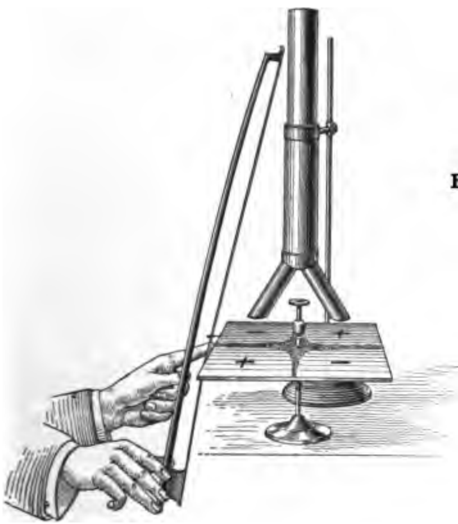

Fig. 128. — Experiment illustrating interference of two sets of sound waves from a Chladni plate meeting in a resonance tube.

Experiment. — Hold a tuning fork over a resonant air column (Fig. 127) and turn it over slowly. When the fork is oblique so that both prongs act at the same time on the air of the jar, a position will be easily found where the resonance will be destroyed by the interference of the two sets of waves. If while the fork is in this position a cylinder of paper is thrust over one prong so as to keep the waves produced by it from entering the jar, the resonance caused by the waves of the other prong will begin again.

Experiment. — Procure a tin tube like that shown in Fig. 128. This

tube should be about 5 cm. in diameter and 30 cm. long to the point where the branches are attached. Another tube should telescope into the upper end of this tube so that its length may be adjustable.

Support the tube in a vertical position with the branches over a Chladni plate, vibrating in four segments as shown in the figure, and adjust the length of the tube so that resonance occurs. Turn the tube so that its branches are over adjacent segments, those marked + and –, and the resonance will be destroyed by interference. The diagonal segments always vibrate in the same direction and hence both send condensations up the tube at the same time; but adjacent segments always vibrate in opposite directions so that one sends a condensation upward at the same time the other sends a rarefaction. These meet in the larger part of the tube and destroy each other.

The same result may be obtained by using a tube of large diameter without branches. If such a tube is held over one segment only, resonance will occur; but if it is held so that half of it is over one segment and half over the adjacent segment, the resonance is destroyed. It may be restored by placing a card between one segment and the opening of the tube, thus allowing waves from one segment only to enter the tube.

221. Beats.—*Regular throbbing alternations in the intensity of a sound are called beats.*

Experiment. — Cause two forks of the same frequency to sound together. The two sounds will blend into one smooth tone. Cut a hole in a small rubber eraser so that it will fit snugly on one of the prongs of one fork and slide it down to about the middle of the prong. This will cause the fork to vibrate at a slower rate. Now when the two forks are sounded together, they produce a throbbing or pulsating note. If the eraser is moved toward the end of the prong, these throbs or pulsations will be more frequent. If it is pushed down near the base of the prong, the throbs may be so few as scarcely to be heard. These regular throbbing pulsations of sound are beats.

222. Beats are caused by interference. — At regular intervals the two forks of the last experiment vibrate together, both sending out a condensation or a rarefaction at the same instant, so that like phases of the two wave trains coincide and the intensity of the sound is increased.

At other times the forks are in opposite stages of vibration, so that when one sends out a condensation, the other is producing a rarefaction; and opposite phases of the two wave trains coincide, and the intensity of the sound is weakened.

Experiment. — Prepare two singing flames (Fig. 129) as described in § 207, making one sounding tube about 5 cm. shorter than the other. Fit to the shorter one a sliding paper tube so that the sounding tube may be lengthened at pleasure. Adjust the tubes to the same length, and the notes produced will unite into one smooth tone. But when one tube is shortened, beats will be produced. The more the tubes differ in length the more rapid the beats.

FIG. 129. — Singing flames arranged to produce beats.

223. The number of beats per second. — Suppose that two vibrating bodies which have different frequencies start together. Presently, as one gains on the other, they will be vibrating in opposite phases, and the two sounds will weaken each other; but when one has gained a whole vibration, they will be in like phase again and the two sounds will strengthen each other. It is evident, therefore, *that a beat occurs as often as one body gains a vibration on the other.* The number of vibrations gained per second must equal the difference between the frequencies of two bodies, hence *the number of beats per second must also be equal to the difference between their frequencies of vibration.*

224. Discord or Dissonance.—Two notes which are disagreeable to the ear when sounded together are said to be *dissonant*, and they form a discord. Two notes of different frequencies that are agreeable to the ear are said to be *consonant*. Beats are the chief cause of dissonance. The preceding experiments show that they may be very unpleasant. If the number of beats per second is very small, the dissonance is not very marked; but as the beats increase in number the effect becomes more and more unpleasant, until a maximum is reached. From this point on the unpleasantness decreases with an increase in the number of beats until, with a large number of beats per second, two notes become consonant.

VII. CHARACTERISTICS OF SOUND—INTENSITY

225. The characteristics of sound are (1) *loudness*, or *intensity*, (2) *pitch*, and (3) *quality* or *timbre*.

Loudness refers to the degree or amount of the sensation produced in the ear; while the term *intensity*, as applied to sound, refers to the energy of the vibratory motion producing the sensation. Since in physics sound is defined as a vibratory motion, and not as a sensation, and since sensations cannot be measured with definiteness, we shall discuss intensity of sound and not loudness.

Pitch is the characteristic of sound described by the terms *high* and *low* or *acute* and *grave*. Thus, the voice of a soprano singer has a high pitch and that of a bass singer a low pitch, and the keys at the right end of a piano give notes of high pitch, and those at the other end a low pitch.

Quality is the characteristic of sound by which we distinguish different notes from one another, even though they be of the same pitch and intensity. Thus, we distinguish the voices of our friends or the notes of different musical instruments by their quality, not by their loud-

ness or their pitch. When we describe a singer's voice by such adjectives as pure, sweet, rich, thin, harsh, etc., we are describing its quality.

226. Intensity of sound depends on (1) the area of the vibrating body, (2) the density of the medium in which the sound is produced, (3) the amplitude of vibration, and (4) the distance from the sounding body.

227. Effect of area.—A thin body, such as a violin string, when it vibrates, cannot set up well-defined and strong waves in air because the air particles slip around it. It is for this reason that a resonance box or a sounding-board is necessary to many musical instruments. In the violin, for example, the vibrations of the string are transmitted to the sonorous body of the instrument, which in turn transmits the sound to the air. In like manner a tuning fork when held in air is scarcely audible, but when its stem is pressed against the table, the table itself is set in vibration, and because of its greater area gives forth a sound of greater intensity.

228. Effect of density.—The experiment of an electric bell placed under the receiver of an air pump (§ 208) shows that the less dense the air about the bell the less intense the sound produced by it. Observe that in this experiment the change of density occurs in the air about the bell, not in the medium about the hearer of the sound, nor, in the main, in the transmitting medium. If the receiver is filled with hydrogen or some other gas lighter than air, the effect is the same as when the air is rarefied. This effect of rarefied air on the intensity of sound is strikingly illustrated upon the tops of high mountains, where the report of a rifle sounds little louder than the breaking of a twig.

The reason for this is that the mass of air or gas set in motion at each vibration of the sounding body is less in a

rare medium than in a dense one, and consequently less energy is imparted to the medium at each vibration.

229. Effect of amplitude. — *The intensity of sound varies as the square of the amplitude of vibration.*

It has been shown (§ 194) that the period of vibration is independent of the amplitude. This means, for instance, that if the amplitude is made twice as great as at first, the body completes its vibration in the same time as before, and consequently it must move twice as fast to do it. In other words, the velocity of vibration must vary directly as the amplitude. But we have learned (§ 92) that the energy of a moving body varies as the square of its velocity. Therefore, since the velocity varies as the amplitude and the energy varies as the square of the velocity, the energy of vibration or intensity of sound varies as the square of the amplitude of vibration.

230. Effect of distance. — *The intensity of sound varies inversely as the square of the distance from its source.* This is an example of the law of inverse squares. It has been shown that sound waves are expanding spherical shells. As a consequence of this, the energy of sound which is carried outward by the wave is distributed among more and more particles as the wave advances, and the share of each particle decreases as the number of particles increases.

Geometry teaches that the surfaces of spheres vary as the squares of their radii ; hence, the surface of a sound wave increases, not at the same rate as its distance increases, but as the square of its distance increases. Obviously the number of particles composing the surface of a sound wave must increase at the same rate. Therefore, since the number of particles increases as the square of the distance and the energy of each particle decreases as the number of particles increases, the energy or

intensity of sound varies inversely as the square of the distance from its source.

This is the theoretical law. In reality the energy of sound is converted into the energy of heat as it is transmitted outward, and consequently the intensity decreases at a greater rate than is stated in this law. Of course, if the waves are prevented from expanding in the form of spheres, as they are by speaking tubes and to some extent by the megaphone, the intensity of the sound will decrease at a less rate.

VIII. PITCH — MUSICAL SCALE

231. The pitch of a sound *depends on its vibration frequency; the greater the frequency the higher the pitch.* With a given velocity the wave length varies inversely as the frequency (§ 198), hence the higher the pitch the shorter the wave length.

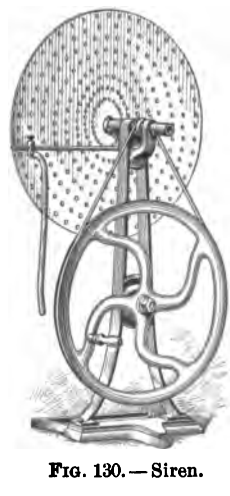

FIG. 130. — Siren.

Experiment 1. — Let a circular disk having in it eight concentric circles of holes, 24 holes in the inner circle and 27, 30, 32, 36, 40, 45, and 48 respectively in the remaining circles, be mounted on a whirling machine (Fig. 130). While it is in rotation direct a jet of air through the holes of each circle in rapid succession. It will be found that each circle gives a musical tone, and the greater the number of holes in the circle the higher the pitch. Such a disk is called a *siren.*

Experiment 2. — In place of the siren disk use a toothed wheel, touching its edge while it is revolving with the edge of a card. It will be found that the greater the rate of rotation or the finer the teeth of the wheel the higher the pitch.

232. Noises and musical sounds distinguished. — Many sounds, such as the report of a gun or the roar of street

traffic, *have no definite pitch* and are for that reason called *noises.* A noise generally consists of a mixture of unrelated sounds. An illustration of this occurs when a number of persons are talking in a room; a by-stander hears only a noise because, even though each voice may have a definite pitch, the sound as a whole has none that the ear can detect.

A *musical sound*, on the other hand, *is one which has a definite pitch and which is pleasing to the ear.* It is characterized by regularity and simplicity of vibration.

The ear is not equally sensitive to all rates of vibration, and when the rate exceeds a certain limit, there seems to be no sound at all. The limit at which this occurs has been placed at about 40,000 vibrations per second, but it differs with the individual, being lower for old people than for young people. It may happen, therefore, that what is sound for one person is not sound for another. Most musical sounds are comprised between 27 and 4,000 vibrations per second.

233. Musical intervals. — When two notes are sounded together, or one immediately after the other, the ear perceives a certain relationship between them. This relationship, determined by the relative frequencies of the two tones, is termed a *musical interval.* It is expressed as a simple ratio of their vibration frequencies. For example, the interval between two tones having frequencies of 240 and 360 is $\frac{3}{2}$, and the ear recognizes the same relationship or the same interval between any two tones whose frequencies are to each other as 3 to 2, as two tones having respectively frequencies of 400 and 600, or 1000 and 1500.

The interval between two tones whose frequencies have the ratio $\frac{1}{1}$ is called in music *unison;* $\frac{2}{1}$, an *octave;* $\frac{3}{2}$, a *fifth;* $\frac{4}{3}$, a *fourth;* $\frac{5}{4}$, a *major third;* $\frac{6}{5}$, a *minor third;* $\frac{5}{3}$, a *major sixth;* $\frac{8}{5}$, a *minor sixth.* The interval expressed by the ratio $\frac{9}{8}$ is called a *major tone;* $\frac{10}{9}$, a *minor tone;* $\frac{16}{15}$, a *major semitone;* $\frac{25}{24}$, a *sharp;* and $\frac{24}{25}$, a *flat.*

234. The diatonic scale. — Most of us are familiar with the succession of eight notes often designated by the names, *do*, *re*, *mi*, *fa*, *sol*, *la*, *si*, *do*. This series of eight notes is the *diatonic scale*. The first one of the series is the *keynote*. It may have any vibration frequency at pleasure, but the other notes must have a certain relationship to the keynote and to one another to form a perfect diatonic scale; the same intervals must occur and always in the same order. The siren (§ 231) gives this scale exactly. The intervals between the keynote and each of the other notes of the scale in their order are expressed by the following scale ratios:

do	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do*
$\frac{1}{1}$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$

Although the diatonic scale used among all civilized peoples has a scientific basis, yet it is not the only possible scale. In fact, music, like the other arts, is a product of evolution, and the diatonic scale and the music based upon it is of comparatively recent growth or development. Other times and other peoples have used different scales. The great fundamental law of music seems to be simplicity of relation, or that notes when sounded together should have relations which may be expressed by simple ratios.

235. Table for illustrating the diatonic scale.

Number	1	2	3	4	5	6	7	8
Letter	C	D	E	F	G	A	B	C
Syllable	*do*	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do*
Scale Ratios	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
1st Octave	240	270	300	320	360	400	450	480
2d Octave	**480**	540	600	640	**720**	800	900	960
3d Octave	**960**	1080	**1200**	1280	**1440**	1600	1800	**1920**
Intervals	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	

240 × 7 = **1680**, 6th harmonic.

The above table is based upon a keynote having a frequency of 240 per second, this number being chosen because fractions are avoided by its use. The series of notes having frequencies of 240, 270, etc., up to 480, which are obtained by multiplying 240 by the scale ratios, constitutes a perfect diatonic scale. The last note of one octave forms the keynote of the succeeding one. This table explains some of the names of the intervals (§ 233). For example, the interval called the *fifth* is so called because it is the interval between the first and the fifth note of this scale, and the *octave* is so named because it is the interval between the first and the eighth note of the scale.

The ratios in the last line express the intervals between the successive notes of the scale. Observe that there are three major tones, two minor tones, and two major semitones among them.

236. Basis of the diatonic scale. — When three notes whose frequencies are to one another as 4 : 5 : 6, are sounded together, they produce a most pleasing effect. Three notes having such a relationship are called a *major triad.* We are, perhaps, familiar with them under the names *do*, *mi*, and *sol*. When the octave of the first is added, the *major chord*, *do*, *mi*, *sol*, *do*, is formed.

An analysis of the diatonic scale shows that it is a combination of three related major triads. The major triad may, therefore, be said to be the basis of the diatonic scale, and consequently of all modern music. This may be illustrated by the numbers given in the table (§ 235).

1st major triad, C, E, G, $240 : 300 : 360 = 1 : \frac{5}{4} : \frac{3}{2} = 4 : 5 : 6.$

2d major triad, G, B, D′, $360 : 450 : 540 = \frac{3}{2} : \frac{15}{8} : \frac{18}{8} = 4 : 5 : 6.$

3d major triad, F, A, C′, $320 : 400 : 480 = \frac{4}{3} : \frac{5}{3} : 2 = 4 : 5 : 6.$

Observe that these three major triads include all the notes of the first octave except the second, but that 540, the octave of that, is used. Also observe that the second triad is related to the first by having one note, G, in common, and that the third is related to the first by the fact that C is used in the first triad and its octave in the third.

237. Transposition. — It is often desirable to change the keynote from one letter to another. This is called transposition. Suppose D, with a frequency of 270, is selected as the keynote instead of C. Applying the scale ratios to this number and computing the frequencies of the several notes, a new series is obtained as follows:

C	D	E	F	G	A	B	C	D
	do	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do*
	270	303¾	337½	360	405	450	506¼	540
240	270	300	320	360	400	450	480	540
		F♯ = 333⅓			C♯ = 500			

The lower line gives the frequencies when C was the keynote. E and A differ slightly for the two keys, but F and C are much too low. Sharping them by multiplying 320 and 480 each by $\frac{25}{24}$, we obtain 333⅓ and 500. Even then they do not quite agree with the frequencies required for the key of D.

It is evident from the above comparison that an instrument with a fixed keyboard, like the piano or organ, if tuned correctly for the key of C, would be incorrect for the key of D or any other key, the sharps and flats not wholly overcoming the errors.

238. The tempered scale. — That a piece of music might be rendered correctly in any key upon a piano would require the addition of a large number of keys to each octave, so many, indeed, that it would be practically impossible to manipulate them. To avoid this difficulty the scale of *equal temperament* has been adopted for such instruments. In this scale the whole tones are all equal and the semitones are exactly half of the whole tones. Five notes are added to the octave in this scale, making thirteen notes and twelve intervals which are all equal. The value of the interval is $\sqrt[12]{2}$, or 1.05946+, which is a little smaller than a true major semitone. The only correct intervals on the piano and organ are the octaves, all others are slightly false; hence with such instruments music of the highest artistic excellence is impossible. With instruments like the violin, however, in which the pitch is wholly under the control of the musician, it is possible to render music correctly in any key.

239. Standard pitch. — Any vibration frequency at pleasure may be used for the keynote, but at present the A′ fork having a frequency of 435 vibrations per second is the (international) standard of pitch most widely used. This gives 261 as the frequency of "middle C" in the diatonic scale, or 258.7 in the scale of equal temperament. Using this standard, the frequencies for an octave in the two scales are as follows:

	C	D	E	F	G	A	B	C′
Diatonic	261	293⅝	326¼	348	391½	435	489⅜	522
Tempered	258.7	290.3	325.9	345.3	387.6	435	488.3	517.3

In 1751 Handel used an A′ fork having a frequency of 422.5 a second. For many years after this, however, there was a tendency toward a higher standard of pitch. In 1857 a frequency of 448 was in use at Paris, and at one time in London 455 was the standard for the A′ fork. A high pitch adds to the "brilliancy" of instrumental music, and this fact may have been the chief cause for the rise in pitch; a high pitch is objectionable, however, to the vocalist, and hence a reaction against such a high pitch set in which led to the adoption of the present standard.

The successive octaves above "middle C" are designated by C′, C″, C‴, etc., and the successive octaves below it by C_1, C_2, C_3, etc.

IX. QUALITY OF SOUND

240. Fundamental tone. — In general a large body vibrates at a slower rate than a small one, and the whole of a body vibrates slower than a part of it. This is illustrated by the fact that the pitch of a small bell is higher than that of a large one. Bodies usually vibrate in parts, and the greater the number of parts or the smaller the parts, the higher the pitch. The experiments with the Chladni plate (§ 206) furnish excellent illustrations of this principle.

The lowest tone that a body can give, usually that produced by it when vibrating as a whole, *is called its fundamental.* Those tones of a higher pitch produced by the vibrations of the parts of a body are called *overtones* or *upper partials.*

Those overtones whose frequencies are exact multiples of the fundamental are called *harmonics* or *harmonic partials.* If in the table (§ 235), 240 be taken as the frequency of a fundamental, the numbers in heavy type will be the frequencies of its first seven harmonics. The *first* harmonic has a frequency of *twice* that of the fundamental, or 480; the *second*, *three* times that of the fundamental, etc. Observe carefully the relation of these harmonics to the

fundamental, and especially that the sixth has no place in the table.

241. Quality. — It rarely, or never, happens that a sounding body gives only one tone at a time, either its fundamental or a single overtone; but it produces a note which is a combination of several tones, usually its fundamental with some of its harmonics. The ear ascribes to the note the pitch of the most prominent tone, most often but not always the fundamental.

Quality of sound depends upon what tones are combined to produce it and upon their relative intensities. It is possible that two notes may be composed of the same tones and yet differ in quality because the component tones differ in relative intensity. In one, for instance, the third harmonic might be very prominent, while in the other the fourth might be prominent and the third weak.

A note composed of only one or two tones would be described as thin, flat, or poor in quality. The lower harmonics are often very prominent, and when the attention of a person has once been called to them, he learns to detect them, especially the second, fourth, and fifth, in the sounds of bells, whistles, human voices, and even of some animals, with astonishing facility. Fortunately the higher overtones are generally weak and die out very quickly, otherwise many sounds which are pleasant to the ear would be very harsh and unmusical.

242. The composite character of sound may be made visible to the eye by the *manometric flame*. The apparatus for producing this flame (Fig. 131) consists of a chamber or cavity *A* closed on one side by a thin, flexible membrane, *cc*, of gold-beater's skin or of thin sheet rubber. Two tubes enter this cavity. Through one, *D*, illuminating gas is admitted. The other, *E*, serves as an exit for the gas, its outer end being drawn down to form a jet.

The figure represents a manometric capsule made from a round wooden pill box. An inch hole is bored through the top and the bottom. A rubber stopper carrying the two tubes is placed through the hole in the cover, and the rubber membrane is clamped between the cover and the body of the box. The sound waves which enter *B* through the funnel *F* strike the membrane and cause variations of pressure in the gas in *A*. A condensation increases the pressure and lengthens the gas flame, and a rarefaction relieves the pressure and shortens the flame.

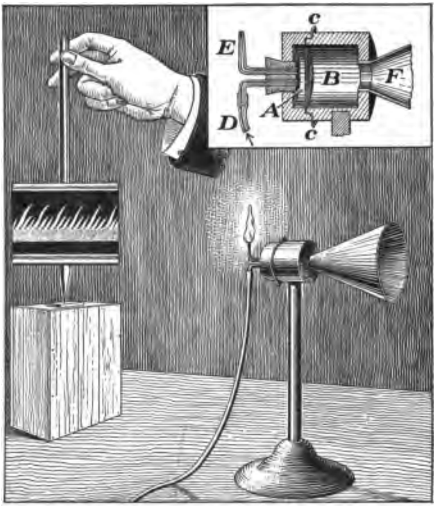

Fig. 131. — Apparatus for producing the manometric flame.

The variations of the flame are made visible to the eye by being observed in a revolving mirror (§ 207). If the flame is steady, its image forms a smooth band of light. If an organ pipe is blown near the funnel so as to give only its fundamental, the band of light in the mirror is changed into a series of equidistant tongues of flame shown at 1 (Fig. 132). With a given rate of rotation of the mirror, the higher the pitch the finer the tongues become. If two pipes of different pitch are sounded at the same time, or if one is blown harder so as to give an overtone as well as its fundamental, the flame 3 or 4 will show the composite character of the sound. Likewise the presence of overtones in the human voice may be shown when different vowel sounds are sung into the funnel of the instrument.

We have seen (§ 224) that discords are due to beats. It often happens, however, that the discordance is due, not to beats between the fundamentals, but to those caused by the harmonics or overtones of the two notes, or by the fundamental of one and an overtone of the other. For example, *C* and *B* of the piano are discordant because of the

beats formed by C and the first harmonic of B, and bells or chimes whose fundamental tones are in perfect harmony produce discords when sounded together because of their overtones. These overtones are not harmonics, their frequencies not being simple multiples of their fundamentals.

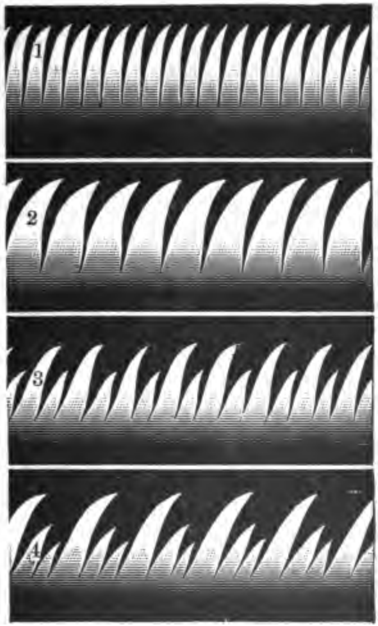

FIG. 132. — A manometric flame, showing the composite character of sound.

243. Analysis of sound. — We have seen that a resonator will respond only to a tone of some particular frequency. Helmholtz first made use of this principle in analyzing sound. He used a number of air resonators of different sizes and of a form shown in Figure 133. Each one had a small opening *a* which was applied to the ear, and a larger opening *B* at which the sound waves entered. The silence of a resonator indicated the absence of its tone from the note being tested, and resonance in any particular resonator indicated the presence of a tone corresponding to that of the resonator. After analyzing a note, Helmholtz was able to reproduce or imitate it by combining the tones found in it by analysis.

FIG. 133. — A Helmholtz resonator.

Problems

1. The frequency of a fork is 435. What is the length of a resonant air column reënforcing it at 20° C., the diameter of the tube being 6 cm.? *Ans.* 17.6 cm.

2. A tuning fork having a frequency of 384 produces resonance in a tube 19.1 cm. long and 5 cm. in diameter. What is the velocity of sound?

3. A fork held over a tube 5 cm. in diameter produces resonance when the air column is 17 cm. deep. What is the frequency of the fork, the temperature of the air being 15° C.?

4. The waves of one musical note are 120 cm. and those of another 114 cm. long. If the two notes are sounded at the same time in air at 16° C., how many beats will be produced per second?

5. Using 256 as the vibration frequency of the keynote, calculate the frequencies for the notes of one octave in the diatonic scale.

6. Three notes have respectively frequencies of 240, 360, and 540. Compare the interval between the first and second with that between the second and third.

7. The interval between *re* and *sol* in the diatonic scale is $\frac{4}{3}$. How many more intervals of the same value are there in the scale?

8. The interval between two notes is a "fifth" and the frequency of the lower note is 256. What is the frequency of the other note?

9. What harmonics of two notes whose fundamentals are produced respectively by frequencies of 256 and 400 would produce a maximum discord if this maximum occurs at about 32 beats per second?

X. MUSICAL INSTRUMENTS — STRINGS

244. Most musical instruments are included in the two classes, *stringed* instruments and *wind* instruments. The violin, the guitar, the harp, and the piano are illustrations of the first class; and the flute, the cornet, the trombone, and the organ, of the second class.

The strings of musical instruments are made to vibrate by means of a bow as in the violin, or by being plucked with the fingers as in the guitar and harp, or by being struck with a hammer as in the piano.

245. Laws of vibrating strings. — The laws of vibrating strings are studied by the use of a *sonometer*, which consists of a long resonance box (Fig. 134) with one or more strings or wires stretched over it from end to end. These wires, which rest upon bridges, one at each end, are made taut, and the stretching force or tension is regulated by passing them over pulleys at one end of the sonometer

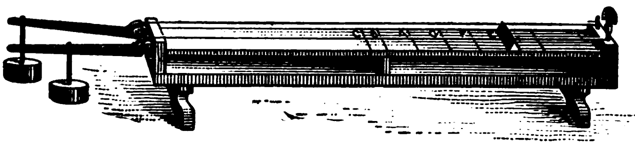

FIG. 134. — Sonometer.

and attaching weights to them, or often a lever of some form is used for the purpose. Perhaps the simplest method is to fasten the sonometer in a vertical position against the wall, and to stretch the wires by weights attached to their lower ends. With such an instrument the following laws may be illustrated, but exact determinations are better suited to the laboratory than to the classroom.

First Law. — *The tension being constant, the frequency of a string varies inversely as its length.*

Let l represent the length of a string and n its frequency, and l' another length of the string and n' its frequency; then this law may be expressed as follows:

$$n : n' = l' : l.$$

Second Law. — *The length of a string being constant, its frequency varies directly as the square root of its tension.*

Let n represent the frequency of a string when its tension is t, and n' its frequency when its tension is t'. Then this law may be expressed as follows:

$$n : n' = \sqrt{t} : \sqrt{t'}.$$

Third Law. — *The length, tension, and material of two strings being the same, their frequencies vary inversely as their diameters.*

Let n represent the frequency of one string and d its

diameter, and n' the frequency of another string and d' its diameter. Then this law may be expressed as follows:

$$n : n' = d' : d.$$

Fourth Law. — *The length and tension of two strings being the same, their frequencies vary inversely as the square roots of their masses per unit length and are independent otherwise of their material.*

This law means, for example, that if two strings are stretched equally and are made of equal length, and one of them weighs four times as much as the other per foot of its length, the lighter one will make twice as many vibrations per second as the other even if they are made of different material.

Experiment 1. — Stretch a single wire upon a sonometer, and observe its pitch. Then place a movable bridge under it exactly midway between the other bridges. Each half of the string will now be found to give a note an octave higher than the whole string did; that is, half of the string has twice the frequency of the whole.

Make the string $\frac{2}{3}$ as long as at first. Observe that its note is a fifth above its original note; that is, $\frac{2}{3}$ of a string makes $\frac{3}{2}$ times as many vibrations as the whole string does. These experiments illustrate the first law.

Experiment 2. — Stretch two similar wires upon the sonometer, making their lengths equal. Attach a 4-lb. mass to one and a 16-lb. mass to the other. The latter string will be found to be an octave higher than the other. That is, a string with 4 times the tension of another has twice the frequency of the other. This illustrates the second law.

Experiment 3. — The diameter of No. 11 piano wire is $\frac{3}{4}$ of that of No. 16, and that of No. 10 is $\frac{4}{5}$ of that of No. 13. Place a No. 11 and a No. 16 wire or a No. 10 and a No. 13 wire upon a sonometer, making their lengths and tensions equal. If the first pair is used, they will give two notes corresponding to *do* and *fa* of the diatonic scale. If the second pair is used, the notes will correspond to *do* and *mi*. This illustrates the third law. If their masses per unit length are compared, it will be found that it illustrates the fourth law also.

Since they are of the same material their masses per unit length will be to each other as the squares of their diameters.

In a general way musical instruments illustrate these laws. In the piano and the harp the short thin strings give the notes of high pitch, and the long thick ones the notes of low pitch. It is well known that stringed instruments are tuned by changing the tension of the strings, which is increased to raise the pitch and lessened to lower it. The violinist illustrates the first law as he rapidly changes the length of his strings by pressing upon them with his fingers at different places.

246. Corollary to the First Law. — *The lengths of two strings in unison are directly proportional to the frequencies of the two strings when their lengths are made equal.*

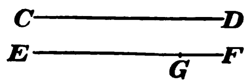

FIG. 135.

Proof. — Let CD and EF (Fig. 135) be two strings equal in length, and let EG, a part of EF, be in unison with CD.

(1) $\dfrac{\text{Frequency of } EG}{\text{Frequency of } EF} = \dfrac{\text{Length of } EF}{\text{Length of } EG}$ by the first law of strings.

(2) Frequency of CD = Frequency of EG by hypothesis.

(3) Length of CD = Length of EF by hypothesis.

Substituting these values in the proportion, we have

(4) $\dfrac{\text{Frequency of } CD}{\text{Frequency of } EF} = \dfrac{\text{Length of } CD}{\text{Length of } EG}$ Q. E. D.

This principle is of great use in proving the other laws of strings upon the sonometer.

247. The harmonics of a string. — When a string is bowed or plucked at its middle point, it vibrates as a whole, giving its fundamental tone; but if it is touched or "damped" at that point with a feather and is bowed near one end, both of its halves will vibrate, giving the octave of the fundamental, the feather causing a node at the middle. When the string is damped at one third of

its length from the end and the short portion bowed, it divides itself into three equal parts, each part vibrating with a frequency three times that of the fundamental, producing a tone an octave and a fifth above it or its second harmonic. In a similar manner a string may be made to vibrate in four, five, or more equal segments, producing a series of harmonics of the fundamental tone.

These vibrations may be made visible to the eye by placing light paper riders astride the wire. That at the

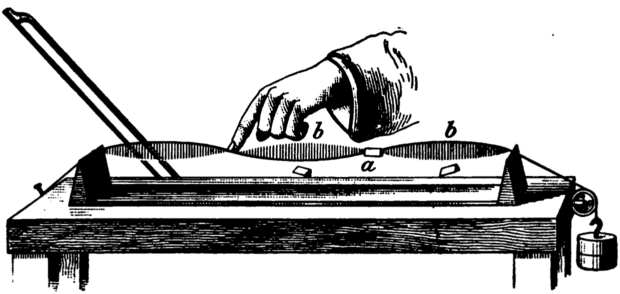

FIG. 136. — Apparatus for showing a string vibrating in segments.

nodal point *a* (Fig. 136) will remain stationary and those on the ventral segments *bb* will be thrown off.

248. Notes of a string composite. — In the last paragraph it was shown how a string may be made to vibrate as a whole or in equal parts. Usually, however, when a string vibrates as a whole it also vibrates in parts, *at the same time* giving a note composed of a fundamental and several harmonics or overtones. Its actual motion is a resultant of all its vibratory motions and is very complex in character. The parts in which a string vibrates and hence the quality of its note depend very largely upon how and where the string is bowed or struck.

Experiment 1. — Damp the wire of a sonometer at the middle by touching it with a feather, and bow one of its halves. Remove the

feather while bowing the string. It will begin to vibrate as a whole, giving its fundamental, but the ear will be able to detect the continuation of the tone produced by the halves of the string. This will show that a string may vibrate as a whole and in halves at the same time.

Again, damp the wire at one third its length from the end and bow the short portion. Remove the feather while bowing the string and presently the fundamental will be heard and with it the tone produced by the vibrating thirds of the string. This shows that a string may vibrate as a whole and in thirds at the same time. With long strings the experiment may be carried further, showing that a string may vibrate in fourths, fifths, etc., at the same time that it is vibrating as a whole.

XI. WIND INSTRUMENTS—PIPES

249. In wind instruments the air column which produces the sound is usually contained in tubes or pipes. The French horn, for example, is a long coiled tube terminating in a wide bell, while an organ pipe and a flute are straight tubes.

Pipes are of two kinds, *reed* pipes and *flute* pipes. In a reed pipe the air is made to vibrate by a thin flexible tongue or reed which is itself set in vibration by a blast of air. A clarionet and a ragman's horn are illustrations of a reed pipe.

In flute pipes the air column is set in vibration by a thin ribbon of air blown across an opening. The flute, fife, and a boy's willow whistle, which is much like an organ pipe in structure and action, are illustrations of a flute pipe. Figure 137 shows such a pipe. The air entering through *C* passes through the narrow slit *a* across the opening *D*, striking the sharp edge *b*.

FIG. 137.—Section of an organ pipe.

A pipe that is open at the upper end *M*

is called an *open* pipe; one closed at the upper end is called a *closed* or a *stopped* pipe. The former is open at both ends, but the latter is open at one end only, that is, at *D*.

250. Vibration of air in pipes. — When a pipe is sounding, the air particles in it vibrate longitudinally and sound waves travel to and fro along the tube. An example of this is furnished by the resonant air column (§ 218); it is that of a stopped pipe sounding its fundamental.

There is always a node at the closed end of a pipe, because the air cannot vibrate there, and an antinode at the open end. In an open pipe sounding its fundamental, there is a node in the middle, and an antinode at each end. In all pipes, as in the case of the resonant air column, the distance from node to antinode is always one fourth of a wave length; an open pipe sounding its fundamental is therefore a half wave length long. When the harmonics are sounded by a pipe, there is more than one node, but always one at the closed end and antinodes at the open ends. The experiment with the vibrating spiral spring (§ 203) illustrates the action of the air in a pipe, when several nodes and antinodes exist in it.

251. The laws of vibrating air columns. —

1. *The frequencies of air columns vary inversely as their lengths.*

2. *The frequency of a closed pipe is half of that of an open pipe of the same length.*

3. *In open pipes a complete series of harmonics is possible, but in closed pipes only those harmonics are possible whose frequencies are odd multiples of that of the fundamental.*

These laws are roughly illustrated by many wind instruments. Thus, the fundamental note of a flute is obtained by closing all of the openings by the fingers and blowing softly. By raising each finger

in succession beginning at the lower end a series of notes of higher and higher pitch is given. Uncovering an opening has the same effect as cutting off the tube at that point. This illustrates the first law.

Again, by closing all the openings and blowing harder the harmonics are obtained. Thus the third law is partially illustrated.

Experiment 1.—Close one end of a glass tube about 1 cm. in diameter and 24 cm. long with the finger, and blow gently across the open end to produce a musical note. Try the same again with a tube half as long and with another two thirds as long. The second will give a tone an octave higher than the first and the third a note a fifth above the first. This illustrates the first law.

Experiment 2.—Prepare a singing flame apparatus (§ 207) using a tube 48 cm. long. The tube constitutes an open pipe. Compare its note with that of the 24-cm. tube used as described in the last experiment. The two will have the same pitch. Also compare the note of a 24-cm. tube when placed over the singing flame with that of the 48 cm. tube used in the same manner.

Experiment 3.—Hold a closed tube opposite the air jet of a siren (§ 231) so that the jets of air may pass through the holes of the disk into the tube. By trial, tubes of different lengths being used and the jet being placed opposite different circles an intense sound may be produced by resonance. A 4-in. or a 6-in. test tube often answers the purpose admirably. When resonance has been established in this way, substitute for the closed tube an open one twice as long. It will give the same note as the closed tube.

Problems

1. A string vibrates as a whole and makes 300 vibrations per second. If it also vibrates in thirds at the same time, how many vibrations per second has each third of the string? What harmonic is produced? If the fundamental is called *do*, what name should be given to the harmonic?

2. To what note on the piano would the fifth harmonic of "middle C" correspond?

3. Two strings, one 80 cm. and the other 64 cm. long, are in unison. The first one has a frequency of 280. What is the frequency of the other when it is 80 cm. long?

4. If the note of a violin string 36 cm. long is taken as the key-

note, how long would it have to be to produce *sol* in the same octave? How does the player alter the length of the string?

5. What law of strings does the violin player illustrate by the use of the fingers of his left hand?

6. If the vibration number of a string 60 cm. long and under a tension of 100 Kg. is 480, what will its frequency be if its length is changed to 40 cm. and its tension to 64 kg? *Ans*. 576.

7. What is the rate of vibration of a string 80 cm. long, 0.8 mm. in diameter, and under a tension of 25 Kg., if the frequency of a string of the same kind 96 cm. long, 1.2 mm. in diameter, and under a tension of 36 Kg. is 256?

8. What is the length of a closed pipe whose fundamental is produced by 435 vibrations per second when the velocity of sound in air is 1125 ft. per second?

9. How long must an open organ pipe be if its frequency is to be 64, the velocity of sound being 1120 ft. per second?

10. Calculate the lengths of a series of closed tubes which will produce the diatonic scale, the one producing *sol* being 36 cm. long.

11. Will the pitch of a pipe organ be raised or lowered by a rise in temperature? Why?

12. A closed pipe is 12 ft. long. What must be the length of an open pipe to give a note two octaves higher?

CHAPTER III

LIGHT

I. NATURE OF LIGHT

252. The ether. — It is the accepted belief among men of science that all space is filled with something so rare and subtle that it cannot be weighed or indeed perceived by any of our senses, and to this all-pervading medium the name *ether* has been given. It is believed to fill not only the immense spaces between the stars or between the earth and the sun, but also to exist between the molecules of all matter, whether solid, liquid, or gaseous.

This concept of the ether may strike the beginner in science as fanciful, yet very many facts, especially in Light, Heat, and Electricity, impel one to the belief that something, whatever its name or nature, by which energy is transmitted through the enormous spaces of the universe, fills all space.

253. Nature of light. — Practically all the energy we have comes to us from the sun; and the question arises, how is it transmitted across the millions of miles of intervening space? There are two ways in which we may conceive of this being done. One consists in the transmission of energy by the bodily transfer of matter itself, just as a bullet carries energy when it is shot from a gun. The other method of transferring energy is by wave motion in some medium, as we have learned in the study of sound.

The *corpuscular* or *emission* theory of light accorded with the first method of transferring energy. This theory,

which was elaborated and advocated by Newton (1642–1727), held that light consisted of very small corpuscles or particles shot out by luminous bodies, like the sun or the flame of a candle, in all directions. These particles, by impinging on the retina of the eye, were supposed to cause the sensation of sight. This theory was not entirely discarded till well into the nineteenth century.

The *undulatory* or *wave* theory of light, first stated in definite form by Huygens in 1678 and now universally accepted, accords with the second method of transmitting energy. This theory assumes that light is a form of energy due to vibratory disturbances in the ether which are transmitted through space by ether waves, the vibrations being transverse to the line of propagation. These vibratory disturbances are electromagnetic in their nature.

Not all waves in the ether are light waves, but only those which are of the proper length to affect the eye and cause the sensation of sight. Some are too short and others are much too long to be light waves, although differing from them in no other respect except length.

All energy of wave motion in the ether is called *radiant energy;* hence, light is radiant energy, but not all radiant energy is light.

Note 1. — The student should bear in mind two important distinctions between sound waves and light waves: (1) Sound waves are longitudinal; light waves are transverse. (2) Sound waves are waves in ordinary matter only, as in air, wood, water, etc.; light waves are in the ether only. Light waves exist in air or glass only because the ether exists between the molecules of those substances; but light is propagated with the greatest facility through the best vacuum attainable. Sound, on the other hand, cannot pass through a vacuum.

Note 2. — Just as there are two distinct and correct definitions of sound, so also there are two of light. In addition to the one given above, light may be defined as a *sensation* which is reported to

the brain by the optic nerve. This is the definition of the physiologist and the psychologist; but the physicist also uses the term *light* with this meaning at times in discussing the subject of color.

II. TRANSMISSION OF LIGHT

254. Radiation. — If a tuning fork were set in vibration and dropped from a height, it would have two kinds of motion at the same time, — its vibratory motion and its downward motion toward the earth. Likewise molecules are supposed to be in ceaseless vibration while at the same time they may be moving from point to point within the body. Now as the tuning fork by its vibrations may set up waves in the air about it and transmit its energy of vibration to a distance by the waves in the air as a medium, so the molecules or the parts of molecules of a body by their vibrations set up, in the ether surrounding them, waves which transfer energy away from the body outward into space in all directions.

This production of waves in the ether by the vibrations of molecules or their parts and the transmission of energy away from the body by the ether waves thus formed is called *radiation.*

The waves emitted by most bodies at ordinary temperatures are too long for light waves ; but if a body is heated more and more, shorter and shorter waves are produced until finally a point is reached (525° C.) where it begins to give off waves short enough to be light. A body heated to such a degree that it emits light is said to be *incandescent.*

Some bodies, like the firefly, in a way not yet understood, emit light when they are not heated.

255. Effect of matter on light waves. — Light travels most readily through space devoid of ordinary matter, and the presence of air or other substances obstructs or modi-

fies the ether waves more or less. Many substances which, unless extremely thin, entirely prevent the passage of light waves are called *opaque;* others, like air, water, and glass, which allow the waves to pass so that objects can be seen through them distinctly, are termed *transparent.* Bodies like oiled paper, ground glass, and porcelain, which allow light to pass but not so that objects can be seen distinctly through them, are called *translucent.*

Some very long ether waves, many times too long to be light waves, pass through a brick wall as readily as light waves pass through glass.

256. Reflection and absorption. — The question arises, what becomes of the energy of light which strikes a body and does not pass through it?

First, some of it bounds off again into space; that is, it is *reflected.* Evidence of this exists on every hand. We can see an object only when light enters our eye from that object, and hence most objects are seen by us simply because they reflect light to our eyes.

Secondly, some of the waves of light in attempting to penetrate and pass through a body set the molecules of the body in more rapid vibration, and thus the waves, losing their energy, are destroyed. This transference of energy from the ether to the molecules of matter, the radiant energy being transformed into kinetic energy of molecular motion, is called *absorption.*

Absorption is the converse of radiation. In radiation kinetic energy of molecular motion is transformed into radiant energy, being at the same time transferred from matter to the ether.

Bodies visible by the light which they radiate are termed *luminous;* but those visible by the light which they reflect are said to be *illuminated.*

In general, when light strikes a body, some of it is

always reflected, almost always some of it is absorbed, and sometimes some of it is *transmitted.*

257. Velocity of light. — Roemer, a Danish astronomer, first gave a definite proof (1676) that it takes time for light to travel through space. He was engaged in observing the eclipses of one of the moons of the planet Jupiter at the Paris observatory. This moon, like our own, is visible by the sunlight which it reflects, but it disappears suddenly from sight at every revolution about Jupiter, because it passes into the shadow of that planet and is eclipsed. This enabled Roemer to determine the time required for it to go around Jupiter once and, having done this, to calculate the exact time of each eclipse for a year or more in advance. He did this, but found as he continued his observations that the observed time kept falling behind the computed time, — a few seconds at first, but more and more at each succeeding eclipse, until at the end of about six months the discrepancy amounted to nearly 1000 seconds.

Let S (Fig. 138) represent the sun, and the circle EE' the orbit of the earth, J Jupiter, and the small circle the

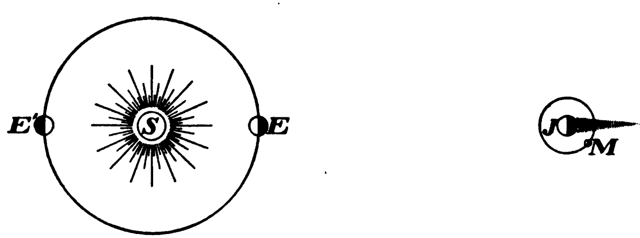

Fig. 138. — Diagram illustrating Roemer's method for determining the velocity of light.

path of the moon M around that planet. When Roemer made his first calculations, the earth was at E on the same side of the sun as Jupiter, but during the next six months

it passed halfway around the sun to E'. He reasoned that the eclipses occurred at the computed times, but that he did not see them until later because the earth was moving continually farther away from Jupiter. He reasoned further that, if his explanation was correct, during the next six months, while the earth was going toward Jupiter, the observed times would gain on the computed times and agree with them when the earth reached E again. This he found to be true. He concluded, therefore, that it took about 1000 seconds longer for light to travel from M to E' than from M to E.

If we assume that ES equals 93,000,000 miles, which is very nearly correct, then the distance EE' will be 186,000,000 miles and we have as the velocity of light (186,000,000 ÷ 1,000) 186,000 miles per second.

Roemer's actual result was about 192,000 miles per second. His work, disregarded at the time, was confirmed fifty years later by another astronomer who was working out an entirely different problem.

Fizeau, in 1849, measured the velocity of light by finding the time required for it to traverse a few kilometers, and in 1850 Foucault was able to make the measurement within the limits of a large room. Michelson, in 1878, modified and improved Foucault's method. It is now possible to show that light travels with different speeds through different substances, its speed being less, for example, in water or glass than it is in air. Modern research gives the velocity of light as about 299,860 kilometers (186,324 miles) per second. It may be stated in round numbers as 186,000 miles or 300,000 kilometers per second.

258. A ray of light is a mere direction or line along which a small piece of a wave front travels. It is perpendicular to the wave front. When light spreads outward from a point P (Fig. 139), the wave fronts are convex.

FF and *GG* represent wave fronts and the lines *Pa*, *Pc*, and *Pe* the rays or paths along which the points *a*, *c*, and *e* travel. These rays are divergent, and the light forms a *diverging pencil.*

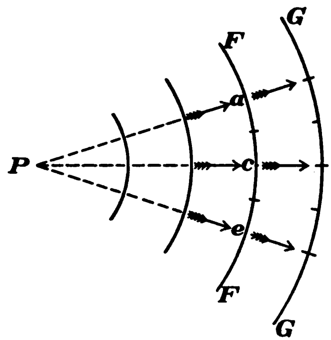

FIG. 139. — Diagram of a diverging pencil of light.

When light is going toward a point (Fig. 140), the wave fronts are concave, the rays are convergent, and the light constitutes a *converging pencil.* In sunlight the source of light is so far distant that the wave fronts are practically plane (Fig. 141) and the rays being perpendicular to them are parallel to one another. A pencil of light in which the wave fronts are plane and the rays are parallel constitutes a *beam* of light.

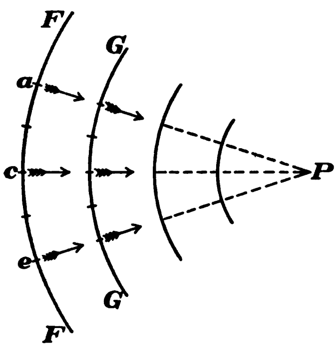

FIG. 140. — Diagram of a converging pencil of light.

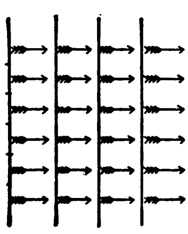

FIG. 141. — Diagram of a beam of light.

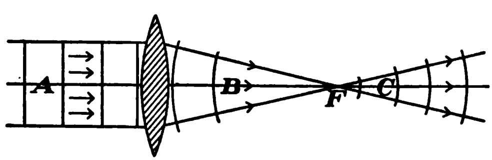

FIG. 142. — Diagram of a beam of light (*A*) transformed by a convex lens into a converging pencil (*B*) and a diverging pencil (*C*).

Experiment. — Pass sunlight through a convex lens, such as a reading lens (Fig. 142), and scatter chalk dust in the path of the light

by striking two erasers together above it. The light A, before passing through the lens, illustrates a beam of light; the light B, going toward the point F, is a converging pencil; and the light C, going on from the point F, is a diverging pencil.

259. Light travels in straight lines. — In a medium alike in all directions the rays of light are straight, and light is said to be propagated in straight lines. This fact is recognized by the surveyor and the artisan in sighting, and by the marksman in taking aim.

260. Shadows. — When an opaque object intercepts light from a luminous body, the space behind the opaque object from which the light is excluded is called a shadow. Since the space has length, breadth, and thickness, a shadow is a geometrical solid, not simply a surface.

Experiment. — Hold a small ball or a disk like a silver dollar between the flat side of a gas flame and a white screen so as to cast a shadow upon the screen. If the distances are right, the shadow will have a dark central portion surrounded by a less dense border.

A shadow then may consist of two parts, the *umbra* and the *penumbra*. The umbra is the dark part which receives light from no part of the luminous body. The penumbra is the lighter part of a shadow which receives light from a portion, but not the whole, of the luminous body.

Let S (Fig. 143) be a luminous sphere, such as the sun, and E an opaque body like the earth. The figure represents a section of the

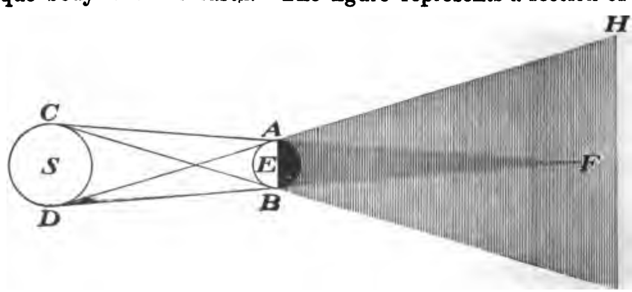

Fig. 143. — Diagram of the shadow cast by the earth.

bodies and the shadow cast. It is evident that the space $FABG$ can receive no light from the point C; and that the space $HABF$, no light from D. The space FAB is common to these two spaces, and hence can receive no light from either C or D. It is evident, further, that it can receive no light from points between C and D, and hence from no part of S. The space FAB is therefore the umbra, and in this case it is a cone.

The space HAF GBF evidently receives light from a portion of S and hence constitutes the penumbra. In this case it is a frustum of a cone containing the umbra within it, and having AB as its upper base.

The earth, the moon, and the planets cast shadows of this form in space, and in studying eclipses of the sun and moon it is well to remember it.

Imagine E and S to be of the same size. In that case the lines CAF and DBF would be parallel, and the umbra would be a cylinder.

If S were smaller than E, the lines CAF and DBF would diverge, and the umbra would be the frustum of a cone.

If S were to become a point, evidently the lines CAF and DAH would coincide, also the lines DBF and CBG, so that there would be no penumbra, the shadow being all umbra.

Observe the lack of distinctness in the outline of the shadows of such objects as the leaves or limbs of a tree in sunlight; and, if you have opportunity, the distinctness of outline of the shadows of such objects in the light of a single arc lamp when the latter is not surrounded by a translucent globe. Can you explain the difference?

III. INTENSITY OF LIGHT

261. Intensity of light. — The word *intensity*, as applied to light, has two uses. First, we may speak of the intensity of the *source* of the light itself, meaning the rate at which it is radiating light. Secondly, we may speak of the intensity of *illumination*, meaning the amount of light received by an illuminated body per unit of area.

Common experience teaches us that different sources of light vary greatly in their intensities; an electric arc lamp, for instance, is much more intense than a candle flame; but we also know that the intensity of illumina-

tion upon the page of a book produced by a candle close at hand may be greater than that caused by a very brilliant but distant arc lamp.

262. Laws of intensity of light. —

I. *The intensities of illumination of any given surface are inversely proportional to the squares of the distances of that surface from the source of light.*

II. *The intensities of two sources of light are directly proportional to the squares of the distances at which they give equal illumination.*

The first law is the same as that given for sound (§ 230), and follows from an exactly similar line of reasoning. It may, however, be illustrated experimentally as follows:

Experiment. — Place a card *A* (Fig. 144) or piece of tin having a hole in it about 0.5 mm. in diameter close to a gas or candle flame so that the light shall shine through the hole in a horizontal direction. Consider this hole as the source of light. Place a second card *B* having an opening 5 cm. square (area 25 sq. cm.) in a vertical position 30 cm. from the first card, and receive the light shining through it upon a third card *C*. Measure the side of the square of light on *C* when it is respectively 60, 90, and 120 cm. from *A*. It will be found that when the distance from the source of light to *C* is 60 cm. or 2 times that to *B*, the light upon it is a square 10 cm. on a side, its area being 100 sq. cm.; at 90 cm. or 3 times the distance, 15 cm. on a side, area 225 sq. cm.; at 120 cm., 20 cm. on a side, area 400 sq. cm. That is, when the distances from *A* to *C* are respectively 2, 3, and 4 times the distance from *A* to *B*, the areas of the squares of light are respectively 4, 9, and 16 times the area of the opening in *B*. Obviously the light passing through the opening in *B* is spread over a greater surface on *C*, and as the area of this surface

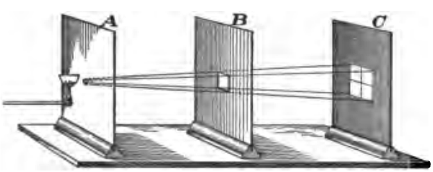

FIG. 144. — Apparatus for showing how illumination on a surface decreases with the distance from the light.

increases as the square of the distance from the source of light, the amount of light per square centimeter must decrease in like ratio.

263. Photometry treats of the measurement of the intensity of sources of light, and a photometer is an instrument for making the measurements. Many kinds of photometers have been invented, but the Bunsen has perhaps been more widely used than any other. It consists of a card of thick white unglazed paper about 10 cm. square, made translucent at its center by greasing a spot about 2.5 cm. in diameter. It is usually mounted in a small car rolling on a graduated bar, and two mirrors are so placed that both surfaces of the card can be seen at the same time.

Such an instrument, when used to compare the intensities of two lights, is placed between them (Fig. 145), and the distances from it to

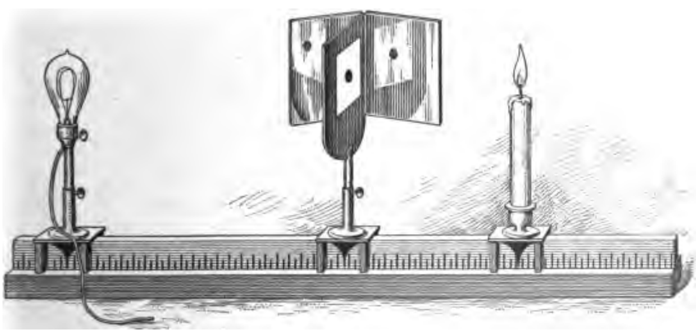

FIG. 145. — The Bunsen photometer.

the two lights so adjusted that both sides of the card are equally illuminated. In theory this is accomplished when the translucent spot cannot be distinguished from the rest of the card. In practise this is never quite attained, but the adjustment is made by observing when both sides of the card viewed in the two mirrors look exactly alike.

The second law may be illustrated with such an instrument by placing four candles on one side of it at a distance of 100 cm., and a

single candle on the other side at a distance of 50 cm. The illumination on the two sides of the photometer will be found equal. That is

$$\frac{\textit{the intensity of } 4 \textit{ candles}}{\textit{the intensity of } 1 \textit{ candle}} = \frac{100^2}{50^2} = \frac{4}{1}$$

264. The unit of intensity of source of light. — No unit of intensity or standard light thoroughly satisfactory has yet been found, but the one in general use is the *candle power*, or the light of a sperm candle which weighs one sixth of a pound (for that reason called "sixes"), and which burns at the rate of 120 grains an hour. When the candle power of a light is to be measured, the light is placed on one side of the photometer and the standard candle, or usually two of them, on the other. The photometer is then adjusted for equal illumination. The candle power of the light is then computed by the second law. For example, suppose a gas light at 210 cm. gives the same illumination as 2 standard candles at a distance of 70 cm. What is its intensity in candle power? By the second law we have

$$2 : x = 70^2 : 210^2. \qquad \text{Solving, } x = 18 \text{ c.p.}$$

Problems

1. What is the intensity of a lamp which at a distance of 15 ft. gives the same illumination as a standard candle at a distance of 2 ft.?

2. A 16 c. p. electric lamp is placed 240 cm. from the photometer on one side. At what distance on the other side must 3 standard candles be placed to balance it?

3. A 5 c. p. lamp was placed 400 cm. from one of 12 c. p. At what point between them must a photometer be placed to be equally illuminated?

4. A photometer is placed between two sets of candles which are 180 cm. apart. There are 3 candles on one side, and 2 on the other. At what point is the photometer when equally illuminated on both sides?

5. When an arc light gives the same intensity of illumination at a distance of 45 meters as a 16 c. p. incandescent lamp does at 100 cm., what is the candle power of the arc lamp?

IV. REFLECTION OF LIGHT

265. Reflection of light is of two kinds, *regular* and *diffused*. When a beam of light enters a darkened room through a small opening and strikes a mirror, a reflected beam will be seen traveling along some definite path. This is called regular reflection. Should the light, however, fall upon a piece of white paper, it would be reflected and scattered in all directions. This is called diffused reflection, and it is caused by the inequalities of the reflecting surface.

A *mirror* is any surface smooth enough to reflect light regularly. To do this the inequalities of the surface must be small in comparison with the wave-lengths of the light incident upon it.

All objects, even the best of mirrors, to some extent, reflect light diffusely, scattering it in all directions. It is by such light that we see objects that are not self-luminous; a perfect mirror would be invisible. It is by diffuse reflection of light, not only by large objects, but also by dust particles and moisture floating in the air, that light is so generally diffused. If it were not so, it would be dark everywhere except in the direct path of light from some luminous body.

Experiment. — Fill a glass jar with water which contains a little sediment, or which is slightly roily. Shield the jar from the light by placing near it a board having a hole through it about 2 cm. in diameter, at such a height that a horizontal beam of light may be passed through it into the lower part of the jar. If a beam of light from a porte lumière or a lantern is focused by a lens through the opening into the water, the whole of the water will be illuminated by the light diffused by the foreign particles in suspension in it. If the

experiment is conducted in a darkened room, the jar may be made to appear full of a red or green liquid by covering the hole in the board with a piece of red or green glass.

266. Law of reflection. — The light falling upon any surface is called the *incident* light, and the point where it strikes the *point of incidence*. The angle between the direction of the incident light and the perpendicular, or as it is often called, the *normal*, to the reflecting surface at the point of incidence, is termed the *angle of incidence*. The light which is reflected is called the *reflected light*, and the angle between its direction and the normal at the point of incidence is named the *angle of reflection*. Law of reflection: *The angle of reflection is equal to the angle of incidence, and the two angles are in the same plane.*

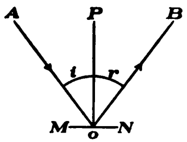

Fig. 146. — Diagram of the angle of incidence and the angle of reflection.

Let Ao (Fig. 146) be a ray of light incident on the mirror MN at o, and let Po be a perpendicular or normal to the mirror at the point of incidence o. Then the light will be reflected in the direction oB, forming the angle of reflection PoB equal to the angle of incidence PoA.

Experiment. — The optical disk (Fig. 147) may be used to illustrate the law of reflection. It consists of a disk about 40 cm. in diameter, graduated around its circumference in degrees, and mounted so as to

Fig. 147. — Hartl optical disk, for illustrating the law of reflection.

turn on a horizontal axis through its center. Let a plane mirror MN be fastened to the disk, as shown in the figure, the zero line Pc being normal to the mirror at the center of the circle. Let a beam of light Lc from a lamp pass through a slit in the shield D so as to strike the mirror exactly at c. It will be reflected in the line cR, and the angle of reflection PcR will be found equal to the angle of incidence PcL

The figure shows the reflected beam to be 60° from the incident beam. If the mirror and disk are turned 10°, making the angle of incidence 40°, the angle of reflection will be 40°, and the two beams will be 80° apart; that is, when the mirror turns 10° the reflected beam moves 20°.

This illustrates the following law: —

When a mirror is revolving, the angular velocity of the reflected beam is twice the angular velocity of the reflecting mirror.

NOTE. — The Hartl optical disk is a most useful piece of apparatus for illustrating in the classroom the laws of reflection and refraction, and showing the action of mirrors, lenses, and prisms; and considering all that can be done with it, is inexpensive.

V. REFRACTION OF LIGHT

267. Refraction is the bending of a ray of light when it passes from one medium to another.

Experiment. — Place in a cylindrical quart bottle of clear white glass, which has a smooth, flat bottom, sufficient water to make it exactly half full, or so that the surface of the water shall exactly coincide with the horizontal diameter of the bottle (Fig. 148) when it lies on its side.

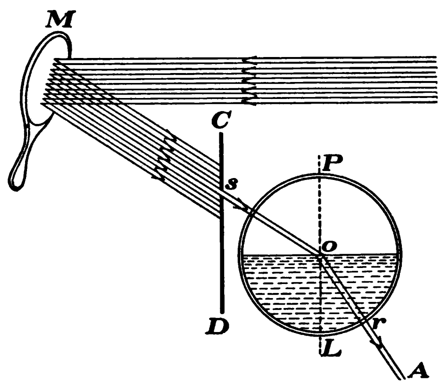

FIG. 148. — Diagram illustrating the refraction of light in passing from air into water.

Support the bottle in a

horizontal position, and place near it a cardboard screen CD having in it a horizontal slit s about 8 cm. long and 0.3 cm. wide. By a mirror M reflect light from a lantern or a porte lumière through the slit into the water, as shown in the figure, having it fall obliquely upon the water exactly at the center of the bottle. At o, where it strikes the water, the light will be bent or refracted *toward* the normal oL, taking the path oA.

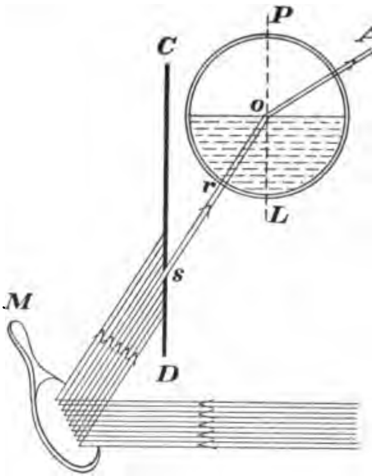

Fig. 149. — Diagram illustrating the refraction of light in passing from water into air.

Again, place the mirror M below the bottle, and arrange the apparatus (Fig. 149) so that the light shall pass obliquely up through the water and come out into the air at the point o. It will be refracted at o, being bent *away from* the normal, taking the path oA. If the water is slightly clouded by a little soap or sulphate of quinine, and the upper part of the bottle is filled with smoke before the stopper is inserted, the path of the light through the bottle can be more easily seen in these experiments.

Fig. 150.—Optical disk arranged to show the refraction of light by glass.

The same experiments are more easily performed by means of the optical disk which has fastened upon it a semicylindrical block of glass (Fig. 150).

The ray So in each case is the incident ray and oA is the refracted ray. The angle between the incident ray and the normal at the point of incidence, SoP (Fig. 148), or SoL (Fig. 149), is the *angle of incidence*. The angle between the refracted ray and the normal, AoL (Fig. 148), or AoP (Fig. 149), is the *angle of refraction*.

268. First law of refraction. — *Light on passing obliquely from a vacuum into any medium, or from air into any liquid or solid, is bent toward the normal.* In this case the angle of refraction is always less than the angle of incidence.

Conversely, *light on passing obliquely from any medium into a vacuum, or from a solid or liquid into air, is bent from the normal.* The angle of refraction is then greater than the angle of incidence.

When the angle of incidence is zero, that is, when the incident ray is not oblique to the surface, there is no refraction. For example, the light is not bent at the point *r* in either experiment, as the path *ro*, being a radius, is normal to the circumference at that point. The greater the angle of incidence, the more the light is bent in passing from one medium to another.

When light passes from one medium to another, it may be bent toward the normal, or from the normal, or it may not be bent at all. Generally, in passing from one medium to another more dense, it is bent toward the normal, but this is not always true. How it is bent depends upon its relative speed in the two media. If the speed of light is less in the second medium than in the first, it is bent toward the normal; if greater, away from the normal; if the speed is the same in the two media, it is not bent at all.

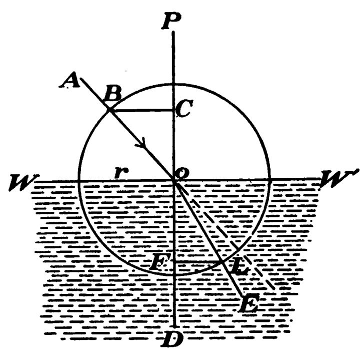

Fig. 151. — Diagram illustrating the index of refraction.

269. Index of refraction. — Let *Ao* (Fig. 151) be a ray of light incident on

the water WW' and let oE be the refracted ray, PD being the normal at the point of incidence. With the point of incidence as a center and with any convenient radius, as r, describe a circle. From the point B, where the incident ray cuts the circle, draw BC perpendicular to the other side of the incident angle AoP. In the same manner draw LF in the angle of refraction. The ratio of BC to the radius of the circle, $\frac{BC}{r}$, is defined in trigonometry as the *sine* of the angle AoP. Likewise the ratio $\frac{LF}{r}$ is the *sine* of the angle EoD.

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is called the *index of refraction* of the two media. $\frac{BC}{r} \div \frac{LF}{r} = \frac{BC}{LF}$. Hence $\frac{BC}{LF}$ is the index of refraction in this case.

Experiment. — Cut from a piece of cardboard a ring about 2.5 cm. wide, just large enough to slip over the bottle used in the last experiment. Graduate this ring in degrees and place it in position (Fig. 152). Paper protractors already graduated can be obtained from instrument dealers for this purpose. Repeat the first experiment in refraction again, measuring the angles of incidence and refraction. Find in trigonometrical tables the sines of the angles measured and determine the index of refraction, or measure the distances corresponding to BC and LF.

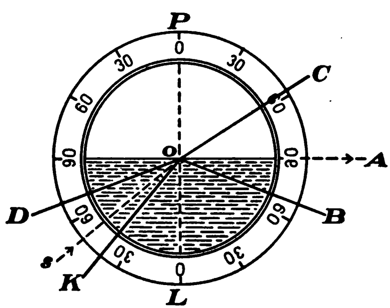

Fig. 152. — Diagram of apparatus for measuring the index of refraction.

If the index is determined several times in this way, the size of the incident angle being changed each time, it will be found to

be nearly the same each time; great accuracy cannot be obtained with such apparatus.

This illustrates a second law of refraction: *Whatever the angle of incidence, the index of refraction is constant for the same two media.*

When the light passes from water to air, the index is the reciprocal of that when the light is passing in the opposite direction.

The more a substance bends the light, the greater its index of refraction. For example if *Ao* (Fig. 151) were bent more at *o*, the angle *EoD* would be smaller and the line *LF* shorter; consequently the quotient of $\frac{BC}{LF}$ would be greater.

270. Absolute and relative indices of refraction. — The absolute index of refraction of a substance is that obtained by passing light into it from a vacuum. The relative index of refraction is that obtained when light passes from one substance into another. The index of a substance relative to air differs so little from its absolute index that usually no distinction is made between them.

The following table gives the indices of refraction of a few substances when light passes into them from the air.

Water	1.33	Benzene	1.50
Carbon bisulphide	1.64	Crown glass	1.52
Turpentine	1.47	Flint glass (averages about)	1.62
Alcohol	1.36	Diamond	2.47

Ordinarily the index of water is said to be $\frac{4}{3}$, crown glass $\frac{3}{2}$, flint glass $\frac{8}{5}$, and diamond $\frac{5}{2}$. When light passes from these substances into air, the reciprocals of these fractions must be used for the indices.

271. Cause of refraction. — The cause of refraction is the fact that light travels with different velocities in different media. For instance, the velocity of light in water is about three fourths of its velocity in air. In fact the

index of refraction indicates the relative velocities of light in the two media.

Let the short parallel lines (Fig. 153) represent the wave fronts of a beam of light passing obliquely from air into water. Evidently when a wave front reaches the water, one part of it, *a*, enters the water before the other part *i*, and as it travels more slowly than before, it can go only as far as *c* while *i* is going to *e*. Thus all the waves as they enter the water are given a new direction and the beam is bent. Conversely, if the light is going in the opposite direction, *e* will emerge before *c* does and gain upon it so that *e* goes to *i* in the time *c* goes to *a*. Thus the light is refracted away from the normal on passing out of water into air, just as much as it is refracted toward the normal on passing into the water.

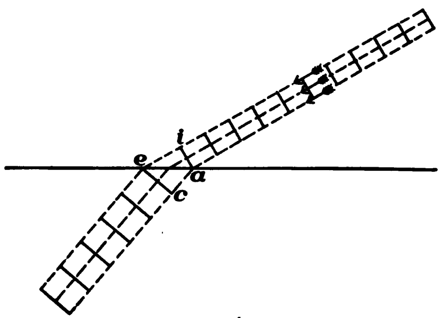

FIG. 153.—Diagram illustrating the cause of refraction.

272. Total internal reflection.—A study of Figures 149 and 152 shows that when light passes out from water into air the angle of refraction is greater than the angle of incidence. It is further evident that the angle of refraction cannot be greater than 90°, or greater than *PoA* (Fig. 152), for if it were, the light would be bent back again into the water and there would be no refraction. For example, the ray *Ko* is refracted along *oC*, but as the angle of refraction increases more rapidly than the angle of incidence, the ray *so* just grazes the surface, after re-

fraction, taking the path oA. For the ray Do refraction is therefore impossible and none of it can pass out into the air. It is all reflected at the point o along the path oB, the angle of reflection BoL being equal to the angle of incidence DoL. Such reflection is called *total internal reflection.*

273. Critical angle. — The angle of incidence that gives an angle of refraction of 90° is called the *critical angle*, soL (Fig. 152). Whenever the incident angle exceeds the critical angle, as DoL, and the light is attempting to pass into a less dense medium, total internal reflection occurs. The critical angle of a substance depends on its index of refraction. It is about 48½° for water, 41° for crown glass, 38° for flint glass, and 24° for the diamond. It is the angle whose sine equals the reciprocal of the index of refraction.

Repeat the experiment with the bottle (Fig. 149), trying various angles of incidence greater than 48½°. Observe that angles slightly smaller than that give *partial* internal reflection.

274. Illustrations of total internal reflection. — Nature furnishes many interesting phenomena caused by total internal reflection. The leaves of some plants, for example, glisten like silver when placed under water, because a layer of air adheres to them and the light in trying to pass into this layer of air from the water is totally reflected. If a tumbler of water is held above the head and the water surface viewed obliquely, it presents a brilliant mirrorlike appearance, reflecting objects on the other side of the glass, objects held just above the surface being invisible through it. Because of total internal reflection it is impossible for a person to see through, or for light to pass through, the square corner of a cut-glass inkstand.

Figure 154 represents a *cathetal* prism, a piece of polished glass having one 90° angle and two 45° angles. The incident angle for the ray *ro* and others parallel to it is 45°, or greater than the critical angle for glass; it is therefore totally reflected along *os*, not a trace of the light being able to pass through the polished surface *BC*. Such prisms are often used as reflectors in optical instruments instead of silvered surfaces.

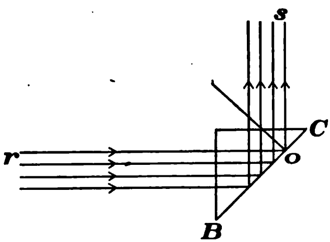

FIG. 154.—Diagram of total reflection in a cathetal prism.

A beautiful illustration of total internal reflection is furnished by a stream of water flowing from an opening in the side of a flask or jar (Fig. 155). If light is passed through the flask so as to emerge at the opening with the water, it will not pass on in a straight line, but will follow the stream down into a beaker held to receive it. This happens because the light strikes the upper curved surface of the stream at an angle greater than the critical angle, so that it is repeatedly reflected along this surface, being imprisoned in the stream. The effect is striking in a darkened room, especially if the light is passed through red or green glass. The beaker will appear to be filled with a red or green liquid. This principle is utilized in the illumination of electric fountains.

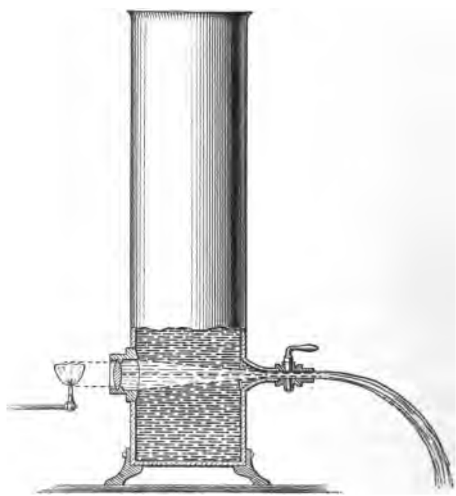
FIG. 155.—Apparatus for illustrating total internal reflection in a stream of water.

275. Refraction through a body with parallel faces. — Let *AF* (Fig. 156) represent a piece of thick plate glass whose faces *AB* and *DF* are parallel. The ray of light *so* on entering the glass obliquely is bent toward the normal *oe* along the path *or*, and on passing into the air at *r*, it is refracted from the normal *rc* in the direction *rt*. Since *or* makes equal angles with *DF* and *AB*, the ray is bent from the normal on leaving the glass exactly the same amount as it is bent toward the normal on entering it; its path *rt*, after passing through the glass, is therefore parallel to its original direction *soa*. Therefore there is no change of direction in light passing through a body when the surface where it enters is parallel to that where it leaves the body.

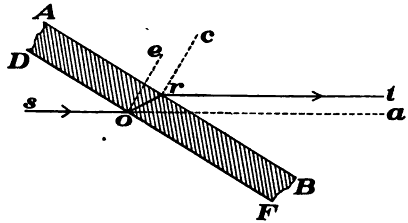

FIG. 156.—Diagram of the path of light through glass having parallel surfaces.

Experiment.— Pass a beam of light through a long narrow slit in a piece of cardboard, the slit being horizontal, and place a piece of plate glass obliquely in this narrow ribbon of light. If some of the light passes by the edge of the glass, it will be seen to be parallel to that which has passed through the glass, but separate from it. To make the two beams visible, darken the room and scatter chalk dust or smoke in their paths.

VI. DEVIATION AND DISPERSION

276. Refraction through a prism. — An *optical prism* is a portion of a transparent substance bounded by two plane surfaces that meet at an angle, that is, by two planes that are not parallel. The angle between the two surfaces is

called the *refracting angle* of the prism. Let ABC (Fig. 157) represent a section of a glass prism. If a ray enters the face AB and emerges from the face AC, the angle A is the refracting angle of the prism; but if the light should pass through the two faces AB and BC, then the angle B would be considered the refracting angle of the prism. The ray Go on entering the glass at o is refracted toward the normal oH, in the direction oE. At E it is again refracted, but away from the normal EF, in the direction EL. With reference to the face AB, Go is the incident ray and oE the refracted ray; and i and r are respectively angles of incidence and refraction; with reference to the face AC, oE is the incident and EL the refracted ray, and i' and r' respectively angles of incidence and refraction.

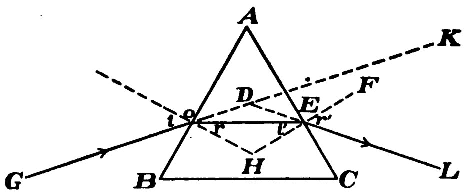

FIG. 157.—Diagram of the refraction of light through a prism.

277. The principal effects of prisms on light are two, *deviation* and *dispersion*.

As it will be necessary in the study of these effects to make use of many experiments with the prism, a description of the proper arrangement of the apparatus for projecting spectra and light through a prism will first be given. Figure 158 shows, in horizontal section, the general plan. Light from a porte lumière or a lantern is admitted to a darkened room through a vertical slit s. This slit should be 2 or 3 cm. long and about 1 mm. wide, cut in a piece of tin or cardboard. L is a convex lens, preferably of rather long focal length (about 60 cm.). A spectacle lens will answer the purpose. The distance from slit to lens must be carefully adjusted so as to give a sharply defined image of the slit on the screen at T before the prism is in place, the distance from L to T being the same as from L to R, the place where

the light will strike when the prism is in position. This adjustment may be made by reflecting the light by one face of the prism at *A* to

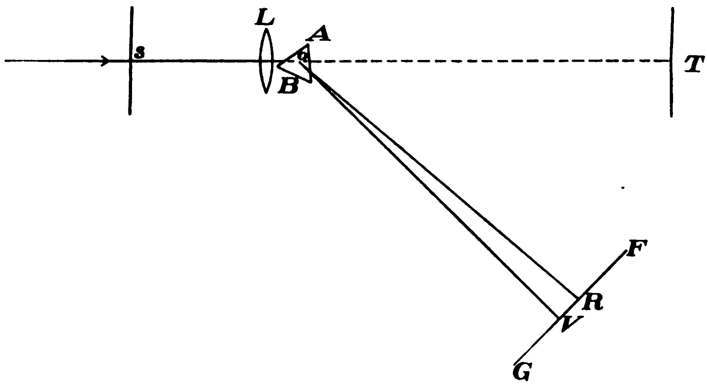

FIG. 158. — Diagram of the projection of a spectrum.

the screen *GF*. The prism *B* stands on end next to the lens. It is to be understood that in all the following experiments with prisms the arrangement is that described here.

278. Deviation. — *A* (Fig. 158) is the refracting angle of the prism. When the prism is removed, the light passes in a straight line to *T*; but when it is in position, the light is bent away from the refracting angle toward the thick part of the prism and goes to *R*. The angle *ToR* or *KDL* (Fig. 157) is the *angle of deviation.* It is the angle between the incident ray produced and the emergent ray, and it measures the amount the light is turned aside, or deviated, by the prism from its original path. By turning the prism back and forth about a vertical axis, the size of the angle of deviation *ToR* is changed, and by trial a position for the prism may be found in which the ray *oR* is turned aside from the line *oT* less than for any other position. When this adjustment is made, the angle of deviation is called the *angle of least deviation.* It occurs when the ray of light through the prism makes equal

angles with its two faces, that is, when the two angles *AoE* and *AEo* (Fig. 157) are equal. In all experiments with the prism, it should be adjusted for the angle of least deviation. The amount of deviation produced by a prism varies with the material of the prism, with its refracting angle, and with the color of the light passing through it.

Experiment. — To show that the deviation depends on the material of the prism, cover the slit with red glass and place in position successively a 60° crown glass prism, a 60° flint glass prism, and a 60° carbon bisulphide prism. Observe and compare the deviations in the different cases. It will be found that the deviation is almost the same for the last two prisms but that it is about three fourths as great for the crown glass as for either of the others, or 4 : 4 : 3.

279. Neutralization of deviation. — If two prisms are placed with their refracting angles *A* in opposite directions (Fig. 159), one will tend to counteract the other, *B* deviating the light to the right, and *C* to the left. If the prisms are of equal deviating power, there will be no deviation. This is the case with two prisms of the same angle and material, also with a flint glass prism combined with one of carbon bisulphide of equal angle.

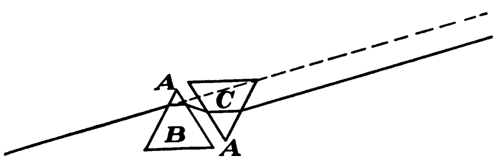

FIG. 159. — Diagram of prisms placed so that the second neutralize the deviation of the first.

280. Dispersion. — When white light, such as sunlight, is passed through the prism *B* (Fig. 158), it is not only deviated as before, but it is also spread out into a beautiful band of colors from *R* to *V*. The colors begin at *R* with red and shade by imperceptible gradations from one color into another, passing through orange, yellow, green, blue,

and ending with violet. The red is the least refrangible and the violet the most. This separation of light by a prism into its various colors is called *dispersion*, and the band of colors formed by the dispersion is called a *spectrum*. A spectrum made of sunlight is called a *solar* spectrum.

The angle between the extreme red and violet RoV measures the amount of the dispersion; it varies with the material and the angle of the prism.

Dispersion is due to the fact that deviation varies with the color of the light; the violet, for instance, is bent or deviated more than the blue, the blue more than the green, etc. It enables us to *analyze* light, that is, to separate any light into its component parts and determine its constituent colors. It shows that sunlight, for example, is not one single color, or is not monochromatic, but is composed of many colors and an infinite number of shades of those colors.

Experiment. — Arrange the apparatus as shown in Fig. 158, carefully adjusting the distance sL, and, using either sunlight or light from a lantern, project a spectrum on the screen FG. Compare carefully the lengths of the spectra produced by different prisms. Try in succession a 60° crown glass prism, a 60° flint glass prism, a 60° carbon bisulphide prism, and a crown or flint glass prism of smaller angle.

It will be found that the spectrum formed by the 60° crown glass prism is about half as long as that formed by the flint glass prism of 60°, while that formed by the carbon bisulphide is about twice as long as that formed by the flint glass, or four times as long as that formed by the crown glass. It will also be found that the length of the spectrum is very nearly proportional to the angle of the prism. The lengths of the spectra measure roughly the amount of the dispersion.

281. Neutralization of dispersion. — If white light is passed through two prisms of the same material and angle, placed as shown in Figure 159, not only is the deviation neutralized, but the dispersion of one is overcome by that

of the other. C tends to produce a spectrum equal in length to that produced by B, but with its colors reversed. The result is that the second prism recombines the colors, separated by the first, into white light, which strikes the screen (Fig. 158) at T. There is neither deviation nor dispersion.

Experiment. — With a single prism at B (Fig. 158) hold a concave mirror in the path of the diverging rays so as to reflect them back to one spot on a wall or screen. In this way the various colors may be recombined to form white light.

Again, hold a convex lens in the path of the rays so as to converge all the colors to one spot on the screen FG. White light will be formed.

These experiments illustrate *synthesis* of white light and show its composite nature.

282. Dispersion without deviation. — It has been shown that a 60° flint glass prism is equal to a 60° carbon bisulphide prism in its power of deviation, but that its power of dispersion is only about half as great. Hence, if two such prisms are placed together (Fig. 159) and substituted for B (Fig. 158) a spectrum is formed at T. There is no deviation, because one prism neutralizes the other in that respect, but there is dispersion, because the flint glass prism, having only half the dispersive power of the other, cannot entirely overcome its dispersion. The spectrum of the carbon bisulphide prism is shortened about one half. A flint glass prism of about 52° combined with a 60° crown glass would produce the same result. This principle is used in the construction of the direct vision spectroscope.

283. Deviation without dispersion. — A carbon bisulphide prism of 30° would have a dispersive power about equal to that of a 60° flint glass prism, but its power of deviation would be much less than that of the flint glass. Hence, if two such prisms are combined (Fig. 160), there

is deviation without dispersion. If a crown glass prism is combined with a flint glass prism of about half its angle, the same result is produced. Such a combination, producing deviation without dispersion, is called an *achromatic* combination. This principle of achromatism is of very great practical importance, and is applied in the manufacture of lenses for telescopes, microscopes, cameras, and other optical instruments.

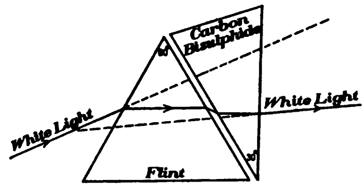

Fig. 160. — Diagram of prisms which deviate the light without dispersing it.

Exercises

With three kinds of prisms three different combinations of two each are possible: (1) crown glass, with flint glass; (2) crown glass with carbon bisulphide; and (3) flint glass with carbon bisulphide.

(1) Draw diagrams for the three combinations in which the prisms have equal angles, and indicate roughly the deviation and dispersion for each combination.

(2) Draw diagrams for the three combinations, making the angles of the prisms such that there shall be dispersion without deviation, indicating the order of the colors. (The size of the angles can be indicated only in a very general way). State the length of the spectrum relatively to that of either prism alone.

(3) Draw diagrams for three combinations, making the angles such that there shall be deviation without dispersion. Indicate the direction of the deviation, and state roughly its amount relatively to what each prism would produce alone.

284. The action of a sphere of water on light. — Let the circle (Fig. 161) represent the section of a raindrop, and *ogS* a line extending through the center of the drop toward the sun. The sunlight will illumine half of the drop *egk*. Let *la* be any ray of this light. Some of it will be reflected and not enter the drop; that which

does enter will be refracted and dispersed at *a*, and fall upon the back of the drop at *c*. Here some of it will pass on through, but a portion will be reflected to *b*, where it will emerge, being again refracted and still more dispersed. In general the various colors are so scattered by this dispersion that at a distance they are too feeble to affect the eye.

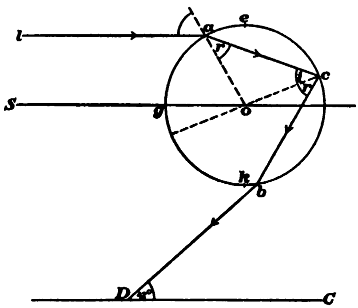

FIG. 161.—Diagram illustrating how a raindrop acts on light to produce the rainbow.

There are certain exceptions, however, to this dispersion which are important, because to them the rainbow is due. If the point *a* is 59° 23′ 30″ from *g*, the light emerges at *b* with the red non-divergent or not dispersed, and hence it can affect the eye at a considerable distance. This red light makes an angle with *DC*, which is parallel to *So*, of about 42°. To an observer at *D* looking in the direction *Db* the drop would appear red. Light entering the drop at some other point will traverse it and emerge with its violet non-divergent, and so with the other colors. The non-divergent violet light crosses *DC* at about 40°, the other colors falling between 40° and 42°. This explains why dewdrops sparkling in the sunlight appear of different colors, some one color and some another, according to the angle at which they are viewed.

If an air thermometer bulb 4 or 5 cm. in diameter is filled with water, and held in a beam of light in a darkened room, this action of a sphere of water upon light may be easily illustrated. Let the light enter the room through a hole in a large cardboard, and the sphere be held about 50 cm. back from the card. A miniature rain-

bow will be thrown back upon the cardboard, forming a circle around the opening; or if the wall of the room is white, the bow may be thrown upon that. The red of the bow will be found upon the outside and the violet toward the center.

285. The rainbow is a solar spectrum produced by the refraction and dispersion of sunlight by raindrops. The *axis* of the bow *DC* (Fig. 162) is a straight line from the sun through the eye of the observer to the center of the circle of which the bow is a part. Each line from the drops forming the red part of the bow to the eye of the observer makes an angle of about 42° with this axis, according to the explanation in the last paragraph; while lines from the drops forming the violet part of the bow make angles of about 40° with this axis. The red is therefore on the outside of this *primary* bow, which is about 2° wide.

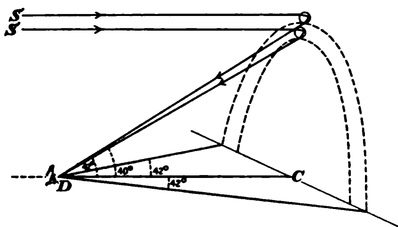

FIG. 162.—Diagram illustrating how a rainbow is produced.

The ordinary conditions for seeing a rainbow are (1) that the sun should be shining in one part of the heavens; (2) that it should be raining in the opposite part of the heavens; and (3) that the sun should not be more than about 40° above the horizon; for example, we may look for one toward evening, if the sun is shining in the west and it is raining east of us. If the earth were a plane and the sun just on the horizon, the bow would be a half circle and the top of it 42° high. A person in a balloon might possibly see a rainbow forming a complete circle.

The *secondary* bow which is sometimes seen outside the

primary is wider and fainter. It is formed by light which is reflected twice inside of the raindrop, and the colors are reversed, the violet being outside.

VII. COLOR

286. Color of light. — Ether waves capable of affecting the human eye, and hence called light waves, vary in length from about 0.00081 to 0.00033 mm.[1] *Color of light depends on its wave length;* it bears the same relation to light that pitch does to sound. The longest waves constitute red and the shortest violet light. Waves longer than the red which cannot be seen are sometimes called infra-red (*infra*, below), and those too short to be visible, ultra-violet (*ultra*, beyond).

Newton named seven distinct colors as existing in the spectrum of white light, namely: red, orange, yellow, green, blue, indigo, and violet; but since there is an infinite number of wave lengths between the longest red and the shortest violet waves, there is an infinite number of colors. The solar spectrum, for instance, beginning with the dark red, passes by imperceptible gradations into lighter red and orange, and so on through the other colors as the wave lengths shorten; there are no sharp lines of demarkation between the various colors.

The short waves, such as the violet and the ultra-violet, are specially active chemically, producing the chemical changes of photography and those that occur in the leaves of growing plants.

287. Measurement of light waves. — The lengths of the waves of light, although exceedingly minute, have been measured with great accuracy. As we have seen, there are many different wave lengths for each hue, so we can-

[1] Drude.

not state definitely the lengths of the waves for any one hue as a whole, but only for some particular part of it.

The wave lengths of light are so very short that it is inconvenient to express them in meters or centimeters. For instance, certain waves of yellow light are 0.0000005896 m. long. To avoid such inconvenient numbers as this, a unit called a *tenth meter* has been adopted for expressing wave lengths of light. A tenth meter is $\frac{1}{10^{10}}$ of a meter. This unit is sometimes called the Ångström unit. To express the wave length given above in tenth meters the decimal point is moved ten places to the right, thus, 5896.

Certain dark lines have been discovered (§ 298) crossing the various colors of the solar spectrum, always occupying the same relative positions; these lines have been designated by the letters A to H. The following table gives the wave lengths for the lines of the solar spectrum from A to H in tenth meters: —

A	in the dark red	7621.	E_1	in the green	5270.495
B	in the red	6870.186	E_2	in the green	5269.723
C	in the orange	6563.054	F	in the greenish blue	4861.527
D_1	in the yellow	5896.357	G	in the deep blue	4340.634
D_2	in the yellow	5890.186	H	in the extreme violet	3968.625

288. The color of a body. — The color of light must be distinguished from the color of a body. The color of light, as we have seen, depends upon its wave length; but the color of a body depends upon the light it reflects, or if it is transparent, upon the light which passes through it. An opaque body is red, for example, because it reflects red light and absorbs the light of other colors incident upon it; or a piece of glass is green, not because it colors the light green, but because green light passes through it, most of the other colors being absorbed.

Experiment. — Project a solar or an electric spectrum upon a white screen, making the room as dark as possible. Place a narrow strip of

red cardboard in the various colors; it will appear black everywhere except in the red, where it will appear red. If a black strip is placed by the side of the red one and both held lengthwise in the spectrum, it will be impossible to distinguish one from the other except in the red light of the spectrum. Red objects are about the only ones that illustrate this principle well in a simple way, although a dark blue card does nearly as well in a spectrum projected by a diffraction grating (§ 303).

An object seldom reflects light of a single hue only, but the reflected light is a mixture of two or more colors in which one color may predominate, and generally some white light is mixed with the reflected light. A color is said to be *saturated* when wholly free from white light. An object may appear yellow because it reflects yellow light only, or because yellow predominates in the reflected light, or it might appear yellow and not reflect any yellow light at all, its color being due to a mixture of red and green light (§ 289). An object is white when it reflects all the colors in the same proportion as they exist in sunlight. Some artificial lights are deficient in certain colors, and hence objects which would reflect those colors, if present, do not appear of the same color in them as when viewed by sunlight. This explains why it is difficult to match certain colors by lamplight or gaslight.

Experiment. — Let a strip of asbestos paper be soaked in a saturated solution of sodium nitrate and bound around a Bunsen burner so that it projects a little above its top. This will give a bright yellow sodium flame. If gas is not available, salt placed on a plate and saturated with alcohol may be used in its place. With this flame in a room from which all other light is excluded, let articles of various colors be examined. Nearly all of them, except the white and yellow ones, will appear black or a dark slate color. White and yellow objects appear yellow because they can reflect yellow light, but a dark red card is black because it cannot.

Experiment. — To illustrate the color of transparent bodies, cover the slit (Fig. 158) with a piece of red glass. Nearly all of the light

will be cut out of the spectrum except the red. A piece of blue glass will transmit blue light mainly, but some green, and a very small amount of red, while a yellow glass will transmit yellow and some green. Consequently, when a yellow and a blue glass are used together, only green can pass through. The same effects may be obtained by using, instead of the pieces of glass, a solution of picric acid and a blue liquid made by adding an excess of ammonia to a solution of copper sulphate.

289. Mixing light of different colors. — We have seen that when all the colors of the solar spectrum are combined, white light results; but it is possible to produce the sensation of white by other combinations. For example, any two colors which are opposite each other (Fig. 163) produce this effect when combined. Any two colors so related are termed *complementary*. When any other two colors are combined, the intermediate color sensation is produced. Green and red, for instance, give yellow. The sensation of white may also be produced by a combination of three colors, as red, green, and dark blue. In fact, all color sensations may be formed by combinations of these three colors in various proportions.

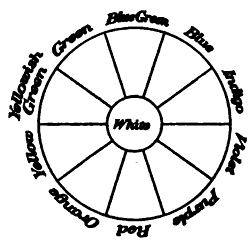

FIG. 163. — Chart of complementary colors.

These facts may be illustrated by Maxwell's color disks. These disks are made of colored cardboard, and are slit from center to circumference so that two or more of them of different colors may be placed together in such a manner as to expose any desired portion of the surface of each to view. They are then mounted on a whirling machine, an electric motor, or a large top and rotated rapidly. The effect of combinations of various colors in different proportions may thus be studied.

290. Mixing pigments. — We have seen that a mixture of blue and yellow light produces white. If, however,

blue and yellow pigments are mixed, the result is green. Every child accustomed to the use of water colors is familiar with this fact. Cloth may be dyed green by being dipped first in a yellow dye and then into a blue one, or the blackboard may be made green by marking upon it first with a yellow crayon, and then with a blue one. A little moisture will intensify the color. These phenomena are explained by the fact that the blue pigment reflects some green light as well as blue, and the yellow pigment also reflects some green as well as yellow; but the blue pigment absorbs the yellow, and the yellow absorbs the blue, hence, when they are mixed, only the green remains unabsorbed.

291. The Young-Helmholtz theory of color sensation. — According to this theory, proposed by Young in 1802, and further developed by Helmholtz later, the nerve ends of the eye are of three sorts, viz. (1) those which are stimulated by red light, (2) those stimulated by green light, and (3) those stimulated by blue light. We may for convenience call them the red nerves, the green nerves, and the blue nerves. When all three are stimulated equally, we have the sensation of white; and the other color sensations are produced by the stimulation of the different sets of nerves in various degrees. Thus, if the red nerves only are stimulated by the ether waves, we say the light, or the body from which it comes, is red; if the red and green nerves are acted upon, we have a sensation which we describe by the term yellow or by the name of some hue between red and green.

This theory, while not fully confirmed, affords a very good explanation of color blindness. Some persons are red blind. Such persons cannot distinguish red from green, a red cherry from the green leaves of the tree except by its form, or they cannot distinguish between the

red danger signal and the green safety signal. To them all colors are varieties of blue and green. This form of color blindness is supposed to be due to the fact that the red nerves are either lacking or are inactive. It is the most common form of color blindness; but there are those who are green blind or blue blind.

VIII. KINDS OF SPECTRA

292. The spectroscope. — The methods so far described for forming spectra, while adapted for producing them on a large scale, are not suitable for a refined and accurate study of them. The spectroscope is an instrument for that purpose. Its essential parts are a collimator *C* (Fig. 164), a prism *P*, and a telescope *T*. The collimator is a tube which has an adjustable slit at one end and a lens at the other. The slit *S* (Fig. 158) and the lens *L* constitute a crude collimator except that they are not mounted in a tube. The light to be examined by the spectroscope, entering the slit, passes through the collimator to the prism and then directly into the telescope. The observer views the spectrum by looking through the telescope toward the prism. Many such instruments are provided with a circle graduated in degrees for the purpose of measuring deviation, dispersion, etc., and in that case they are called spectrometers instead of spectroscopes.

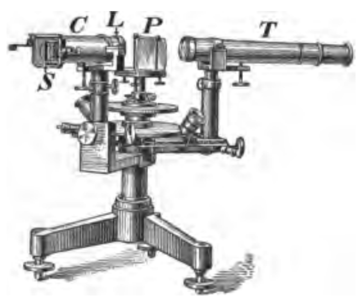

FIG. 164. — Spectroscope.

293. Kinds of spectra. — Spectra are classified as *continuous* and *discontinuous*. In a continuous spectrum

there are no wave lengths or colors lacking from the longest red at one end to the shortest violet at the other end of the spectrum.

A discontinuous spectrum, on the other hand, is characterized by the absence of particular waves so that there is not a continuous band of light from one end of the spectrum to the other, but blank or dark spaces without light occur.

Discontinuous spectra are of two kinds, *absorption* and *bright-line* spectra.

294. Continuous spectra. — *The light from incandescent solids and liquids forms continuous spectra.* When any body is heated to about 525° C., it begins to glow with red light, and it is said to be red-hot; as its temperature rises, the intensity of the shorter waves increases, until at about 1200°C. the body is white-hot.

The light of an arc lamp, as well as that from molten iron, forms a continuous spectrum, — in one case because the source of light is a solid, in the other, because it is a liquid. The light of a candle flame or a gas flame is due to small solid particles of incandescent carbon within the flame, and hence its spectrum is also continuous.

295. Bright-line spectra. — When light from an incandescent gas or vapor is passed through a prism, its spectrum is seen to consist of a number of bright lines. It is discontinuous. Thus the spectrum of sodium vapor consists of a pair of yellow lines; and hydrogen gives four lines, one red, one blue, and two violet. These lines fall in their proper places or colors as if all the other colors were present.

Each element in the gaseous state has its own characteristic spectrum. This fact is of inestimable service to science; it enables the chemist to detect and test new elements, and the astronomer to solve many problems about

the composition, conditions, and motions of the heavenly bodies. Incredibly small amounts of elements may be detected by the chemist with great certainty by means of their spectra.

Experiment 1. — Project a spectrum (§ 277), using an electric lamp as the source of light, and observe the spectrum formed. It will be found to extend continuously from the red through to the violet, with no dark spaces or with no colors lacking. The spectrum is continuous because the source of light is an incandescent solid.

Experiment 2. — Let a sheet of tin with a narrow slit in it be placed in front of an electric lamp, an ordinary gas flame, or a candle flame. If this slit is viewed through a prism held near the eye with its refracting edge parallel to the slit, it will be seen spread out or widened into a spectrum which is continuous, all the colors being present.

Experiment 3. — Let a strip of asbestos paper which has been soaked in a strong solution of sodium nitrate be bound around a Bunsen burner so that its edge projects a little above the top of the burner. This will give a bright yellow sodium flame. If this light is viewed as before through a prism and slit, the slit will not be spread out or widened into a band of different colors. The spectrum formed will be yellow only and consist of a narrow band or line about as wide as the slit itself.

This is a bright-line or discontinuous spectrum, the light coming from the vapor of sodium in the flame.

If light from these same sources is viewed through a spectroscope, the same result will be obtained.

Experiment 4. — Let a carbon which has been saturated with sodium nitrate by being soaked in a saturated solution of that salt be made the upper or positive carbon of the arc lamp of a lantern, and let a spectrum be projected by the light of this lamp on a screen. The spectrum will be a continuous one produced by the light from the ends of the carbons, but superimposed upon this continuous spectrum will be another one consisting of several bright lines. The brightest of these will be the one in the yellow; one or more may also be seen in the green, one in the red, and one in the violet. These bright lines are produced by the light from the vaporized sodium, and if it were not for the light from the ends of the carbons, would constitute the only light on the screen.

296. Absorption spectra. — When light from an incandescent solid or liquid passes through any transparent medium, light waves of some lengths are absorbed and not permitted to pass through. If the spectrum of the light which has thus been deprived or robbed of some of its wave lengths is examined either by a spectroscope or by being projected on a screen, some of the colors are found to be greatly weakened or they are wanting altogether. Dark spaces are thus produced in the spectrum which correspond to the color of the light that is lacking. If the absorption is due to a solid or liquid, the dark spaces may be dark bands or they may cover the greater part of the spectrum; but if it is due to gases, the dark places consist of narrow lines across the spectrum.

Such a spectrum, from which a portion of the light has been abstracted by absorption, is called an absorption spectrum of the body through which the light passes.

The spectra described in the last experiment of § 288 are of this kind.

297. Absorption spectra of gases. — In our study of resonance in Sound (§ 214) we have seen how a tuning fork can take up or absorb motion from sound waves in air, provided the waves are of the same length as the fork itself can produce. Gases behave in a similar way toward light waves in ether. *A gas absorbs light of the same wave length or of the same color as it radiates when it is heated to incandescence.* Hot sodium vapor, for example, radiates yellow light of a definite wave length; it also absorbs yellow light of the same wave length.

The absorption of yellow light by sodium vapor may be shown by experiment, if the vapor is at a lower temperature than the source of the light passing through it.

Experiment 1. — Remove the objective from the lantern and in its stead place a screen provided with a narrow slit. This screen may

be of wood with a hole through it about 3 cm. in diameter, the hole being covered with two pieces of tin which do not quite meet at its center. Project the spectrum of the light of the lantern, which passes through this slit, by means of a lens and a prism in the usual way. This will be a continuous spectrum. Place a Bunsen burner between the condensing lenses of the lantern and the slit, about 6 cm. from the latter. Prepare a compact ball of asbestos wicking about a centimeter in diameter and place it in a loop at the end of an iron wire. This wicking can be obtained from a plumber. Heat the ball in the flame of the Bunsen burner and plunge it while hot into a saturated solution of sodium nitrate, which should stand near by. Now hold the ball in the flame of the burner. This will produce a bright yellow light which, passing through the slit and prism, will give a spectrum of a bright yellow line, provided the light of the lantern is cut off by the interposition of the sheet of tin. This spectrum is due to the sodium of the sodium nitrate. Metallic sodium may be burned in the flame and the same results obtained, but it is not so easily manipulated If the tin is removed while the yellow line is visible, the yellow line immediately changes to a dark line across the otherwise continuous spectrum of the lantern. The ball should be plunged now and then into the solution of sodium nitrate. This reduces its temperature and keeps it saturated. The results will be better after it has been in use a few minutes.

Experiment 2. — The absorption spectrum of sodium is more easily shown as follows: Let the negative or vertical carbon of a right-angled arc lamp be saturated with sodium nitrate, the horizontal carbon being unsaturated. Project a spectrum with this lamp in the usual way, keeping the negative carbon rather high and drawing the positive well back. The spectrum which in a previous experiment (§ 295) had a bright yellow line will now have a heavy dark line in exactly the same place. At times the other lines seen in the other experiment are also reversed.

This dark line is formed because the light from the very hot positive carbon has to pass through the sodium vapor rising from the negative one, which has a lower temperature.

298. The Fraunhofer lines. — When a solar spectrum is projected and great care is taken in focusing, it is found to be crossed by many dark absorption lines, some of them more prominent than others. These

FIG. 165.—Portion of iron vapor spectrum and the solar spectrum. The bright band represents the solar spectrum with its absorption lines.

lines were first discovered in 1802 by Wollaston, but were rediscovered in 1817 by Fraunhofer, who mapped a very large number of them and first noticed that many of them exactly coincide in position with the bright lines of the spectra of certain elements of the earth. He designated the more prominent lines by letters, the *A* line being in the edge of the red and the *H* line at the limit of the violet. The *D* line exactly coincides with the bright yellow line of the sodium spectrum. This fact is taken as proving the existence of sodium in the sun, because just as we can produce this dark sodium line artificially by passing light from a very hot body through cooler sodium vapor, so the light from the hot central body of the sun has to pass through the sodium vapor contained in the sun's atmosphere to reach the earth.

Kirchhoff showed that over four hundred bright lines of the iron vapor spectrum are exactly matched by dark lines in the sun's spectrum (Fig. 165). Thus the presence of sodium, iron, and many other elements familiar to us have been proved to exist in the sun, and thus the stars are made to tell of what they are composed.

IX. INTERFERENCE AND DIFFRACTION

299. Interference of light. — We have already (§ 201) studied interference of wave motion and found in Sound some excellent examples of it. In light also there are many exceedingly interesting and beautiful illustrations of interference which are of great scientific importance and afford the very best proof of the truth of the wave theory of light.

Experiment. — Let two strips of thoroughly clean plate glass be tightly clamped together. When the reflection of sodium light (§ 288) in the plate glass is observed in a darkened room, it will be seen to consist of a series of black and yellow lines or bands. The dark lines occur because at those places the light is destroyed by interference; and the bright bands occur because the light is intensified by interference.

If the glass is viewed obliquely in sunlight, a series of bands of different colors will be observed. These bands, which are sometimes circular, are known as *Newton's rings*.

300. Interference by thin films. — Let *ab* (Fig. 166) represent the very thin film of air between the two pieces of glass, its thickness being greatly exaggerated in the figure, and let *RS* be a ray of light passing through toward *S*. At each of the surfaces *i*, *c*, *e*, and *o* some of the light will be reflected back toward *R*; but we are here concerned only with that reflected at *c* and *e* on the two surfaces of the air

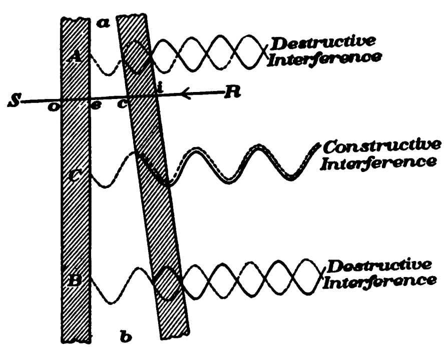

FIG. 166. — Diagram illustrating the interference of light by a thin film of air.

film. It is evident that the part reflected at *e* goes through the film twice and travels that much farther than the part reflected at *c*; the light reflected at the back surface of the air film will be behind that reflected from the front surface because it travels farther. These waves reflected from the two surfaces of the film interfere, sometimes destroying and sometimes intensifying each other. For example, if one is a half wave length behind the other, as at *A*, or a wave and a half as at *B*, destructive interference occurs; but if the difference is one or more whole wave lengths, as at *C*, constructive interference occurs and the light is intensified. The thickness of the film and the length of the waves determine which will occur. If the thickness is such that the red is intensified in white light, other colors, because differing in wave length from the red, will be wholly or partially destroyed; and if the film varies in thickness, it will appear of different colors at different places.

The colors of a soap bubble are caused in this way, the interfering rays being reflected from the two surfaces of the soap film. Oil upon water, films of oxide on steel, and air films in cracks in ice furnish further examples of this phenomenon.

Destructive interference occurs when the thickness of the film is $\frac{2}{4}$, $\frac{4}{4}$, $\frac{6}{4}$, etc., of a wave length, and constructive interference, when it is $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc., of a wave length. One might think that it would be just the reverse of this, but there is a loss of a half wave length at the reflection on the surface *c*, which causes the interference to be the reverse of what one would expect.

301. Diffraction. — Newton objected to the wave theory of light because he thought that if light consisted of waves it would bend around the edges of opaque objects as sound waves do, making shadows impossible. There are many proofs that this bending does occur. It is shown best

when light passes through a very narrow slit, for it bends around the edges of the slit and spreads out in all directions. This bending of light as it passes the edge of an object or goes through a narrow opening is known as *diffraction.*

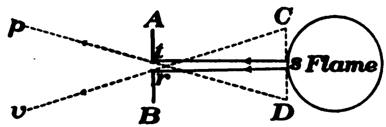

Fig. 167.—Diagram illustrating diffraction.

Diffraction may be observed in the following manner: The flame of a lamp is surrounded by a tin chimney in which there is a vertical slit nearly a millimeter wide and two or three centimeters long. The observer standing at a distance of two or three meters then views this slit through another slit which is close to the eye and parallel to the first slit.

Note.—It is essential that the slit before the eye shall be clean-cut and very narrow. A single cut through a thin card by a sharp knife often makes a suitable slit, or two visiting cards held with their edges nearly touching answer admirably for the purpose, especially because the effect of changing the width of the slit can be easily observed. A cut in the film of a "slow" photographic plate or a scratch by a sharp instrument in the silvering of a piece of German looking-glass also makes an excellent slit for the purpose.

When viewed in this way the slit before the lamp appears very much wider than to the naked eye, and on either side of it there is seen a series of bands of light called *fringes,* separated from each other by intervals of darkness. The appearance changes with the width of the slit before the eye. When the slit is widened, the bands become narrower and crowd more closely together; when it is narrowed, the bands widen and become more widely separated from each other.

To understand this phenomenon let us study Figure 167, in which *tr* represents the slit before the eye and *S* the one in the chimney. The light from *s* on passing through the second slit bends around its edges, *t* and *r*, and spreads out into a wedge-shaped body of light bounded by the rays *tp* and *rv*. These rays entering the eye just

back of *rt* appear to come from the points *C* and *D* and the slit *s* appears to have the width *CD*.

The bands are caused by interference, but for an explanation of them the student is referred to more advanced works on physics. (See Tyndall, *On Light*, pp. 80–90.)

Diffraction furnishes an exception to the statement that "light travels in straight lines," and it disposes of Newton's objection to the wave theory. It is in itself one of the best proofs of the truth of that theory.

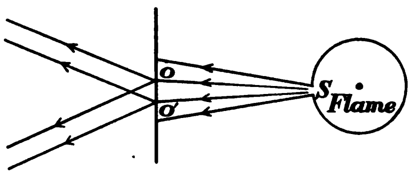

FIG. 168. — Diagram illustrating interference by diffraction.

302. Interference by diffraction. — If instead of a single slit, as *rt* (Fig. 167), two or more slits are placed side by side, as at *o* and *o′* (Fig. 168), the light from one slit spreading out overlaps that from the other and the two sets of waves interfere with each other, the light at some points being destroyed and at others being intensified.

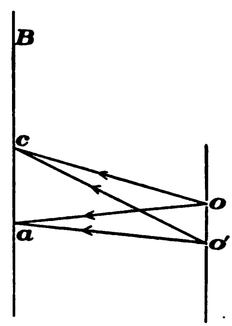

FIG. 169. — Diagram illustrating the production of spectra by diffraction.

In Figure 169 it is evident that the point *a* is equally distant from both slits *o* and *o′*, hence the waves from the two slits will meet at *a* in the same phase and intensify each other; but to points on either side of *a*, as *c*, the paths from *o* and *o′* are unequal, — *o′c* is longer than *oc*. The difference is exceedingly small when *c* is very near *a*, but increases as *c* is moved away from *a* toward *B*. When *o′c* is one *whole* wave length longer than *oc*, as it must be at some point between *a* and *B*, then constructive

interference occurs, forming a bright line. The same result happens at other points when the ray from one slit is two or three or more *whole* wave lengths longer than that from the other. When the difference is a half wave length, or $1\frac{1}{2}$, $2\frac{1}{2}$, etc., wave lengths, then the waves destroy each other. If the light is of one color only, as red, then a series of red bands is formed on each side of *a* with dark spaces between them; but when white light is used, each color is intensified at different places so that several complete spectra are formed on each side of *a*.

303. Diffraction gratings are made by ruling upon glass by a diamond point thousands of parallel lines per inch. The spaces between these lines form very narrow slits for the light to pass through. If such a grating (or a photograph of one) is substituted for the prism (Fig. 158), spectra may be projected on the screen which in several respects are superior to those produced by a prism. Beautiful diffraction spectra may be seen by holding a fine bird's feather close to the eye and looking at the sun through an opening among the leaves of a tree. The feather forms the grating.

X. IMAGES FORMED BY PLANE MIRRORS

304. Formation of images. — An image is an apparent object formed by light. If the light actually passes through the points of the image to form it, the image is *real;* but if the light only seems to pass through it but does not, it is *virtual.* Real images may be caught upon the hand or projected on a screen; virtual images cannot.

When a room is darkened and there is a small opening through the window shade, real inverted images of objects outside may often be seen on the walls and ceiling of the room. Figure 170 shows how such images are formed.

The ray *co*, passing through the aperture *o* in the screen *G*, forms an image of the point *c* at *s* on the screen *H*. In the same way images of *e* and *r* are formed at *a* and *i*,

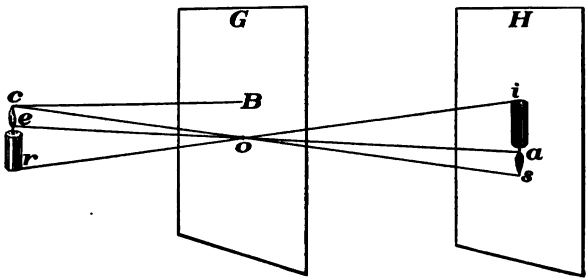

Fig. 170.—Diagram of the formation of an image through a small aperture.

and so of all other points of the candle. The image is inverted because the rays all cross at the aperture.

If there were another aperture at *B*, the ray *cB* would form another image of *c* on *H*, and for each new opening made in *G*, a new image of *c* would appear. In fact, if the screen *G* were removed entirely, *H* would be covered with a multitude of images of *c*, and not only of *c*, but also of all other points of the candle, and hence of the whole candle. These images would not be visible, however, because, there being so many of them, they would overlap and obliterate one another.

Experiment.—Procure a box large enough to place within it with safety a lighted candle. It should be ventilated at top and bottom in such a way as not to allow much light to escape from it. Bore a hole about 2.5 cm. in diameter in one side of the box on a level with the candle flame, and cover the hole with tin foil. Place a white screen a meter or so from the box and darken the room. Prick a hole in the tin foil with a hat pin. An inverted image of the candle will appear on the screen. Prick another hole and a second image will be formed. After forming several images in this way, hold a convex lens near the foil, and by moving it back and forth gather all the images formed into one image.

Remove the lens and multiply the images by pricking holes in the foil until it is all cut away. As more and more images are formed, they overlap and obliterate one another, and finally nothing will appear but a patch of light of the same shape as the hole in the foil. The experiment shows, however, that this patch of light is composed of a multitude of images of the candle.

NOTE. — This experiment may be performed much more satisfactorily by the use of the electric lamp and the lantern. Remove all the lenses, and cover the opening with a metallic cap having an opening in it about 2.5 cm. in diameter. Cover this opening with tin foil and proceed as above.

From the above discussion and experiment, we learn that each ray of light from a point in an object forms an image of the point from which it comes, and that to form a perfect image there must be but one image of each point, so that there shall be no overlapping of images. A very small aperture gives an image distinct in outline but dim. Increasing the size of the aperture makes the image brighter but less perfect in outline. Very good photographs can be taken by "pin hole" cameras, which make use of this method of forming images.

We shall learn later how many rays from each point of an object can be concentrated at one point, and thus images be formed which are both bright and distinct.

305. Mirrors classified. — Mirrors may be divided into two classes, *plane* and *curved;* and curved mirrors may be classified as spherical, cylindrical, parabolic, etc. Again, curved mirrors may be either *concave* or *convex.* Only plane and spherical mirrors will be considered in this book. A plane mirror is one whose reflecting surface is a plane. A spherical mirror is one whose reflecting surface is a portion of a sphere. It is concave if it reflects light toward the center of the sphere, and convex if it reflects light away from the center of the sphere.

306. Image of a point in a plane mirror. — Let *MN*

(Fig. 171) be a plane mirror, and O a point whose image formed by the mirror is to be located. *In general the position of the image of a point is located by finding the intersection of any two rays that form the image.* Let OF and OE be any two rays from O incident on the mirror. OF is reflected in the direction FC, making the angle of reflection BFC equal to the angle of incidence BFO. Therefore the image of O lies in the line FC, or FC produced.

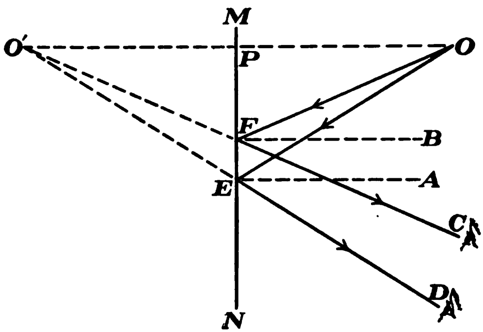

FIG. 171.—Diagram of the image of a point formed by a plane mirror.

Likewise the ray OE is reflected in the line ED, making the angle AED equal to the angle AEO, and the image of O, therefore, lies in the line ED, or ED produced. Since it lies in the line FC, or FC produced, and ED, or ED produced, it must be at their intersection, O'.

An observer at C or D would receive the light as if it came from O', but since the light does not actually pass through O', the image is virtual.

Starting with the fact that the angles OFB and OEA are respectively equal to the angles BFC and AED, it is easy to prove by geometry that the two triangles OEF and $O'EF$ are equal, and consequently that the two lines OF and OE are equal, respectively, to the two lines $O'F$ and $O'E$. Hence MN is perpendicular to OO' at its middle point, and the points O and O' are equally distant from any point in the mirror MN. Therefore, *the image of a point in a plane mirror lies in a line drawn from that*

point perpendicular to the mirror, and is as far back of the mirror as the point is in front of it.

307. Image of an object formed by a plane mirror. — The image of a straight object like *AB* (Fig. 172), formed by a plane mirror, may be located by locating the images of its extreme points *A* and *B*. The image of *A* is at *C*, its position being found by drawing *AC* perpendicular to the mirror *MN*, and making *oC* equal to *oA*. The image of *B* is found to be at *D* in the same manner, and the image of the whole object is found by connecting the points *C* and *D*.

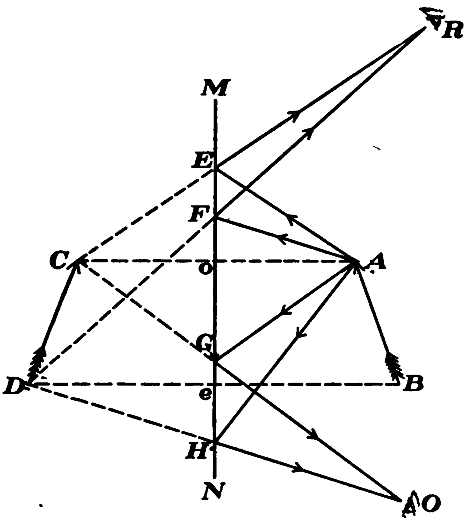

Fig. 172. — Diagram of the image of an object formed by a plane mirror.

To trace the path of the ray by which an observer at *R* sees the image of *A*, draw a line from the image to the eye, as *CR*. From *E*, the intersection of this line with the mirror *MN*, draw a line to the object, as *EA*. *AER* is the path of the ray by which the eye at *R* sees the image of *A*. In like manner the paths of the rays by which an observer at *O* sees the images of *A* and *B* may be drawn. This construction is the converse of that demonstrated in the preceding section.

Query. — What must be the length of a mirror which, in a vertical position, permits a person to see his whole figure?

308. Multiple reflection. — When two mirrors are placed so that light may be reflected back and forth from one to the other, several images of an object placed between them may be formed, the number of images depending upon the angle which the mirrors make with each other. When the angle is a right angle, there are three images; and when it is 60°, there are five.

Let AC and BC (Fig. 173) represent two mirrors whose planes intersect at C, the angle ACB being 60°. The image of the point O in AC is located at I' by drawing OI' perpendicular to AC and making rI' equal to rO. I'' is the image of the image I' located in the mirror BC as I' was in AC. I''' is the image of I'' in AC. Again, starting with O, an image is formed in BC at I_1. I_2 is the image of I_1, and I_3 of I_2. The image I''' of the first series exactly coincides with I_3 of the second series, the two uniting to form one image. For that reason there are five images instead of six when the angle between the mirrors is exactly 60°. There can be no image of this last image, since it is back of both mirrors.

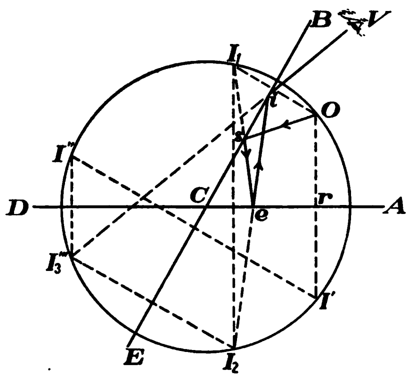

FIG. 173. — Diagram of multiple images formed by two mirrors.

It has been shown (§ 306) that a point and its image are equally distant from any point in the mirror. It follows therefore that O and I' are equally distant from any point as C in the mirror AC; and I' and I'' are equally distant from any point as C in the mirror BC. Likewise I''

and I''', O and I_1, $'I_1$ and I_2, and I_2 and I_3 can be shown to be equally distant from C. Therefore, the point O and all of its images lie in the circumference of a circle whose center lies in the intersection of the planes of the two mirrors.

The path of the light by which an observer V sees the image I_3 is from O to s, s to e, e to i, and i to V. This path is found by the continued application of the method given in § 307. First, draw a line from the image I_3 to the eye; from the intersection of this line with the mirror at i a line is drawn to the preceding image I_2, and from the intersection of this line with the mirror at e a line is drawn to the next preceding image I_1, and finally from s to O.

The kaleidoscope furnishes an interesting illustration of multiple reflection. It consists of a tube in which three mirrors of equal width make angles of 60° with one another so that five images of broken bits of colored glass are symmetrically placed around the intersections of the mirrors. These five images with the objects themselves form a regular six-sided figure.

Experiment. — Join two thin boards (10 × 20 cm.) together with hinges and fasten upon the face of each of them a mirror. The angle between these mirrors can be adjusted to any size at pleasure. Place a lighted candle between the mirrors and observe the position and number of its images for different angles.

309. Multiple reflection by a single mirror is very common. On looking into a plate-glass window one can see that the images of the gilt letters of the signs on the opposite side of the street are double. This is due to the reflection from the front and back surfaces of the glass. If a small gas flame is viewed very obliquely in a thick glass mirror, several images will be seen. These images are formed by repeated reflections of the light back and forth between the two surfaces of the glass, some of the light passing out from the front surface at each reflection on that surface.

XI. IMAGES FORMED BY SPHERICAL MIRRORS

310. Spherical mirrors are either concave or convex. A concave spherical mirror consists of a portion of the inside surface of a hollow sphere, and a convex mirror is a portion of the outside surface of a sphere.

Let MN (Fig. 174) be a section of a concave mirror. C, the center of the sphere of which it forms a part, is called the *center of curvature* of the mirror, and V, which is the center of the mirror surface, is called the *vertex*. Any straight line passing through the center of curvature to the mirror is termed an *axis*, and the one passing through the vertex is named the *principal axis*. All others are termed *secondary axes*. The angle between two opposite axes that touch the edge of the mirror is the *aperture* of the mirror. The line DCV is the principal axis, and ECB, GCA, HCM, and KCN are secondary axes. The angle MCN is the aperture of the mirror.

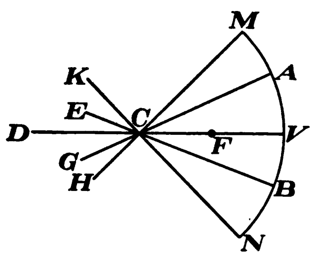

FIG. 174. — Diagram illustrating the spherical mirror.

311. A focus is a point to which rays of light converge or from which they diverge. The *principal focus* of a spherical mirror is the point at which rays originally *parallel to the principal axis* meet after reflection. With concave mirrors it is real, but with convex mirrors it is virtual. It is situated on the principal axis, halfway between the center of curvature and the vertex of the mirror, as at F (Fig. 174). The distance from the vertex to the principal focus is called the *focal length* of the mirror.

312. To locate the principal focus of a spherical mirror. — Let MR (Fig. 175) be a concave mirror and AB and DE

be any two rays parallel to the principal axis Cv. The point of their intersection after reflection is by definition the principal focus. CB, being a radius, is normal to the reflecting surface at B, hence ABC is the angle of incidence. Construct the angle of reflection CBH equal to ABC. AB is reflected in the line BH and the principal focus must lie in BH or BH produced. In like manner DE can be shown to be reflected in the line EK, and the principal focus must lie in EK or EK produced. Since it lies in each of them, it must lie at their intersection, or at F.

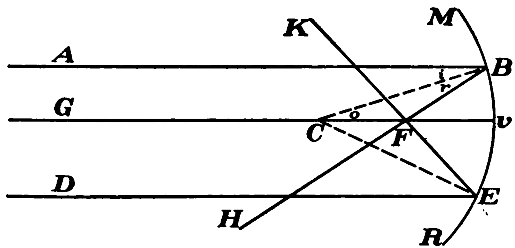

Fig. 175.—Diagram showing the position of the principal focus of a concave mirror.

In Figure 176, which represents a convex mirror, the reflected rays BH and EK are found by constructing the angles of reflection LBH and KEP respectively equal to the angles of incidence ABL and DEP. The principal focus must lie at the intersection of BH and EK or these lines produced, or at F. In this case the rays themselves do not meet

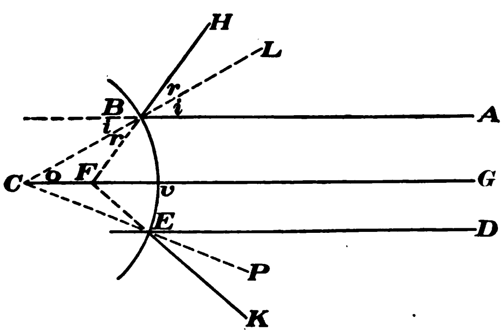

Fig. 176.—Diagram showing the position of the principal focus of a convex mirror.

at F. Hence, with convex mirrors the principal focus is virtual.

In either case the triangle CFB is isosceles because the angles i, r, and o are all equal (Why?) and CF equals FB. When the point B is very near v, Fv and FB are practically equal; therefore CF and Fv are equal and the principal focus of a spherical mirror lies on the principal axis, halfway between the center of curvature and vertex of the mirror.

(Let the pupil prove that rays diverging from the point F are reflected parallel to the principal axis.)

313. To locate the image of a point in a spherical mirror. —As in plane mirrors, the image of a point in a spherical mirror is located by the intersection of any two rays from the point after they are reflected.

Let A (Fig. 177) be a point whose image is to be formed by the mirror, and let AB be any ray from A incident on the mirror. Draw the normal CB and construct the angle of reflection CBE equal to the incident angle CBA. The ray AB is reflected in the line BE, and the image of A must lie in that line or that line produced. In like manner find the line DH, in which AD, another ray from A, is reflected. The image of A must also lie in DH or DH produced. Since it lies in both BE and DH, or these lines produced, it must be at their intersection o. Therefore o is the image of the point A. (Let the pupil prove by the same figure that A may be the image of o.)

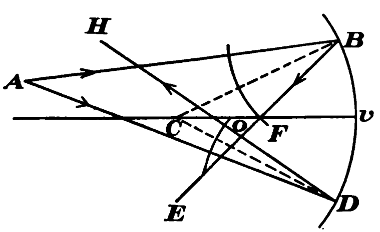

FIG. 177.—Diagram for locating the image of a point formed by a concave mirror.

It is possible, however, to locate the image of a point

without constructing the angles of incidence and reflection. If the ray AB (Fig. 178) be taken parallel to the principal axis, we know without constructing the angles that it will be reflected through F. If the ray AD passing through F be taken, we know that it will be reflected parallel to the principal axis; and if the ray AE passing through the center of curvature be taken, we know that it will be reflected back along the same path through C because it is normal to the mirror. Hence, by taking any two of these rays from a point, the image of the point may be easily and quickly located without constructing any angles of incidence or reflection.

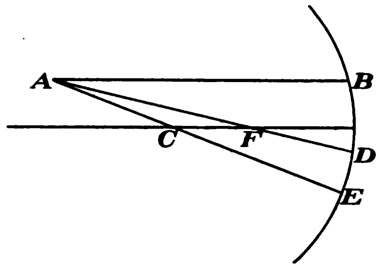

FIG. 178. — Diagram of simple method of locating the image of a point formed by a concave mirror.

314. Conjugate foci. — A careful study of the preceding paragraph will show that the point source and its image are interchangeable. Two points so related that each is the image of the other are called *conjugate foci.* Rays diverging from either one of them toward the mirror will converge after reflection at the other. A and o (Fig. 177) are conjugate foci. Conjugate foci always lie on the same axis. This may be shown by drawing a straight line through the two points; it will pass through the center of curvature and be normal to the mirror.

315. Spherical aberration. — In the formation of images of points in spherical mirrors, it has been assumed that because two rays from one point are reflected to another point, all other rays from the first point meet after re-

flection exactly at the second or conjugate point. For example, because AB and AD (Fig. 177) are reflected to the point o, it is assumed that all other rays from A, incident on the mirror, are also reflected to o. This is not strictly true, especially when the aperture of the mirror is large. If the student should construct these figures accurately, using three or four rays from each point instead of two, he would find that all do not meet exactly at a common point after reflection. This causes images formed by spherical mirrors to be more or less blurred or indistinct. *This inability of a spherical mirror to reflect all the rays coming from a point exactly to its conjugate point is called spherical aberration.* The greater the aperture of the mirror, the greater the spherical aberration.

In § 312 we assumed that $Fv = FB$ to prove that F is halfway between C and v. This assumption, however, is not true when the aperture of the mirror is large, and in that case all rays parallel to the principal axis are not reflected through the principal focus. This is a special case of spherical aberration.

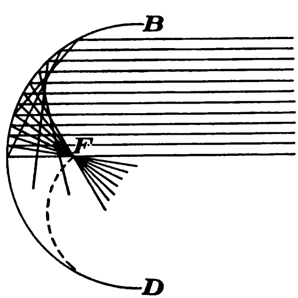

FIG. 179.—Diagram illustrating spherical aberration.

Figure 179 shows how rays parallel to the principal axis are reflected when the aperture is large. Very few of them actually pass through F. The crossing reflected rays form a peculiar curve from B to F and F to D, called the *caustic curve.* It may be observed at the tea table on a glass of milk when the light striking the inside of the rim of the glass is reflected on the surface of the milk. It may also be formed by a plain gold finger ring on a sheet of paper, or by bending a strip of bright tin into a semicircle and placing it so as to reflect light from a window upon a sheet of paper.

316. Images of objects formed by spherical mirrors.— The image of an object is found by finding the images of points in that object.

Let *AB* (Fig. 180) be an object whose image formed by the mirror *MR* is to be located. First find the image or conjugate focus of *A*. The ray *AD* will be reflected through *F* (Why?) in the line *DL*, and the image of *A* must lie in *DL* or *DL* produced; the ray *AG*, being normal to the mirror, will be reflected back through *C* and the image of *A* must lie in the line *GC* or *GC* produced. Since it lies in these two lines, it must lie at their intersection *o*. In like manner the image of *B* can be shown to be at *i*, and *io* is the image of the arrow *AB*. (Let the pupil show that if *io* is taken as object, *AB* is its image.)

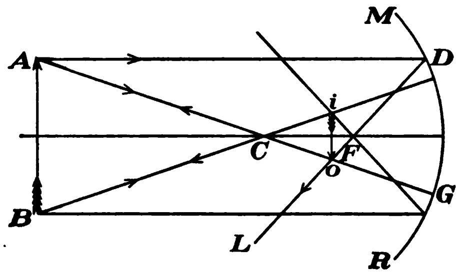

FIG. 180.—Diagram illustrating the formation of the image of an object by a concave mirror.

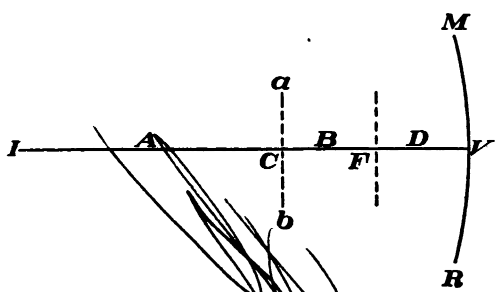

FIG. 181.—Diagram of the five positions in front of a mirror.

The fixed points, center of curvature, principal focus, and vertex, or planes passing through these points perpendicular to the principal axis, serve to divide the space before the mirror into three parts, as *A*, *B*, and *D* (Fig.

181). Space *A* extends from the plane *ab* for an infinite distance in the direction of *I*.

These points and spaces give rise to four cases in the formation of images by concave mirrors.

CASE I. The object may be between the center of curvature and an infinite distance, space *A*. The image is then real, inverted, smaller than the object, and situated between the center of curvature and the focus, space *B*.

CASE II. The object may be at the center of curvature, in the plane *ab*. The image is then real, inverted, of the same size as the object, and in the same plane *ab*.

CASE III. The object may be between the center of curvature and the focus, space *B*. The image is then real, inverted, enlarged, and beyond the center of curvature, space *A*. This is the converse of Case I (Fig. 180).

CASE IV. The object may be between the focus and the mirror, space *D*. The image is then virtual, erect, enlarged, and behind the mirror.

There is no image of an object in the plane of the principal focus because the rays from any point of it are reflected parallel to one another and do not meet.

There is only one case with the convex mirror. An image formed by a convex mirror is always virtual, erect, diminished in size, and back of the mirror, between *C* and *V*.

The pupil can prove all these cases by constructing the figures for them. They can also be easily verified by experiment by means of a candle or lamp placed in the different positions before a mirror in a darkened room. In the first and second cases the image of the candle may be received on a small card, and in the third case it may be projected on the wall of the room. In the fourth case one must look into the mirror to see the image; it cannot be projected because it is virtual.

Experiment. — Let a piece of crayon be supported in a box as at *a* (Fig. 182). This may be done with glue. Let a concave mirror *M* be placed so that the distance from it to the crayon equals its radius. By careful adjustment the image of the crayon may be formed in the air at *b* on top of the box and will be visible to a person standing several feet away, if he is in line with *b* and *M*. The image at *b* is real and appears like a real object. The illusion may be made almost perfect by placing the apparatus so that light strikes the crayon from back of *M* and the mirror is partially hidden so that its presence is not suspected. This may be done by placing it in a black shallow box with a round opening to permit the reflection.

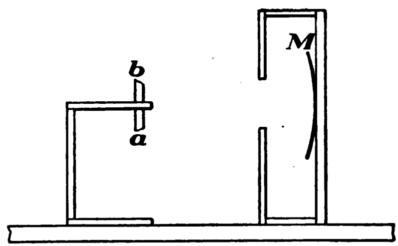

Fig. 182. — Diagram of an apparatus for showing a real image.

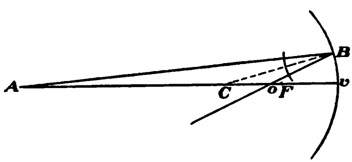

Fig. 183. — Diagram for calculating the focal length of a spherical mirror.

317. Focal length of a spherical mirror. — The focal length of a mirror can be calculated if the distances of a pair of conjugate foci from the mirror are given, or if the positions of an object and its image are known.

Let A and o (Fig. 183) be two conjugate foci, o being the image of A. Let $v =$ the distance Av, or object distance; let $u =$ the image distance, ov; let $f =$ the focal distance, Fv; and let $r = Cv = 2f$. By geometry $\frac{AB}{Bo} = \frac{AC}{Co}$. When B is very near v, AB may be considered equal to Av or v, and Bo to ov or u. $AC = v - r$ and $Co = r - u$.

Substituting these values in the above proportion, we have $\frac{v}{u} = \frac{v-r}{r-u}$, or $rv - uv = uv - ru$ and $rv + ru = 2uv$.

Dividing the last expression by ruv gives $\frac{1}{u} + \frac{1}{v} = \frac{2}{r} = \frac{1}{f}$, the formula used in calculation. With convex mirrors u and f are negative, and with concave mirrors u is negative or back of the mirror when the image is virtual.

Problems

1. An object is 10 cm. from a mirror, and its image is 30 cm. from the mirror on the same side. Is the mirror concave or convex? What is the focal length of the mirror?

2. A concave mirror has a focal length of 15 cm. Find the position of the image of an object 40 cm. from the mirror.

3. An object and its image are both 20 cm. from a concave mirror. What is its focal length?

4. A concave mirror has a focal length of 3 cm. An object is placed 9 cm. in front of it. Find the position of its image by construction, and verify the result by the formula.

5. When the object distance for a concave mirror is 48 cm. and the image distance is 24 cm., what is the focal length of the mirror?

6. When the focal length of a concave mirror is 8 cm. and the object is 4 cm. from it, what is the object distance? What kind of an image is it?

7. A concave mirror has a focal length of 12 cm. How much does its image distance differ from its focal length, when the object distance is 96 meters?

Could the focal length of such a mirror be determined by measuring the image distance for a distant object?

8. The radius of curvature of a concave mirror is 40 cm. Find the conjugate focus of a point 90 cm. from the mirror.

9. When the image distance for a concave mirror is half of the object distance, how does the focal length compare with the object distance?

10. What is the sign of the image distance for a concave mirror, when v is less than f? What does it signify?

11. The object distance of a spherical mirror is 10 cm. and the image distance is -60 cm. Find the value of f. What can you say of the size of the image, when f is positive? When f is negative?

XII. LENSES AND IMAGES FORMED BY LENSES

318. **A lens** is a portion of a transparent substance bounded by two curved surfaces or by one curved and one plane surface. Lenses depend for their action upon refraction.

In this book only spherical lenses will be considered, although those with cylindrical surfaces are not uncommon and other forms occur.

319. Lenses classified. — There are two general classes of lenses: (1) convex lenses, which are thicker at the center than at the edges; and (2) concave lenses, which are thinner at the center than at the edges. Convex lenses are termed *converging* lenses because, like concave mirrors, they tend to converge light to a focus; while con-

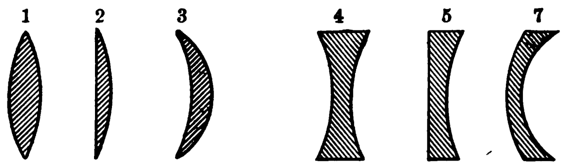

FIG. 184. — Diagram of sections of six forms of lenses.

cave lenses are called *diverging* because, like convex mirrors, they diverge light. There are three kinds of each class of lenses. Figure 184 represents sections through the centers of the different kinds.

Convex Lenses:
- (1) double-convex, — both faces convex.
- (2) plano-convex, — one surface convex, one plane.
- (3) concavo-convex, — one surface convex, one concave.

The double-convex may be taken as a type of these.

Concave Lenses:
- (4) double-concave, — both surfaces concave.
- (5) plano-concave, — one surface concave, one plane.
- (6) convexo-concave, — one surface concave, one convex.

The double-concave may be taken as the type of these.

Lenses, like prisms, tend to bend the light passing through them toward the thicker part.

320. Terms relating to lenses. — The centers of the spherical surfaces of the lens, as c and c' (Fig. 185), are called the centers of curvature. For every lens there is a point through which light may pass without having its

direction changed by the lens; this point is called the *optical center.* In double-convex and double-concave lenses whose surfaces have equal curvature the optical center is at the center of volume of the lens, as *o* (Figs. 186 and 187). In plano lenses it is on the curved surface of the lens. The *principal axis* of a lens is a straight line through its optical center and its centers of curvature, as *AP*. Any other straight line through its optical center, as *SD*, is a *secondary axis*. *ct* is a normal to the surface *lsn* at the point *r*, *cr* being a radius.

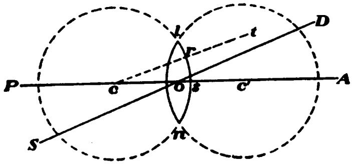

FIG. 185. — Diagram of the parts of a lens.

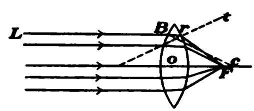

FIG. 186. — Diagram showing the principal focus of a convex lens.

321. Principal focus. — The principal focus of a lens is the common point for all rays parallel to its principal axis after they are refracted by the lens, as *F* (Figs. 186 and 187). It is real for convex lenses, but virtual for concave lenses. This point may lie on either side of the lens, according to the direction in which the light passes through it. The *focal length* of a lens is the distance from its optical center to its principal focus, as *oF*. Observe (Fig. 186) that the ray *LB* is bent toward the normal *cB* on entering the lens at *B*, and away from the normal *rt* on leaving it at *r*; also in Figure 187 it is bent toward the normal *Bt* at *B*, and away from the normal *cr* at *r*.

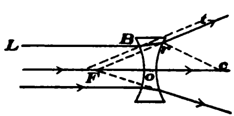

FIG. 187. — Diagram showing the principal focus of a concave lens.

When rays diverge from the principal focus of a convex lens, or pass through it, and then pass through the lens,

they are parallel to the principal axis after refraction. Figure 186 illustrates this if the rays are considered as going in the opposite direction.

NOTE. — The focal length of a double-convex lens whose two surfaces have the same curvature may be calculated by the formula $f = \frac{r}{2(\mu - 1)}$ in which r is the radius of curvature, μ the index of refraction of the glass composing the lens, and f the focal length to be found. For example, lenses are commonly made of crown glass, which has an index of refraction of $\frac{3}{2}$. Substituting this value for μ in the formula we find that $f = r$. This places the principal focus of a double-convex lens made of crown glass half the thickness of the lens beyond its center of curvature.

Experiment. — Hold a convex lens, such as a reading lens, in direct sunlight and bring the light and heat of the sun to a point upon a piece of paper. The distance from the lens to the paper is the focal length of the lens, since the sun's rays are parallel. Or, standing in the darker part of a room, allow the light coming through the window from a distant object to pass through the lens and fall upon a card. When the card is placed at the right distance, a distinct inverted image of the object will be formed upon it, and the distance from card to lens is the focal length of the lens.

Theoretically the distance from the lens to the image is greater than the focal length, but the angles between the rays entering the lens *from any one point* in a *distant* object are so small that the rays are *practically* parallel and the image formed is in the focal plane of the lens.

322. Images formed by lenses. — Experiment. — Having determined the focal length of a convex lens, let a candle or a lamp be placed a little farther from the lens than the focal length. If the room is darkened, a real, enlarged, inverted image of the candle will be seen on the wall several feet distant.

Move the candle slowly away from the lens and receive its image on a large card. As the distance from the lens to the candle increases, the image becomes smaller and approaches the lens. To a person standing back of the image it may be visible suspended in air.

When the distance from candle to lens becomes exactly twice the

focal length, the image will be found to be the same distance from the lens on the other side, and the same size as the candle itself.

Let the candle be moved still farther from the lens. The image becomes smaller and comes nearer the lens; but it will never approach nearer than the principal focus, no matter how far distant the object may be.

Again, let the candle be placed between the lens and the principal focus. No image will be formed on the opposite side of the lens; but on looking through the lens toward the candle, the observer will see a virtual, erect, magnified image of it.

323. The points of reference for a lens. — When rays diverge from a point on the principal axis of a convex lens which is twice the focal length from its optical center, they converge, after refraction, at a point similarly placed on the other side of the lens. Thus (Fig. 188) oS and oS' are twice oF, and rays diverging from either of these will converge at the other.

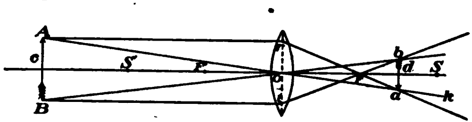

FIG. 188. — Diagram showing the image formed by a convex lens.

The points S and S' are called *secondary foci.* These five points, S, F, o, F', and S', are used as points of reference in locating images formed by lenses. We know in regard to them that (1) rays parallel to the principal axis will pass, after refraction, through F or F'; (2) conversely, rays passing through F or F' will be parallel to the principal axis after refraction; (3) rays passing through o suffer no change of direction; and (4) rays passing through S will pass after refraction through S', the converse of this also being true.

324. To construct an image formed by a lens. — First, draw the principal axis and locate the points of reference. F may be placed at any convenient distance, or its position may be determined by formula (§ 321).

Let AB (Fig. 188) be the object. Two rays from the point A will determine its image. Draw from A the ray Ar parallel to the principal axis; it will pass through the focus F after refraction, and the image of A must lie in the line rF, or rF produced. (The bending of the ray is represented as taking place at r. This, of course, is not correct, but little error is introduced by so doing.) Draw another ray from A, as Ao. Since this ray passes through the optical center, its direction will not be changed, and the image of A must lie in ok or ok produced. Since it lies on rF and ok, or these lines produced, it must be at their intersection, a. In like manner the image of B can be found to be at b, and ab is the image of AB.

Let the pupil take ab as the object and AB as the image, and explain the construction of the figure.

325. Conjugate foci are two points so related that rays diverging from either one will meet at the other after refraction, each being the image of the other. A and a (Fig. 189), B and b, and S and S' are conjugate foci. Two conjugate foci always lie on the same axis.

326. Four cases exist in the formation of images by a convex lens, which correspond to those of the concave mirror. They are illustrated by the experiment (§ 322), and they can be verified by construction.

I. The object may be at a distance from the lens greater than twice the focal length. Its image is real, inverted, smaller than the object, and situated between the focus and twice the focal length on the other side of the lens (Fig. 188). The eye and the camera illustrate this case.

II. The object may be at twice the focal length from the lens. Its image is then real, inverted, of the same size as the object, and at the same distance from the lens on the other side.

III. The object may be between the focus and twice the focal length of the lens. Its image is real, inverted, larger than the object, and at a distance greater than twice the focal length on the other side of the lens (Fig. 188). This is the converse of Case I. The magic lantern illustrates this case.

When the object is at the principal focus, there is no image, since the rays from each point are parallel on leaving the lens.

IV. The object may be between the lens and its principal focus. Its image is then virtual, erect, magnified, and on the same side of the lens, but farther back from it than the object (Fig. 189). The "reading lens" illustrates this case.

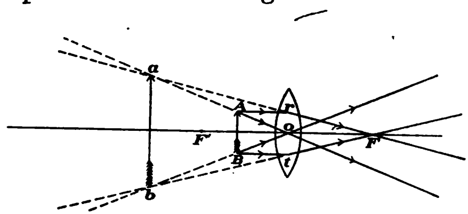

FIG. 189.—Diagram of a virtual image formed by a convex lens.

With concave lenses there is one case only, the images are always virtual, erect, smaller than the object, and between the focus and the lens on the same side as the object (Fig. 190).

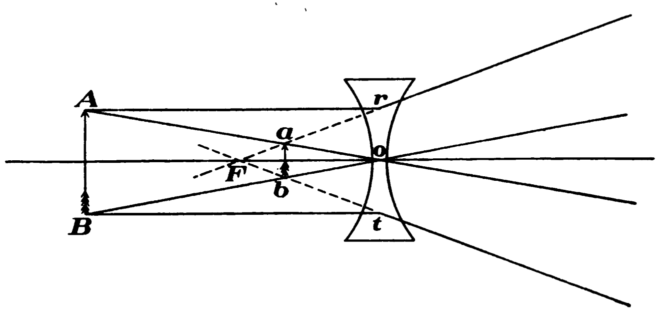

FIG. 190.—Diagram of the image formed by a concave lens.

327. The formula for lenses is the same as that for mirrors, $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. In Figure 188 let u = the image distance, od; v, the object distance, oc; and f, the focal distance, oF. By the similar triangles, AoB and aob, $\frac{AB}{ab}=\frac{oc}{od}$; and by the similar triangles, rFt and aFb, $\frac{rt}{ab}=\frac{oF}{Fd}$. Since $rt = AB$, the first members of these two proportions are equal; hence, $\frac{oc}{od}=\frac{oF}{Fd}$. Substituting in this proportion the values given above, we have $\frac{v}{u}=\frac{f}{u-f}$. Simplifying and dividing it by uvf, we obtain $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

For concave lenses u and f are negative.

From the proportion $\frac{AB}{ab}=\frac{oc}{od}$ the following law is derived: —

The size of the object is to the size of the image as the distance of the object to the lens is to the distance of the image to the lens.

328. Spherical aberration. — It has been assumed, so far, that two rays from any point locate exactly the image of that point; but it is not true with spherical lenses that all rays from a given point meet exactly at another point after refraction, because two of them do. For example (Fig. 188), it is assumed that all rays from A passing through the lens meet at a, because Ao and Ar do. Those rays passing through a lens near its edge focus at a point nearer the lens than those passing through its central part. The result is that the image of each point, and consequently of the whole object, is more or less blurred. *This blurring of an image, due to the failure of different parts of a lens to focus all rays from a point exactly at its conjugate point, is called spherical aberration.* It is less with plano than with double lenses, and may be more or less avoided by changing the curvature of the lens, or by covering the outer part of the lens by a diaphragm.

329. Chromatic aberration. — It has been shown (§ 280) that when white light is bent or deviated by refraction, different colors are bent to different degrees and dispersion is produced; dispersion accompanies deviation. This is true when light passes through a lens as well as when it passes through a prism. As a consequence of this, red light is not focused to the same point as blue light or light of other colors, and images formed by lenses are more or less colored or fringed with color. *This fringing of an image with color, due to dispersion by a lens, is called chromatic aberration.*

Newton thought it was impossible to have deviation without dispersion. Deviation is absolutely necessary in a lens, dispersion is not desirable. In 1757 Dolland invented the *achromatic lens*, that is, a lens in which the dispersion is neutralized without completely destroying the deviation, so that chromatic aberration is more or less completely eliminated.

FIG. 191. — Section of an achromatic lens.

It has been shown (§ 280) that flint glass has about twice the dispersive power of crown glass, while its deviating power is only a very little greater. Hence, by combining a double-convex lens of crown glass with a plano-concave lens of flint glass as shown in Figure 191, the dispersion of the convex lens is neutralized by the concave lens, while its deviation is not. All good telescopes, microscopes, cameras, opera glasses, and many other optical instruments are provided with achromatic lenses.

XIII. OPTICAL INSTRUMENTS

330. The camera. — The camera consists of a dark box with an opening or tube in front in which a convex lens is placed. The camera lens is seldom a simple convex lens,

but is usually a combination of lenses equivalent to one. Several lenses are combined into a system of lenses to correct chromatic and spherical aberration and other defects. The lens of a camera generally has a focal length of a few inches at most, hence the object to be photographed is at a distance greater than twice the focal length of the lens, and an image is formed according to Case I, or as shown in Figure 188. This image is projected upon the film or plate at the back of the box. The light forming the image produces chemical changes in the compounds of silver with which the plate is covered, these changes being due, however, to the light of the shorter wave lengths such as the blue and violet, the red and orange having little effect.

Focusing consists in adjusting the distance between the lens and the plate so that the image formed upon the latter is distinct. A "pin hole" camera has no lens, but the image is formed through a small aperture (§ 304).

331. The eye, optically considered, may be called a small camera, since it consists of a small dark chamber with an opening at the front at which a convex lens is placed, and a screen or network of nerves at the back upon which the image is formed. The human eye (Fig. 192) is a ball about an inch in diameter, the outer shell of which, constituting the white of the eye, is called the *sclerotic* coat. The front transparent part of this outer coat is called the *cornea*, and it fits into the main body of the eye much as a watch crystal fits its case. The *iris* is that which gives color to the eye. It is a diaphragm or curtain just

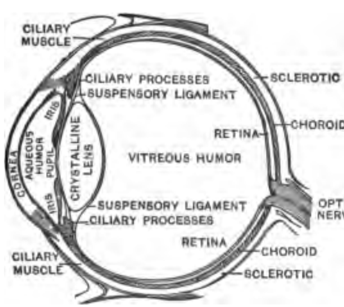

FIG. 192. — Diagram of a section of the human eye.

back of the cornea, and the circular opening or hole through its center is the *pupil* of the eye. The iris may expand and contract, changing the size of the pupil so as to adapt the eye to different intensities of light. The *crystalline lens* is immediately back of the iris, covering the pupil. The main body of the eye back of the lens is filled with a transparent, jellylike substance called the *vitreous humor*, and the space between the lens and cornea with a fluid called the *aqueous humor*. The *optic nerve*, consisting of a large bundle of nerve filaments, connects the eye with the brain. This nerve enters the eye at the back, and the filaments, spreading out, form a network of nerves called the *retina*.

332. Vision. — When we see an object, light from that object enters the eye, and the crystalline lens projects a real inverted image of it upon the retina, just as the camera lens forms an image on the photographic plate. The eye resembles the camera further, in that there is in the retina a sort of coloring matter called the *visual purple*, which is affected chemically by light. The apparent size of an object depends upon the size of the image on the retina, which in turn depends on the size of the *visual angle* ACB (Fig. 193). If the object comes nearer the eye, the visual angle $A'CB'$ and the image on the retina become larger. If the eye were a mere machine, not backed by the reason and experience of the observer, a man at a distance of 10 feet would be taken to be ten times as large as one at 100 feet.

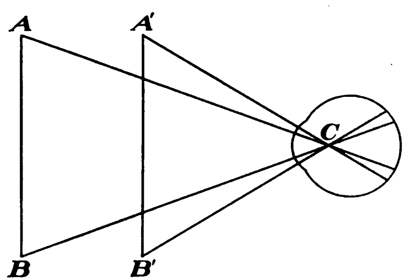

FIG. 193. — Diagram of the visual angle.

The distance between the lens and the retina cannot be changed in the eye as in the camera, but the eye adapts itself for seeing objects at different distances by a change in the form of the lens. When we wish to see an object near the eye, a muscle within the eye contracts, allowing the lens to bulge out at the center and become more convex; thus the image is focused on the retina. This ability of the eye to change its focus is called *the power of accommodation.* The eye at rest, or viewing distant objects, or looking through an optical instrument such as a microscope, is adjusted for parallel rays.

In order to secure very distinct vision, the object is brought as near to the eye as possible. For a normal eye the distance may be as short as four or five inches, but the distance of 10 inches or 25 cm. has been taken as the standard *distance of distinct vision.* In estimating the power of optical instruments, the apparent size or the visual angle of an object at 25 cm. is compared with the apparent size or visual angle as seen through the instrument.

333. Defective vision. — In the normal eye in youth the power of accommodation is very strong, but with advancing age the lens of the eye loses its elasticity and in time ceases to change form when the muscle acts, and as a consequence objects near by are seen indistinctly. Eyes having this defect are termed *presbyopic* or farsighted. This defect is remedied by convex glasses.

Defective vision often occurs because the eyeball is not perfect in shape; it may be too long or too short from front to back or its curvature may not be equal in all directions. When the eyeball is too long, the person is said to be nearsighted and the eye is *myopic.* This defect is remedied by concave glasses. When the eye is too short from front to back, the person is farsighted and the eye is *hypermetropic* and requires convex glasses. When

the curvature of the eye is imperfect, the figures on one diameter of a clock dial, for instance, are distinct, while those on another diameter are indistinct. This defect is known as *astigmatism* and is corrected by cylindrical lenses.

334. The lantern, or stereopticon, is an apparatus for projecting large images or pictures upon a screen or curtain. The fundamental principle of the lantern is the forming of a real, inverted, magnified image of an object by a convex lens according to Case III (§ 326), the object being between the focus and twice the focal length of the lens. The lens *O* (Fig. 194), which projects the image, is called an *objective*. It is contained in a tube and is a system of lenses equivalent to a single convex lens.

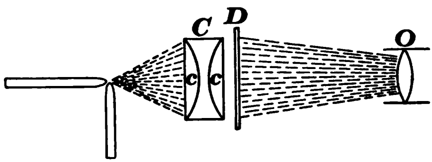

FIG. 194. — Diagram of a section of the lantern or stereopticon.

The object *D* to be shown is usually a photograph upon glass. Such photographs prepared for use in a lantern are called *slides*. A photographic negative is black where the object photographed is white and *vice versa*, but a lantern slide being a negative of a negative, as it were, is a positive.

The slide or object to be projected must be strongly illuminated. The source of light most often used is an electric arc lamp or the calcium light. This light should be as small in area as possible, hence the flame of an oil lamp which is sometimes used is not satisfactory. The light from the lamp is concentrated upon the slide by the *condenser C*, which consists of two large plano-convex lenses *cc* placed with their convex faces toward each other. Sometimes direct sunlight is used for projecting and in

that case the condenser is unnecessary. If the object to be shown is an opaque picture, a very strong light is thrown upon it from in front, and the light reflected from the picture passes through the objective and is projected on the curtain.

335. The simple microscope. — When an object is looked at through a convex lens (Fig. 189), the object being placed between the lens and its principal focus, the lens constitutes a simple microscope. This is an illustration of Case V (§ 326). A magnified, erect, virtual image of the object is formed. The rays, which really diverge from A, appear to the eye to come from a. But since the eye should be adapted for parallel rays in using the lens, the object is placed at its focus. This increases the visual angle over what it would be at 25 cm. as many times as the focal length of the lens is contained in 25, or the magnifying power of the lens is $\frac{25}{f}$.

336. The compound microscope consists of two lenses or systems of convex lenses, the *eyepiece* and the *objective*, mounted at the extremities of a tube. The larger lens, to which the eye is applied, is the eyepiece; and the smaller lens, near the object, is the objective. The latter is the most important and expensive part of the instrument. The smaller it is and the shorter its focal length the higher its magnifying power.

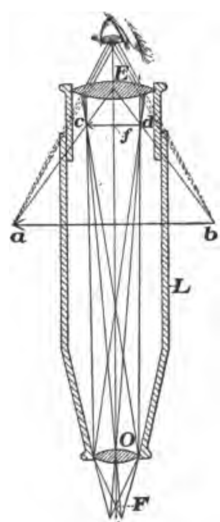

Fig. 195. — Diagram of the compound microscope.

The principle of the compound microscope may be explained by reference to Figure 195. Let O be the objective and F its principal focus. The object

is placed just beyond *F* and hence a real, magnified, inverted image of it is formed at *cd*, Case III. The eyepiece *E* is so placed that this image *ab* falls between it and its principal focus *f*. The eyepiece acts upon the real image *ab* just as a simple microscope acts upon a real object, forming a magnified virtual image of this real image, Case IV. Often only a small portion of this image *ab* can be seen by the eyepiece. If f is the focal length of the objective, f' that of the eyepiece, and l the length of the tube, the magnifying power of the microscope $= \frac{25\,l}{ff'}$, l, f, and f' being expressed in centimeters.

337. The astronomical telescope. — This instrument, like the compound microscope, consists of an *eyepiece* and an *objective* mounted at the ends of a tube. The eyepiece has the same function in both instruments; but while the objective of the microscope is small and of short focal length, that of the telescope is large and of long focal length. Since objects seen through a telescope are at a distance, they are of course always beyond twice the focal length of the objective, and a real, inverted image smaller than the object is formed according to Case I. This falls practically at the principal focus of the objective, just within or at the focus *f* of the eyepiece. The tube of the telescope must therefore be as long as the sum of the focal lengths of the eyepiece and the objective. The objective of the telescope is made very wide so as to gather in a large amount of light to form a very bright image of the object and to give a high resolving power. Figure 196 shows the relative positions of object, lenses, and images.

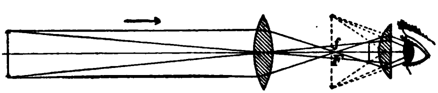

Fig. 196.—Diagram of the astronomical telescope.

The image seen by the eye is inverted. For terrestrial telescopes, which are used for viewing objects on the earth's surface, inversion would not be permissible, and two more lenses, called erecting lenses, are placed in the tube. These invert the image again before it is formed at the eyepiece.

Let f and f' represent respectively the focal length of objective and eyepiece. The objective increases the visual angle of the object $\frac{f}{25}$ times, and the eyepiece increases it $\frac{25}{f'}$ times; hence the two lenses increase it $\frac{f}{25} \times \frac{25}{f'} = \frac{f}{f'}$. The magnifying power of an astronomical telescope therefore equals the quotient of the focal length of the objective divided by that of the eyepiece.

338. Galileo's telescope has a concave lens for an eyepiece, which is placed between the objective and the real

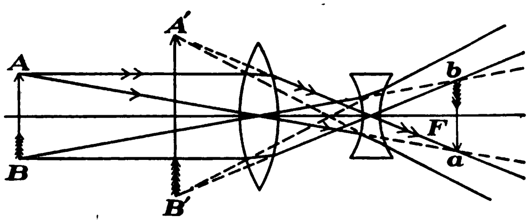

FIG. 197.—Diagram of Galileo's telescope or opera glass.

image *ab* (Fig. 197) that would be formed by the objective if the eyepiece were not there. The focus of the eyepiece *F* is nearly at the place where the real image *ab* would be formed, but the eyepiece, intercepting the rays which are converging to form the image *ab*, causes them to diverge, making a virtual upright image $A'B'$. The length of the tube is the difference of the focal lengths of the two lenses, instead of their sum as with the

astronomical telescope. Opera glasses and field glasses consist of a pair of such telescopes placed side by side. The advantages of this telescope are its convenient length, simple construction, and the fact that it does not invert the image. Its magnifying power is the same as that of the astronomical telescope, $\frac{f}{f'}$.

339. The Zeiss binocular field glass (Fig. 198) combines the advantages both of the Galilean and of the terrestrial telescope. It is short and compact like the former and has the wide field of view of the latter. The objective and the eyepiece are the same as in the astronomical telescope, but the necessary distance between them is obtained by causing the light to travel the length of the tube three times in passing from objective to eyepiece. This is accomplished by the total internal reflection of the light by two prisms as shown in the figure. These reflections also reinvert the image formed by the objective at the focus of the eyepiece, making it erect.

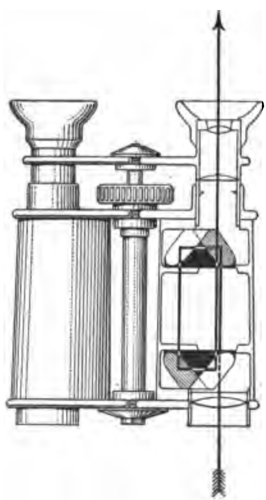

FIG. 198.—Diagram of the Zeiss binocular field glass.

Another advantage of this instrument is the greater stereoscopic effect secured by having the two objectives farther apart than the two eyes of the observer. When we look at an object with two eyes, the images on the two retinas are not exactly alike because the two eyes see the object from slightly different positions. This gives the appearance of depth or solidity to the object. It is this effect which is enhanced by the greater distance between the objectives of the Zeiss instrument.

Problems

1. The focal length of a convex lens is 15 cm. and an object is placed 60 cm. from it. Find the position of the image.

Ans. 20 cm. from lens.

2. An object is 12 cm. from a convex lens whose focal length is 3 cm. Find the image distance. How much larger is the object than the image? Verify the results by construction.

3. An object is 30 ft. from a camera whose focal length is 6 in. If the object is 15 ft. high and 20 ft. long, how large will its picture be?

4. If the camera lens of the last problem has a focal length of 4 in., what is the size of the picture?

5. If in the last problem the object is 15 ft. from the lens of the camera instead of 30 ft., what is the size of the picture?

6. A convex lens has a focal length of 6 cm. If a distinct image of an object 48 meters from the lens is formed on a white card and the distance from the card to the lens is measured accurately, how much would it differ from the focal length of the lens? If it were called the focal length, what would be the percentage of error?

7. If a lantern slide is 4 in. long and the focal length of the lantern objective is 6 in., what must be the width of the curtain to receive the picture at a distance of 30 ft. from the objective?

[First find the distance from the slide to the objective, or object distance, and then apply the principle of § 327. Compare your answer with one obtained as follows:

Size of image : size of slide = curtain distance : focal length.]

8. In a certain school it was desired to project pictures by a lantern on a screen 18 ft. square at a distance of 80 ft. If the slides are 4 in. wide, what focal length was required for the objective?

9. Name five cases in which virtual images are formed and seven cases in which real images are formed.

10. What two effects may be noted when light passes through a prism and what explanation may be given of them?

11. Define the focal length of a lens and of a curved mirror.

12. In what direction is an oar in water apparently bent? Explain by a diagram.

CHAPTER IV

HEAT

I. TEMPERATURE AND THERMOMETERS

340. Temperature. — We may be said to possess a *sense of heat* as well as a sense of touch or of sight. By this sense we distinguish between the hotness or coldness of different bodies which we touch or by it we perceive the warmth of the sun or of a fire. The words *hot* and *cold* with which we are all familiar refer to the state of a body when judged by this sense.

To estimate the hotness of a body we necessarily refer to some standard, and in doing this scientifically, we use the word *temperature*, rather than the word *hotness*. Temperature may be defined as the degree of hotness of a body measured according to some arbitrarily chosen scale; or, it is the thermal condition of a body which determines the flow of heat between it and other bodies in contact with it.

341. Measurement of temperature. — The sense of heat is not at all reliable for measuring temperature. To show this, place one hand in warm water and the other in ice water for a few moments and then place both hands in tepid water. This tepid water will seem warm to the hand taken from the ice water, but cold to the other hand. Again, the unreliability of the sense of heat for estimating temperature is shown when we touch various objects in a cold room; some of them will seem cold and others warm, while in fact all are of the same temperature.

The well-known fact that bodies expand or increase in volume as they become warmer is the basis of most methods of measuring temperature. The common instrument for measuring temperature is called a *thermometer*. As usually constructed it depends on the fact that the liquid or gas in it expands more than the glass of which it is made.

342. The mercurial thermometer. — The thermometer best suited for most practical purposes consists of a glass tube with a capillary bore ending in a bulb filled with mercury. A form called a chemical thermometer is shown in Figure 199.

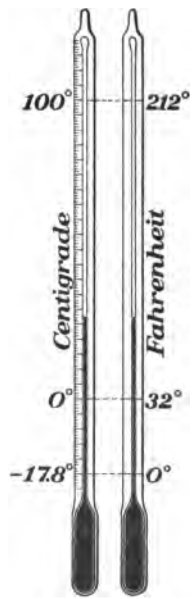

FIG. 199. — Diagram of chemical thermometers.

The mercury is introduced into the thermometer by first heating the bulb to expel some of the air by its expansion and then dipping the open end of the stem into mercury. As the bulb cools and the air in it contracts, a small amount of mercury is forced into it by the pressure of the outside air. The bulb is again heated until the mercury boils, the air being expelled by the vapor of mercury, and then the open end is again dipped into mercury. As it cools and the vapor condenses, more mercury is forced in by atmospheric pressure, and in this way the bulb and the stem are completely filled. The instrument is then raised to the highest temperature it is designed to register and sealed by melting the open end of the glass tube in a hot flame. On cooling the mercury contracts, leaving a vacuum in the top of the tube.

343. The fixed points of a thermometer. — Since the temperatures of melting ice and of steam rising from boiling water are invariable under a constant pressure of 76 cm., these two temperatures are taken as standard temperatures and are called the *fixed points* of a thermometer. They are known as the *freezing point* and the *boiling point*.

To determine the freezing point, the bulb and stem as far up as the top of the mercury column are packed in moist pulverized ice. When the mercury becomes stationary in the tube, the top of the column is marked by a scratch on the glass.

To determine the boiling point, the bulb and stem are immersed in steam issuing from water boiling under a pressure of 76 cm., and the top of the column marked as before. Since the barometer seldom reads exactly 76 cm., it is generally necessary to find a correction for the boiling point on account of this variation in pressure. A variation of 1 cm. from 76 makes a difference of .37° C. in the boiling point. The boiling point of water when the pressure is 73.5 cm. is found as follows: $76.0 - 73.5 = 2.5$; $2.5 \times .37 = .925$; $100 - .925 = 99.075°$ C. When the pressure is above 76, the correction is added instead of subtracted.

344. Graduation of thermometers. — The space between the fixed points of a thermometer is divided into equal parts called degrees, and the scale thus formed is extended beyond these points as far as desirable. Of course, if the bore of the tube is not uniform, the degrees will not be accurately equal. The errors in common thermometers from this cause and also because the fixed points are not accurately placed are often considerable; indeed, for accurate work it is always necessary to test the instrument and correct the readings for errors due to these causes.

The number of divisions between the fixed points may be chosen at pleasure, but three scales are in use: the Fahrenheit, the Centigrade, and the Réaumur. In the Fahrenheit, which was introduced early in the eighteenth century and is in common use among all English-speaking people, there are 180 divisions between the fixed points, which are marked 32° and 212°.

In the Centigrade scale, first constructed by Celsius in 1742, the freezing point is marked 0° and the boiling point 100°, thus making 100 spaces between them. This scale is almost universally used for scientific purposes.

The Réaumur scale has 80 divisions between the fixed points, the lower one being marked 0° and the upper one 80°. This thermometer is used in some countries in Europe for household purposes.

In all these scales, readings below zero are designated by the negative sign. Thus, $-15°$ C. means 15 degrees below 0° Centigrade.

345. Conversion from one scale to another. — Since the same space is divided into 180 equal parts in the Fahrenheit scale, and 100 in the Centigrade, 180 Fahrenheit degrees are equal to 100 Centigrade degrees; or 9 F. degrees = 5 C. degrees, or 1 F. degree = $\frac{5}{9}$ C.

To obtain the number of degrees from any point on the Fahrenheit scale to the freezing point, 32° must always be subtracted from the reading. For example, 40° F. is only 8° above the freezing point, and $-10°$ F. is $(-10 - 32 = -42)$ 42° below the freezing point. The following formula may be used to convert readings in one scale to the other:

$$\frac{F-32}{180} = \frac{C}{100},$$

in which C and F represent the readings on their respective scales.

346. The air thermometer. — Mercury is the most suitable liquid for ordinary thermometers for many reasons, but when extreme scientific accuracy is required, or when very high or very low temperatures are to be measured, a thermometer is used which is filled with air or hydrogen. Figure 200 illustrates a simple form of the air thermometer. It consists of a glass bulb or flask with a long tube of small bore which dips into a colored liquid. If the bulb is slightly warmed, some of the air is expelled; but when it cools again, the pressure of the air outside forces the liquid up the stem. Such an arrangement is very sensitive and is useful as a *thermoscope*, that is, an instrument for *detecting* small changes of temperature; but not as a thermometer which measures temperature. Its construction needs to be modified considerably to adapt it to the latter purpose.

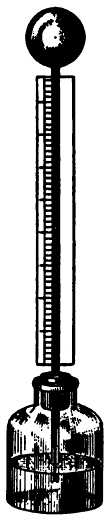

FIG. 200. — Air thermometer.

Problems

1. To how many Centigrade degrees are 54 Fahrenheit degrees equivalent? 63°? 72°? 27°? 108°?

2. To how many Fahrenheit degrees are 15 Centigrade degrees equivalent? 20°? 25°? 30°? 35°? 45°? 60°?

3. What temperature on the Centigrade scale is 54° on the Fahrenheit scale?

4. To what temperature Centigrade is – 40° F. equal?

5. Convert the following readings on the Fahrenheit scale to Centigrade readings: 41°, 50°, 59°, 68°, 77°, and 95°.

6. Convert the following readings on the Centigrade thermometer to Fahrenheit readings: – 10°, – 20°, – 25°, 10°, 15°, 20°, 25°, 30°, and 60°.

7. Draw a figure of a thermometer tube, select two points on the stem for the fixed points, and graduate it, on one side, in the Fahrenheit scale, and on the other, in the Centigrade scale.

8. What should be the temperature, Centigrade, of a sitting room in winter?

9. What temperature, Centigrade, would be considered a very warm day in summer time? Of a very cold day in winter?

10. Express as nearly as you can in both Centigrade and Fahrenheit degrees the extreme range in temperature during the year of the locality in which you live.

II. EXPANSION BY HEAT

347. Expansion. — When a body is heated, it expands, or becomes larger; and when it is cooled, it contracts, or becomes smaller. Increase of length of a body is called *linear* expansion; increase in area, *superficial* expansion; and increase of volume, *cubical* expansion. All three of these terms apply to solids, but cubical expansion only applies to liquids and gases.

Different solids and liquids expand unequally for like changes of temperature, and solids expand so little that some magnifying device is usually necessary to make the expansion apparent.

Experiment. — To show the linear expansion of a solid, place an iron or brass rod upon two blocks about 25 cm. high. Fix one end of the rod by placing a weight upon it (Fig. 201) and let the other end

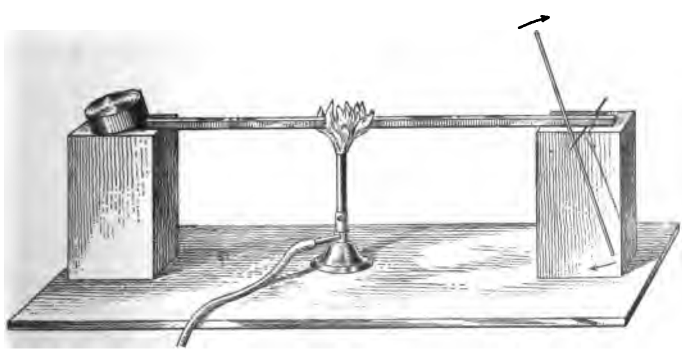

FIG. 201. — Apparatus for illustrating linear expansion of a rod by heat.

rest upon a needle which has been thrust through a straw or long light splinter. Heat the rod by passing a Bunsen flame along it. The expansion will roll the needle over, the movement of the pointer making this movement evident. Allow the rod to cool and the contraction will be apparent from the movement of the pointer in the opposite direction.

Experiment.—Figure 202 represents a fine iron wire *AB* about a meter long, fastened at *A* to a binding post and at *B* to a long light lever of the third class. Let the wire be heated by passing an electric

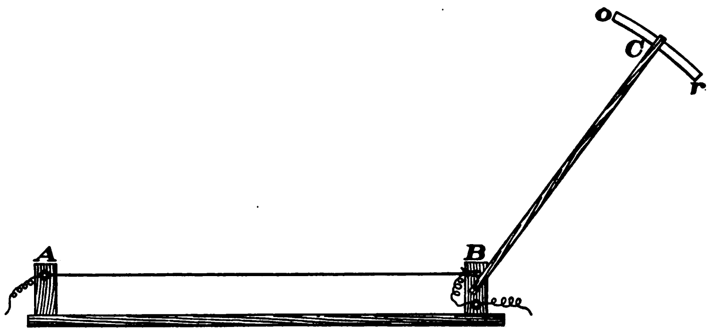

FIG. 202.—Apparatus for illustrating the expansion of a wire heated by an electric current.

current through it. The movement of the pointer *C* toward *r* shows the lengthening of the wire by the heat; when the wire is cooled by fanning it or by cutting off the current, *C* moves back toward *o*.

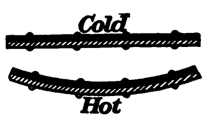

FIG. 203.—Compound bar of two metal strips for illustrating the unequal expansion of solids.

Experiment.—To show the unequal expansion of solids, let a compound bar (Fig. 203) of two metal strips of the same size riveted together, one of iron, the other of brass, be heated gently and uniformly throughout its whole length by passing a Bunsen flame to and fro along it. It will become curved, the brass side being convex. This shows that the brass expands more than the iron. If the bar is made very cold, the iron side becomes convex, showing that the brass contracts more than the iron on cooling. The bending of such a bar may be made evident by clamping it in a vise and making it a part of an electric circuit in which an incandescent

lamp is placed. A needle is placed near the end of the bar so that when the bar is heated it touches the needle and completes the circuit. When the bar cools it bends away from the needle and the lamp goes out.

348. Applications of the unequal expansion of metals are found in the balance wheel of a watch, in thermostats, and in some thermometers.

The rim of a balance wheel (Fig. 204) is double, being brass on the outside and iron on the inside, and it is cut through in two places. If it were not so constructed, warming it would enlarge the wheel so that a greater force would be necessary to make it vibrate as fast as before; but the rise in temperature weakens the hairspring which controls the wheel instead of strengthening it, and consequently the watch would lose time. By this arrangement, however, since the brass expands more than the iron, each half of the rim becomes more curved when it is warmed, so that the wheel becomes smaller rather than larger. Thus the effects of temperature changes on the watch are counteracted.

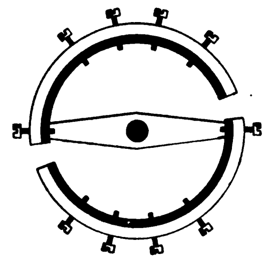

FIG. 204. — Diagram of the balance wheel of a watch; the inner part of the rim is iron, the outer part is brass.

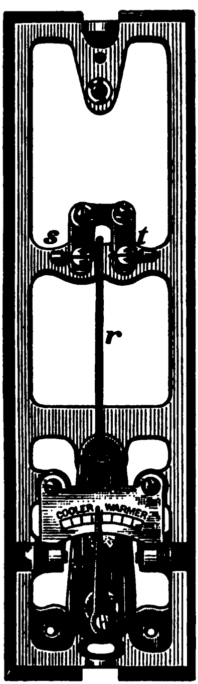

FIG. 205. — A thermostat for regulating the temperature of a room.

Figure 205 represents a thermostat in which *r* is a piece composed of two metals. When the room in which the instrument is placed becomes too warm, *r* bends toward the left and touches the screw *s*. This closes an electric circuit and starts a motor which closes the drafts of the furnace. When the room becomes too cold, *r* bends to the right and touches the screw *t*. This causes the motor to open the drafts.

In dial thermometers a compound metal spring coils and uncoils with changes of temperature, and this motion of the spring is communicated to the pointer of the instrument.

349. Cubical expansion. — **Experiment.** — To illustrate cubical expansion of a solid, use is made of a metallic ring and ball which just passes through the ring at ordinary temperatures. When the ball is heated, it becomes too large to pass through the ring.

Liquids expand much more than solids on being heated. The rise of the mercury in a thermometer shows this.

Experiment. — To illustrate the expansion of liquids, place several different liquids, such as water, kerosene, glycerine, etc., in narrow glass tubes closed at one end by fusion. Color the liquids with aniline and fill the tubes to the same depth. Support these tubes in a vessel of hot water. The expansion will be found to differ for the different liquids.

350. Exceptions to the general rule that a body expands on being heated and contracts when cooled exist. Rubber and iodide of silver are examples, but the most important of all is water between the temperatures of 0° C. and 4° C. When water is cooled, it contracts until 4° C. is reached. At this point, as the cooling goes on, it begins to expand and continues to do so until it reaches 0° C. Water is therefore densest at 4° C.; at 0° C. its density is almost the same as at 8° C.

This fact is of the utmost importance in the economy of nature. Were it otherwise, the waters of our lakes would freeze at the bottom first and probably freeze solid from bottom to top, and never perhaps completely thaw out in summer. As it is, the water at the bottom of deep lakes never becomes colder than 4° C. and in some very deep lakes never warmer than that temperature.

Experiment. — To illustrate the abnormal expansion of water, a little air-tight glass balloon (Fig. 206) can be made with care which will sink in water at ordinary temperatures, but rise as the water approaches 4° C. Place such a balloon in a bottle full of water and

close the bottle with a stopper through which a thermometer extends. Place the bottle in ice water. During the cooling of the water down to 0° C. the balloon will rise and sink again, showing that the water first contracts and then expands on cooling. If the bottle is now placed in a warm room, the balloon will rise as the temperature approaches 4° C. and sink again as it becomes warmer.

FIG. 206. — Diagram of a glass balloon that rises in water at 4° C. but sinks as 0° C. is approached.

351. Coefficient of linear expansion. — The fraction of its length which a body expands on being heated one degree is called its coefficient of linear expansion.

Suppose a bar of metal is heated. If its first length is subtracted from its length after it is heated, the *total expansion is obtained.* Dividing this total expansion by the number of degrees the bar is heated gives the *expansion per degree.* To find what fraction of the original length this expansion for one degree is, we divide it by the original length; the result is by definition the coefficient of linear expansion of the bar.

The coefficient is numerically equal to the expansion of a unit length for a rise in temperature of one degree.

352. The coefficient of cubical expansion of a body is the fraction of its volume which it expands on being heated one degree. The cubical coefficient of a body is three times its linear coefficient of expansion.

The coefficient of cubical expansion is calculated in the same way as that for linear expansion. Subtract the volume of the body when cold from its volume after it is heated to obtain the whole amount of expansion. Divid-

ing the total expansion by the number of degrees the body is heated gives the expansion per degree, and this last result divided by the original volume equals the fraction of its volume it expands on being heated one degree, or the coefficient of cubical expansion.

Strictly speaking, all coefficients of expansion should be reckoned on the size of a body at 0° C., but only in the case of gases is this rigidly adhered to.

For gases the above process may be expressed by the formula,

$$a = \frac{v_2 - v_0}{v_0 t},$$

in which v_0, v_2, t, and a represent respectively the volume of the gas at 0° C., its volume after being heated, the change of temperature, and the coefficient of expansion.

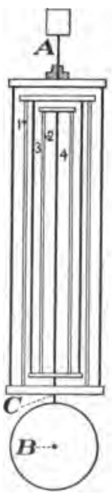

FIG. 207. — Diagram of a gridiron pendulum.

Problems

1. A brass rod 80 cm. long at 9.7° C. expanded .136 cm. on being heated to 99.2° C. What is the coefficient of expansion of brass?

2. A rod of copper at 15° C. is 64 cm. long and at 99° C. it is 64.092 cm. long. What is the coefficient of expansion of copper?

3. If the coefficient of linear expansion of iron is .000012, what will be the length of a railroad rail at 35° C. which is 90 ft. long at −10° C.?

4. A carriage wheel is 7 ft. 6⅜ in. in circumference. An iron tire is 7.5 ft. around its inner circumference at 15° C. To what temperature must the tire be heated to just slip on the wheel?

5. Figure 207 represents a gridiron pendulum, the five heavy lines representing steel rods and the light ones zinc rods; the length from the center of suspension A to center of oscillation B is 99.3 cm. What must be the total length of the two zinc rods 1 and 2 and the steel rods 3 and 4 so that the length from A to B shall remain the same at all temperatures? *Ans.* 70.1 cm.

(NOTE. — The length of $1 + 2 = 3 + 4$, 4 extending to C. Coefficient of zinc is .000029.)

6. A glass flask holds when full 100 cc. at 16° C. How much would it hold at 96° C., coefficient of linear expansion of glass being .0000083?

Ans. 100.2 cc.

7. What should be the correction for temperature for a barometer reading of 74 cm., the temperature of the mercury being 21° C.? (The coefficient of cubical expansion for mercury is .000181.)

Why is this correction subtracted from the reading?

Ans. 0.281 cm.

8. What should be the correction in the above case for the expansion of the brass scale, coefficient for brass being .000019?

This correction is added. Why?

9. Calculate the temperature correction for a barometer reading of 738 mm. taken at 19° C., considering both the expansion of the mercury and the brass scale.

353. Law of Charles. — Gases expand very much more than solids or liquids for a given change of temperature, but especially differ from them in obeying the following laws:

(1) *All gases expand equally for equal increases of temperature, if the pressure remains constant.*

This means that the coefficient of expansion is the same for all gases, being $\frac{1}{273}$ or .00367; that is, any gas expands $\frac{1}{273}$ of its volume at 0° C. for every rise of 1° C. in temperature.

If a gas is inclosed in a vessel so that it cannot expand when heated, then the pressure it exerts on the interior of the vessel increases.

(2) *All gases increase the pressure which they exert on the interior of a vessel equally for equal increases of temperature, if the volume remains constant.*

The pressure of a gas increases $\frac{1}{273}$ of its pressure at 0° C. for every increase of 1° C. in temperature, the pressure coefficient of a gas being practically the same as its

volume coefficient. The name of Gay-Lussac instead of Charles is sometimes given to these laws.

Experiment. — To illustrate the equal expansion of gases, prepare two air thermometers of equal size and with tubes of the same bore (about 8 mm.), the tubes being bent and connected to the flasks by rubber stoppers having two holes (Fig. 208). Mount the thermometers side by side. Each flask should contain about 3 cc. of sulphuric acid to dry the gases, and the ends of the tubes should dip into a dish of such acid. Fill one flask with hydrogen, passing it through the hole in the stopper. The other will be full of air. Close the holes in the stoppers with glass plugs. Set the flasks first in a dish of water at 50° C. to expel excess of gases in them and then the apparatus is ready for use. First, then, place the flasks in water at about 20° C. and observe the height of the acid in the tubes and then in water at 40° C. and note the expansion. It should be equal for both gases.

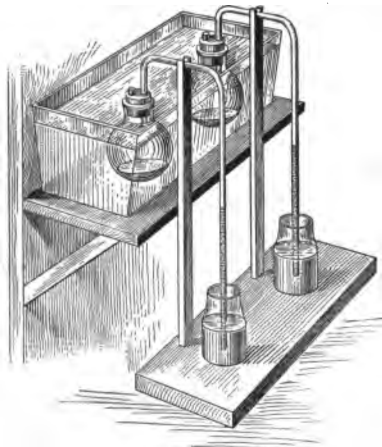

Fig. 208. — Apparatus for showing the equal expansion of air and hydrogen.

354. Absolute temperature. — Since a gas expands $\frac{1}{273}$ of its volume at 0° C. when warmed 1° C., it will contract $\frac{1}{273}$ of its volume at 0° C. when cooled 1° C. Suppose a gas under constant pressure could be cooled 273° below 0° C. and continue to obey this law; then it would lose $\frac{273}{273}$ of its volume at 0° C., that is, lose its whole volume.

Again, suppose the volume is constant and that it is cooled 273° below 0° C. According to the second law it would lose $\frac{273}{273}$ of its pressure at 0° C., that is, its pressure would become zero at − 273° C. Since the

pressure is due to molecular motion (§ 134), no pressure would mean no motion, hence no heat.

Neither of these conditions is possible, for one reason, because the gases would cease to be gases before such a low temperature, even if it were attainable, could be reached; but such considerations have led men to believe that $-273°$ C. is a true or absolute zero of temperature. Temperatures reckoned from this zero are called *absolute temperatures.* Centigrade readings are converted to absolute temperatures by adding 273 to them.

355. Other forms of the gas laws. — When temperatures are expressed in the absolute scale, the laws of Charles or Gay-Lussac may be expressed in very useful forms as follows:

(1) *The volumes of a given mass of gas are proportional to its absolute temperatures, if the pressure is constant.* $\frac{v_1}{v_2}=\frac{T_1}{T_2}$, v_1 and T_1 and v_2 and T_2 being corresponding volumes and temperatures.

(2) *The pressures exerted by a given mass of gas are proportional to its absolute temperatures, if the volume is constant.* $\frac{p_1}{p_2}=\frac{T_1}{T_2}$, p_1 and T_1 and p_2 and T_2 being corresponding pressures and temperatures.

These two laws may be combined into one law if both pressure and volume vary, namely:

(3) *The products of the volumes and pressures of a given mass of gas are proportional to its absolute temperatures.* $\frac{p_1v_1}{p_2v_2}=\frac{T_1}{T_2}$.

Problems

1. 100 cc. of gas at 0° C. was warmed to 30° C. What was its volume at the latter temperature, pressure being unchanged?

2. What is the volume of a quantity of air at 77° C. which has a volume of 300 cc. at 27° C., pressure being constant?

3. What is the volume of a given mass of air at 0° C. which is 300 cc. at 27° C., pressure being unchanged?

4. A gas was heated under constant pressure until its volume became 390 cc. If its volume and temperature were at first respectively 250 cc. and −23° C., to what temperature was it heated?

5. At what temperature will the elastic tension of a gas be 45 lb. per square inch, if it is 20 lb. at 0° C., the volume being unchanged?

6. A gas which at 15° C. was under a pressure of 76 cm. was kept at constant volume and heated to 60° C. What did the pressure become?

7. What will be the volume of a quantity of hydrogen under standard conditions, which has a volume of 32 cc. at 22° C. and 74 cm.?

8. Reduce the volume of a gas which is 28.6 cc. at 19° C. and 75.4 cm. pressure to standard conditions.

9. A chemist collected 21 cc. of a gas in a tube over mercury. At the time the barometer reading corrected for temperature was 73.8 cm. and the temperature of the gas was 24° C. The mercury in the tube stood 13.8 cm. above the mercury in the dish. Reduce the volume of the gas to standard conditions.

10. A quantity of air at 7° C. has a volume of 14 cc. and at 28° C. its volume is 15 cc. Calculate its coefficient of expansion by means of the two equations, $v_1 = v_0(1 + at_1)$ and $v_2 = v_0(1 + at_2)$.

11. A mass of air at 0° C. has a volume of 34.125 cc. and at 63° C. its volume is 42 cc. What is its coefficient of expansion?

12. Calculate the coefficient of expansion of air from the following data: volume at 7° C. = 24 cc. and at 37° C. its volume = 25.4 cc.

III. HEAT AND ENERGY

356. Nature of heat.—Until about the beginning of the nineteenth century the *material* theory of heat was most generally accepted. This theory held that heat is a kind of matter,— a subtle, weightless fluid, which, entering into bodies and uniting with them, makes them warmer. This substance was called *caloric*.

The progress of science during the first half of the nineteenth century was marked by the overthrow of this theory and the adoption of the *mechanical* or *dynamic* theory of heat.

According to this theory the molecules of a body are in a state of rapid vibration and when this motion is increased, the body becomes warmer and when it is decreased, the body becomes colder. Heat, therefore, may be defined as *the energy of molecular motion.* Since it is energy of motion, it is one of the forms of kinetic energy.

357. Development of heat. — Since heat is a form of energy, it can be produced, according to the doctrine of conservation of energy, only from some other form of energy; and when heat is transformed, some other form of energy must be produced. All other forms of energy may be either directly or indirectly transformed into heat, and conversely, heat may be transformed into any other form of energy.

358. Development of heat from mechanical energy. — Instances of the transformation of mechanical energy into heat are so numerous in everyday life that experiments to illustrate it are scarcely necessary. It is done in various ways.

First, by friction. — It is well known that savages were accustomed to kindle fire by heat obtained by rubbing two sticks together. A saw, or a bit, or a gimlet becomes warm when in use, and metal when cut or drilled often becomes too hot to be handled. Illustrations such as these, to which the student can easily add from his own experience, show that friction by destroying mechanical motion transforms the energy of that motion into heat.

Experiment. — Attach a brass tube about 1.5 cm. in diameter and 12 cm. long to a whirling machine. Fill the tube with water or alcohol and close it with a cork. Clasp the tube with a wooden clamp

having two grooves in it to fit the tube, and then revolve the tube rapidly. In a few moments the water will be made to boil by the friction, and the steam will expel the stopper.

Second, by percussion or collision. — By hammering a nail upon an anvil, one can soon make it so hot that it cannot be handled with the fingers. In this case the downward motion of the hammer is suddenly stopped as it hits the nail, but the energy of it reappears in the form of heat, the motion of the molecules in the nail, the anvil, the hammer, and the surrounding atmosphere being quickened by the collision.

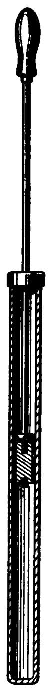

Fig. 209. — Fire syringe, the air in which may be heated by compression so as to ignite tinder.

Third, by compression. — When one pumps air into a bicycle tire, the air is very much compressed and work is done upon it. The energy of mechanical motion which the operator supplies is transferred to the air and changed into heat. This heat is in part transferred to the pump and the tube leading to the tire and is evident to the touch. If the open end of the tube of the pump is placed close to the bulb of an air thermometer, and air blown against the bulb by working the pump rapidly, the heat may be made evident by the thermometer.

In many modern machine shops much use is made of compressed air, and the heat produced in the air pumps is always considerable, and often it is necessary to have special arrangements to convey this heat away.

Experiment. — The fire syringe (Fig. 209) consists of a tube either of brass or glass closed at

one end and having a closely fitting piston. Place a small piece of dry tinder in the cavity in the end of the piston, insert the piston in the end of the tube, and quickly force it in so as to compress the air in the tube. Withdraw the piston as quickly as possible; the heat produced in compressing the air will set the tinder on fire. Care must be taken on repeating the experiment to see that the tube is filled with pure air, and the piston must be removed very quickly or the fire will be quenched by the gases arising from combustion.

When the tube is made of glass, place a drop of carbon bisulphide in the tube to fill it with a mixture of its vapor and air. The heat of compression is made evident by the flash of light which is observed when the piston is forced in. Only a very small quantity of the bisulphide should be used, as there is danger of exploding the tube with too much of it.

359. Development of heat from electrical energy. — Both the electric arc lamp and the incandescent lamp illustrate the transformation of the kinetic energy of the electric current into heat.

Experiment. — Pass an electric current from a battery through a short piece of fine platinum wire. The wire will be heated white-hot; or using the lighting circuit of the building and a bank of lamps in parallel for resistance, pass a current through a piece of fine iron wire. In this way it is easy to melt a piece of wire two or three feet long.

A flash of lightning and the fires that sometimes result from one are evidences of heat obtained by the transformation of electrical potential energy into heat.

360. The transformation of potential energy of chemical separation into heat is a familiar process to us all. Coal and wood possess this form of energy, and when they are burned, heat is produced. Many other chemical actions cause this same transformation of energy. Sometimes, however, chemical changes consume heat, changing it into potential energy.

Can you suggest how the potential energy of a lifted weight may be changed into heat?

361. Radiant energy transformed into heat. — We have seen that radiant energy consists of wave motion in the ether. We say that heat comes to the earth from the sun, but strictly speaking it is not heat that comes from the sun, because on the way it is radiant energy. It may have been heat before it started, and after it reaches us becomes heat again, because the waves impart their energy to the molecules of the matter which they penetrate. This transformation of radiant energy into heat we have learned to call absorption (§ 256).

IV. THE STEAM ENGINE

362. Transformation of heat into other forms of energy. — The changing of heat into other forms of energy, the converse of what we have been studying, is not only possible, but is of very common occurrence. However, it is not possible to reverse the process completely. Other forms of energy are easily transformed entirely into heat; but it is not possible to convert all of a given quantity of heat back again into the other forms of energy. We say, for example, that the sun or a stove emits heat. In reality its molecules set up waves in the ether, sending them outward. This is a transformation of heat into radiant energy, and such bodies do not emit heat, but radiant energy which becomes heat again by absorption.

Again, the steam engine, the gasoline engine, the gas engine, and the hot-air engine are machines for converting the energy of heat into mechanical energy. The proof of this is that we furnish such machines with heat energy and they give us in return mechanical energy, running automobiles, railway trains, and all kinds of machinery for us.

363. The heat engine or hot-air engine transforms heat into mechanical energy by the alternate heating and cool-

ing of an inclosed body of air. Figure 210 represents one form of such an engine. *C* is a hollow cylinder in which there are two pistons, *A* and *B*. *A* is the working piston and fits the cylinder closely. It is attached to the lever *D*, and by moving back and forth in the cylinder turns the fly wheel *E*. The piston *B*, called the displacer piston, fits loosely in the cylinder, and by moving back and forth crowds the air alternately from one end of the cylinder to the other. The cylinder is kept cool at the top either by air or water circulating around it, and at the bottom it is kept hot by a fire placed beneath it. The piston *B* is one quarter stroke ahead of *A*, so that when it is at the end of its stroke, *A* is at the middle of its stroke. The heat at the lower end expands the air in the cylinder and the piston *A* is pushed up, and *B* moves down, crowding the air into the upper end, where it cools and contracts. This reduces the pressure in the cylinder, and *A* is pushed down by the pressure of the outside atmosphere. *B* then moves up, crowding the air again into the lower end of the cylinder, where it is heated and the operations are repeated. Such engines are used where small power is

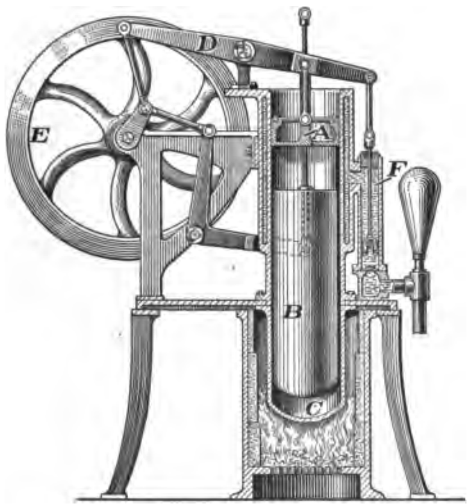

FIG. 210. — Diagram of a hot-air engine.

required and little attention can be given to an engine in operation.

364. The steam engine is a machine for converting the heat energy in steam into the energy of mechanical motion.

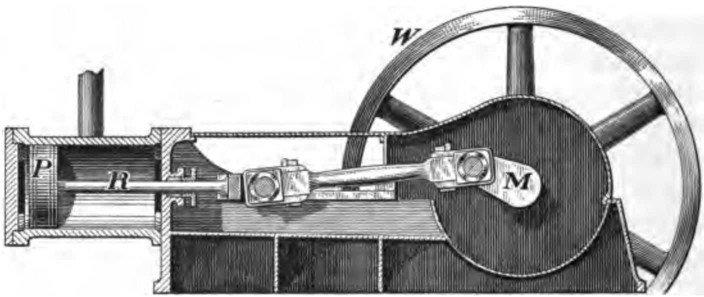

Fig. 211. — Diagram of a steam engine seen from the side.

The boiler, although necessary to the operation of the engine, is not a part of it. Its most essential parts are the *cylinder* (Figs. 211, 212), in which a tight-fitting *piston*

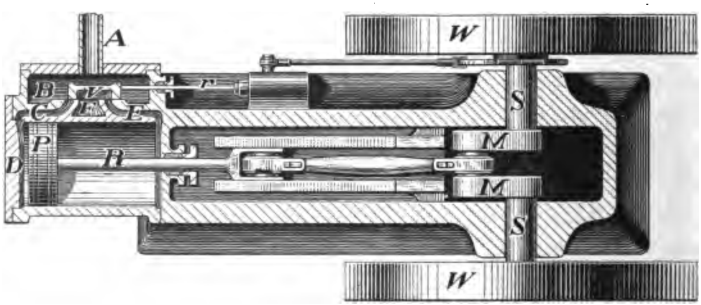

Fig. 212. — Diagram of a steam engine seen from above.

P moves back and forth, and a *steam chest* or *valve chest* *B*, from which the steam enters the cylinder through the openings *C* and *E* called *ports*, which are closed and opened by the *slide valve* *V*.

The steam, which enters the steam chest from the boiler through *A*, is represented as going through the port *C* between the piston head *D* and the piston and pushing the piston toward the right, and the steam used in the preceding stroke is going out through the port *E* and escaping through the *exhaust pipe F*, either into the air or into a vacuum chamber called a *condenser*.

Before the piston reaches the end of its stroke, the valve *V* begins to move toward the left and closes the port *C*; and by the time *P* completes its stroke toward the right, the valve has moved so far to the left that the port *E* is open to the steam chest *B*, and the port *C* is in communication with the exhaust *F*. Steam now enters through *E*, pushing *P* back toward the left, and the exhaust steam through *C*. The piston is thus driven back and forth in the cylinder by the steam which is admitted alternately to its opposite sides.

The motion of the piston *P* is communicated to the *main shaft S* by the *piston rod R* and the *connecting rod* which is attached to the *crank M*, and is changed into rotary motion in the same way as is the motion of the treadle of a sewing machine. The valve *V* is moved back and forth by the *eccentric rod r* and the *eccentric* which is mounted on the main shaft *S*. On the shaft are one or two heavy *fly wheels* the momentum of which serves to give steadiness of motion to the main shaft, and also to carry the crank past the *dead points* or the positions the crank is in when the piston is at the end of either stroke.

365. The compound engine. — In some engines the steam, after leaving the cylinder, enters a second larger cylinder, where it acts on another piston in the same way as before and does more work by expanding into a larger volume. Such an engine with two or more cylinders is called a compound engine. Those with three cylinders

are called *triple expansion* engines and those with four cylinders quadruple expansion engines. In such engines each successive cylinder must be larger in diameter than the preceding one.

366. Efficiency of an engine. — In § 399 we shall learn that when a gas expands and does work, it cools itself, the energy for doing the work being furnished by its own heat. It is on this principle that the steam engine operates. The steam as it leaves the engine is cooler than when it entered because it has done work. To get the greatest possible amount of work from steam it must enter the engine as hot as possible and leave it as cold as possible. The practical limits for these two temperatures set the limit for the efficiency of an ideally perfect engine. Such an engine could utilize scarcely more than 30% of the energy contained in the coal used. The very best engines utilize hardly more than 15% of the coal consumed, and many engines have an efficiency of only 5% or even less. The steam engine at its best is very wasteful of energy.

367. The gas engine. — A mixture of air and coal gas, or air and gasoline vapor, is explosive. In gas engines and gasoline engines such a mixture is introduced into the cylinder and exploded by an electric spark. The explosion generates heat which raises the resulting gases to a high temperature, and they expand and force the piston along. The return stroke of the piston expels the exploded gases, the second stroke allows a new charge of mixed gases to enter the cylinder, the second return stroke compresses them, and then they are exploded by another spark. Only one side of the piston is acted upon. Thus in an engine with a single cylinder the piston is acted on only once in two revolutions of the shaft.

368. Turbine engines. — Engines which have a piston moving back and forth are called *reciprocating* engines. In such engines much energy is wasted in stopping and starting the piston, which often weighs several hundred pounds, at the end of each stroke. At the present day the steam turbine, in which the steam produces rotary motion directly, is being introduced. In December, 1905, the *Carmania*, the first Atlantic liner to be fitted with turbine engines, began to make regular trips across the ocean.

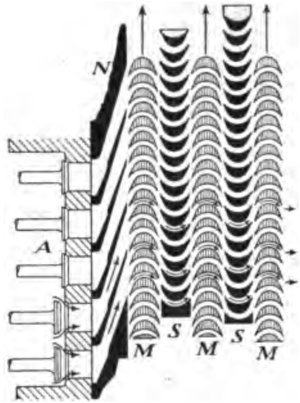

FIG. 213. — Diagram illustrating the action of steam in a turbine engine. *M*, blades on revolving drum; *S*, stationary blades.

The turbine consists of a drum (Figs. 213, 214) whose outer surface is covered with a large number of curved blades or paddles. Jets of steam, striking these blades, cause the drum or shaft to revolve. In one form of the steam turbine the interior surface of the inclosing case is

FIG. 214. — Simple turbine engine, showing a jet of steam striking the blades.

also covered with blades, which alternate with those on the drum, and the steam, rebounding between these stationary blades on the casing and the blades on the revolving drum, is used over and over again in its passage from one end of the drum to the other. At the end of the drum where the steam enters the blades are smaller than at the other end, and the steam expands on its way through, doing work at the expense of its own heat.

Steam turbines are as efficient as the best reciprocating engines, occupy less space, and run more quietly and with less jarring.

V. MEASUREMENT OF HEAT

369. Measurement of heat. — Heat is a thing that can be measured as definitely as a length or an area. We have learned how to measure temperature and know that it is measured in degrees, but temperature and heat are two very different things; this is proved by the fact that a pound of water, for example, contains much more heat than a pound of mercury, although both are at exactly the same temperature.

To measure heat a unit of heat is necessary. Different units are in use, but for scientific purposes the *calorie* is selected. A *calorie is the amount of heat necessary to warm one gram of water* 1° C. This, sometimes called the gram calorie, is the unit that will be used in this book.

In engineering in this country and Great Britain the unit used is *the amount of heat necessary to warm one pound of water* 1° F. This unit is known as the *British Thermal Unit* and is designated by the letters B.T.U. Sometimes the *kilogram calorie* is used in engineering; it is *the quantity of heat necessary to warm* 1 *kg. of water* 1° C. It is one thousand times as great as the gram calorie.

370. The mechanical equivalent of heat. — Since heat is energy and since it is transformable into mechanical energy and mechanical energy is transformable into heat, it follows that a definite quantity of one must be equal to a definite quantity of the other. The amount of mechanical energy to which a unit of heat is equal, is called *the mechanical equivalent* of heat. James Prescott Joule of Manchester, England, after a memorable series of experiments extending over several years, concluded, in 1849, that one B.T.U. of heat was equivalent to 772 foot pounds of mechanical energy. In later years the subject was reviewed by Professor Rowland and others with the result that the mechanical equivalent of heat is now given as 4.187 *joules of work for one gram calorie of heat.* 4.187 joules = 41,870,000 ergs or .427 kilogrammeter of work. This value makes one kilogram calorie equivalent to 427 kilogrammeters and one B.T.U. to 778 foot pounds of work.

Problems

1. If the energy of a 10-lb. mass moving at the rate of 100 ft. per second is all converted into heat, how much heat is produced?

Ans. 2 B.T.U.

2. How much heat is produced when a bullet weighing 15 g. and having a velocity of 600 m. per second strikes a target?

Ans. 645 calories, nearly.

3. When a street car weighing 4 tons and going at the rate of 12 mi. per hour is stopped by the brakes, how much heat is produced?

4. A pound of pure carbon when burned produces about 14,500 B.T.U's. of heat. How much water could be pumped from a river into a reservoir 100 ft. above it by a steam engine if all the heat energy of half a ton of coal could be used to do the work without any waste whatever, supposing the coal to be pure carbon?

5. If the amount of water pumped were 1,400,000 gallons, an engineer would consider that the work had been done quite satisfactorily. How efficiently has the energy of the coal been converted into the work desired to be done?

6. If the food you eat at one meal would, when burned, produce 500 B.T.U.'s of heat, and if the body were able to convert all of the chemical potential energy of this food into mechanical energy and use it all in climbing the stairs of Washington monument (500 ft.), how many times could you climb the stairs, your weight being 125 lb.?

Ans. Over 6 times.

7. What transformations of energy occur when a bullet is fired from a gun?

8. At Niagara the water falls 160 ft. How much warmer should the water at the foot of the cataract be than at the top?

371. The specific heat of a substance is the ratio of the amount of heat necessary to warm any mass of that substance one degree to the amount required to warm an equal mass of water one degree.

For example, it requires 0.113 times as much heat to warm a pound of iron 1° C. as it does to warm a pound of water 1° C.; or 0.113 times as much heat to warm a ton of iron 1° F. as it does to warm a ton of water 1° F.; or 0.113 times as much heat to warm a gram of iron 1° C. as it does to warm a gram of water 1° C. Hence, by the definition, 0.113 is the specific heat of iron.

The specific heat of a substance is numerically equal to the number of gram calories necessary to warm a gram of that substance 1° C. The specific heat of water, which is 1, is much larger than that of most substances. Because of its comparatively large specific heat water requires a relatively large amount of heat to warm it, and in cooling it gives out a relatively large amount of heat. This fact explains the great influence of large bodies of water upon the climate of a country, also why a hot-water bag is better than a hot brick in sickness. The amount of heat that will warm one pound of water one degree will warm over thirty pounds of either lead, gold, or mercury one degree.

372. Thermal capacity. — Experience teaches us that it requires more heat to warm a large mass than a small one,

and we have seen that some substances require more heat to warm them than others.

The amount of heat necessary to warm any body one degree is called the *thermal capacity* of that body. The thermal capacity of a body is obtained by multiplying its mass by its specific heat. For example, the thermal capacity of a mass of iron weighing 1500 g. is $1500 \times 0.113 = 169.5$ calories.

The quantity of heat lost by a body in cooling, or gained by it in being warmed, equals *its mass* $\times$ *its specific heat* $\times$ *its change of temperature*.

Problems

1. If it requires 18 calories to warm a stone 3° C., what is the thermal capacity of the stone?

2. How many degrees will 450 calories warm a body whose thermal capacity is 15 calories?

3. How much heat will a body whose thermal capacity is 180 calories lose in cooling from 36° C. to 20° C.?

4. The thermal capacity of a mass of lead of 80 g. is 2.40 calories. What is the specific heat of lead? Solve by using definition of specific heat.

5. It requires 10.7 B.T.U's of heat to warm 25 lb. of aluminum 2° F. What is the specific heat of aluminum?

6. It requires 10.7 calories to warm 5 g. of aluminum 10° C. What is the specific heat of aluminum?

7. How much heat is needed to warm 80 g. of mercury 4° C., the specific heat of mercury being 0.033?

8. A mass of lead of 480 g. was cooled from 96° C. to 16° C. How much heat did it give out, its specific heat being 0.03?

9. The mass of a body $\times$ its specific heat is sometimes called its *water equivalent.* Explain why.

10. A copper ball weighing 300 g. and having a temperature of 99° C. was placed in a liquid which it warmed to 20° C., the ball itself being cooled to 20° C. How much heat did the ball impart to the liquid, the specific heat of copper being 0.093?

11. In the last problem, what was the temperature of the liquid before the ball was placed in it, if its mass was 400 g. and its specific heat was 0.60?

12. The thermal capacity of a mass of lead is 408 calories and its mass is 13.6 kg. What is the specific heat of lead?

373. How to determine specific heat. — If a known mass of copper, for example, at a known temperature, as 100° C., is placed in a known mass of cold water at a given temperature, the water will be warmed up to a certain temperature and the copper cooled down to the same temperature. The copper gives heat to the water; and, provided no heat is lost to outside bodies, *the amount the water receives is equal to the amount the copper loses.* The amount the water receives = *its mass* × *its specific heat* × *its change of temperature;* the quantity of heat lost by the copper = *its mass* × *its specific heat* × *its change of temperature.* These two quantities are placed equal to each other, and the value of the specific heat of the copper, the unknown quantity, is determined from the equation formed. This method of procedure is called the *method of mixtures.*

In practice, the cup holding the water, called a *calorimeter*, receives some of the heat from the copper, and this must be considered. The cup is warmed the same number of degrees as the water it holds, and the amount of heat it receives = *its mass* × *its specific heat* × *its change of temperature.* The quantity of heat the copper loses is then placed equal to the sum of the amounts of heat gained by the water and the cup.

The mass of the cup × its specific heat is its thermal capacity, and represents its *water equivalent.* Thus, suppose the cup is an iron one weighing 120 g. Its thermal capacity is 120 × 0.113, which equals 13.56 calories; *i.e.*, it requires 13.56 calories to warm the cup 1° C.; but 13.56 calories would warm 13.56 g. of water 1° C. Therefore,

13.56 grams is the water equivalent of the cup because it requires just as much heat to warm the cup 1° C. as it does to warm 13.56 grams of water 1° C.

Table of Specific Heats

Aluminum	.22	Iron	.113
Brass	.094	Lead	.031
Copper	.093	Mercury	.033
Gold	.032	Silver	.056
Ice	.5	Zinc	.094

Air at constant pressure	.24
Hydrogen at constant pressure	3.40
Steam at constant pressure	.5 nearly

Problems

1. A mass of copper weighing 200 g. was heated to 99° C. and dropped into a copper cup holding 400 g. of water at 19.5° C. The cup and water were warmed to 23° C. and the copper mass cooled to the same temperature. The cup weighed 100 g. Find the specific heat of copper and the water equivalent of the cup.

Let s = the specific heat of copper.

The heat lost by the copper $= 200 \times s \times (99 - 23)$.
The heat gained by the water $= 400 \times 1 \times (23 - 19.5)$.
The heat gained by the cup $= 100 \times s \times (23 - 19.5)$.

$$200 \times s\,(99 - 23) = 400 \times 1 \times (23 - 19.5) + 100 \times s \times (23 - 19.5).$$
$$s = .094.$$

Water equivalent $= 100 \times .094 = 9.4$ g.

2. 80 g. of iron at 99° C. placed in 300 g. of water at 21.8° C. raised its temperature to 24° C. The cup used was the one mentioned in the last problem. Find the specific heat of iron.

The water equivalent of the cup is 9.4 g., hence the water and the cup together are equivalent to $300 + 9.4 = 309.4$ g. of water.

$$\text{Heat gained by water and cup} = 309.4 \times 2.2.$$
$$\text{Heat lost by the iron} = 80 \times s \times 75.$$
$$80 \times s \times 75 = 309.4 \times 2.2.$$
$$s = .113.$$

3. A copper ball weighed 240 g. Its temperature was 99.2° C. A copper cup weighing 150 g. contained 480 g. of water at 18° C. The

ball was placed in the cup of water, and the resulting temperature of cup, water, and ball was found to be 21.5° C. What is the specific heat of copper? *Ans.* 0.0927.

4. 4 oz. of iron tacks at 99° C. were placed in 1 lb. of water at 16° C. with a resulting temperature of 18.2° C. Neglecting the calorimeter, calculate the specific heat of iron.

5. 120 g. of shot at 100° C. were placed in 486 g. of water at 18° C., the water being in the calorimeter mentioned in problem No. 3. The resulting temperature was 18.6° C. Calculate the specific heat of lead of which the shot are made.

VI. TRANSMISSION OF HEAT

374. Three methods of transmitting heat. — Heat is transmitted from one place to another in three different ways: by *conduction*, *convection*, and *radiation*. In general there is a tendency for all bodies to become of equal temperature. In any room all bodies are receiving heat from one another, but the warmer ones give heat faster than they receive it, and the cooler ones receive more than they give, until the equalization of temperature is complete. When this equality is reached, the exchange of heat continues, but each receives the same amount it gives out.

375. Conduction is the transmission of heat in matter by invisible molecular motion, the molecules passing their motion along from one to another within the body; thus, if one end of an iron rod is heated in the fire, the heat passes through the rod to the other end by conduction. Conduction is a slow process. In general, solids are better conductors than liquids, and liquids than gases.

376. Conduction in solids. — Solids differ among themselves in their conductivity of heat. Metals are the best conductors; and organic substances, such as wood, paper, and cloth, are poor conductors. Of the metals, silver is the best and copper ranks next. A short piece of glass tubing may be held in the fingers without discomfort while

one end of it is being melted in a flame, but a metal rod under similar conditions soon becomes too hot to hold.

Experiment. — Twist the ends of a copper and an iron wire of the same size together and support them with the junction of the two wires in a Bunsen flame (Fig. 215). After a few moments move a sulphur match along each wire toward the flame until a point is reached where the wire is hot enough to ignite it. This point will be found on the copper wire farther from the flame than on the iron wire. This shows that copper is a better conductor than iron.

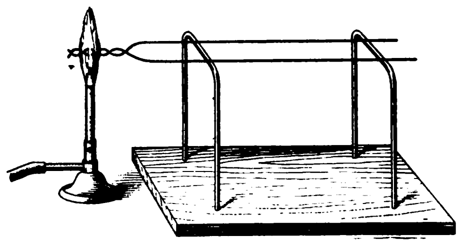

FIG. 215. — Apparatus for showing the difference in conductivity of iron and copper.

Experiment. — Prepare a cylinder, half brass and half wood, and wrap a sheet of paper around it (Fig. 216). Hold this in a Bunsen flame. The paper next to the wood will soon char, while that in contact with the metal will not be injured. This is explained by the fact that the metal conducts the heat away so rapidly that the paper is kept comparatively cool, whereas the paper covering the wood soon becomes hot because the wood is a poor conductor.

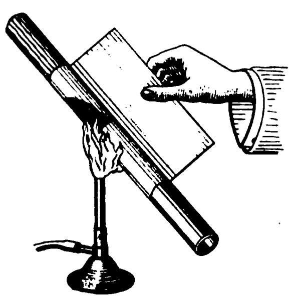

FIG. 216. — Apparatus for showing the difference in conductivity of brass and wood.

377. Conduction in liquids. —

Liquids, with the exception of mercury and molten metals, are very poor conductors of heat.

Experiment. — Fill a test tube full of water and hold it obliquely by the lower end in the Bunsen flame. The water in the top of the tube may be boiled, while that at the bottom is not perceptibly warmed. The tube should be constantly turned in the fingers during the process to prevent the breaking of that part of the tube not in contact with the water.

Experiment. — Surround the bulb of an air thermometer with a funnel (Fig. 217) or a bottle whose bottom has been removed, and fill the funnel with water so the bulb of the thermometer is covered to the depth of about 0.5 cm. Pour a spoonful or two of ether upon the water and set fire to it. The water is such a poor conductor that it will transmit very little heat of the burning ether to the thermometer. If the finger is pressed upon the bulb as soon as the ether ceases to burn, it will probably have more effect upon the motion of the liquid in the stem of the thermometer than the burning ether did.

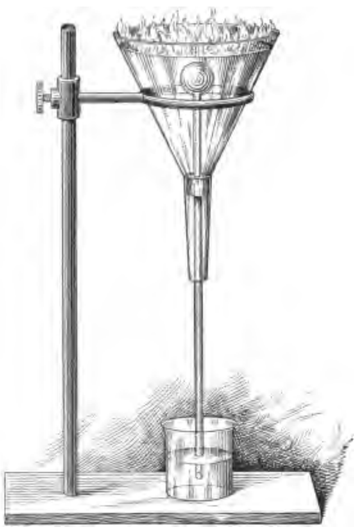

Fig. 217. — Apparatus for showing that water is a poor conductor of heat.

378. Conduction in gases. — Gases are such poor conductors of heat that it has been stated that they do not conduct it at all. This, however, is not true; but it is not easy to prove it because convection and radiation interfere with the experiment. That they are extremely poor conductors is made evident in many interesting ways.

Drops of water rolling about upon a red-hot stove cover are said to be in the *spheroidal state*. The reason why the water is not evaporated almost instantly by the intense heat is because the drop is supported on a cushion of its own vapor which is such a poor conductor that the heat of the stove does not readily reach it.

Experiment. — Heat a copper ball red-hot and plunge it into a basin of warm water. The disturbance and spluttering will not be violent, not nearly so great as when the ball is not so hot. The hot ball is instantly surrounded by a layer of steam, and it is not cooled very quickly because the steam is a poor conductor of heat.

Tyndall, in his *Heat as a Mode of Motion*, speaks of freezing water and also mercury in a red-hot crucible. A hot flatiron is often tested by touching it with a moistened finger without burning the finger. Such things are possible because a gas or vapor is a very poor conductor of heat. The finger is protected from the heat of the iron by the vapor developed, and the water and mercury are protected from the heat of the crucible by the vapor of the solid carbon dioxide used in freezing them.

Powdered substances and porous materials, such as sawdust and charcoal, and such substances as wool, fur, down, and feathers, are poor conductors of heat largely because of the presence in them of interstices filled with air.

379. Convection. — The transfer of heat from one place to another by the movement of heated masses of matter is called convection. According to this definition, heat is transferred by convection when one carries a pail of hot water from one room to another; but this is not the most common meaning of the term. When any portion of a fluid is heated, it expands and becomes less dense than the cooler parts, which, acted on by gravity, sink and crowd the warmer parts upward. Currents of air or water established in this way are called *convection currents*. They may be observed in a beaker of water having solid particles in suspension when it is being heated over a flame. It is in this way largely that the heat of a stove is distributed through a room, and convection currents are depended upon to carry the air up the flues of houses warmed by hot-air furnaces and to cause the draft of a stove or chimney.

Experiment. — Remove the bottom from a bottle and fit it with a stopper and a glass tube bent in the form shown in Figure 218. Fill the bottle and tube with water and support it on a stand. The tube may be filled by placing a rubber tube over one end of it and drawing the water into it by suction. Place some fine shreds of asbestos paper in

the water and then apply heat to the tube as shown in the figure. The convection current in the tube going in the direction of the arrow heads will at once be evident from the movement of the asbestos shreds carried along by the water, and the water in the bottle, beginning at the top, will soon become hot. This experiment illustrates how the water in the kitchen water tank is heated, a pipe extending from the bottom of the tank to the stove or furnace and returning to it again at the top or side.

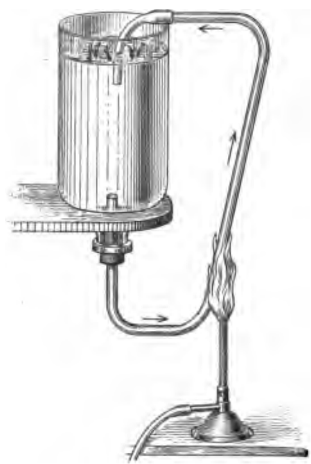

FIG. 218. — Apparatus to show convection in water.

The heating of houses by hot water is an illustration of convection. The cold water, because of its greater density, falls through the pipes to the heater in the basement, and the hot water is thus forced up into the radiators of the rooms above.

Nature furnishes many illustrations of convection on a large scale. Winds, such as land and sea breezes and the trade winds, are true convection currents; and such ocean currents as the Gulf Stream carry enormous quantities of heat from the equatorial regions to colder latitudes.

380. Ventilation. — Convection currents are largely depended upon for the ventilation of houses. The strength of a draft up a flue depends on the difference in weight between the heated gases within it and that of an equal column of air outside. The force with which such currents move, however, is never very great, and they are easily interfered with in many ways. The higher the chimney and the hotter the gases within it, the stronger the draft. Many large buildings are now heated and ventilated by currents of air forced through the flues by blowers or fans driven mechanically.

Experiment. — Fit a wide-mouthed glass jar with a cork through which two glass tubes about 20 cm. long and 2 cm. in diameter extend. These tubes should terminate at the same level within the jar. Pass a stiff wire through the cork at such a place that by turning it about it can support a candle underneath either tube. First light the candle and place it away from the mouths of the tubes, and insert the cork in the jar. The candle will soon go out although both tubes are open. Again, clear the jar of impure air, turn the candle so that it shall be at the mouth of one of the tubes and place it in the jar, closing the jar with the cork. It will continue to burn brightly. To make the convection currents apparent, hold a piece of burning touch paper over the tubes.[1]

381. Radiation, or the transmission of energy by waves in the ether, has already been discussed (§ 254). When we speak of radiation as one of the methods of transferring heat, we must remember that the energy is transmitted as radiant energy and not as heat. As has already been said, the energy before it starts from the heated body is in the form of heat, and becomes heat again on being absorbed by another body, but on the way it is not heat. Waves emitted by a dark object, such as a hot flatiron for example, although too long to affect the eye, are of the same nature as those we call light and obey the same laws; such waves were formerly called "radiant heat."

382. Laws of radiation. — (1) *Radiation travels in straight lines with a velocity of about* 186,000 *miles per second.*

(2) *The intensity of heat received from a body by radiation varies inversely as the square of the distance from the source.*

(3) *Radiation is reflected from a surface so that the angle of reflection is equal to the angle of incidence.*

(4) *The rate at which a body radiates heat varies with its*

[1] This paper is made by soaking blotting paper in a strong solution of sodium or potassium nitrate, and drying it. It burns without a flame, giving off a dense smoke.

temperature, and depends upon the polish of its surface and the nature of its material. The body emitting radiant energy is called a *radiator*. In general, polished metallic surfaces are poor radiators and rough surfaces are good radiators. Lampblack is the best radiator.

(5) *The rate at which a body cools by radiation is roughly proportional to the excess of its temperature above the surrounding medium.* This is known as *Newton's law of cooling*.

Experiment. — Place two large concave mirrors several feet apart and facing each other. Place a candle at the principal focus of one of them, and then adjust them so that the light from the candle being reflected in a parallel beam from the first mirror will strike the second mirror and be reflected upon a card at its principal focus. After the position of the mirrors has been adjusted, place the bulb of an air thermometer at the focus of one of them, and a hot iron ball at the focus of the other. The thermometer will at once give evidence of being heated. This experiment illustrates both the first and the third law of radiation, and shows that the long invisible waves from the dark iron travel in the same way as the short ones of the candle.

Experiment. — Prepare two air thermometers of the same size, smoking the bulbs of each, and place them about 15 cm. apart. Smoke one side of a nickel-plated cup or a bright tin can and fill it with hot water. Place the cup on a stand between the two thermometers with its bright side toward one of them and the smoked side toward the other. The action of the thermometers will show that the side of the cup covered with lampblack radiates heat much more rapidly than the bright polished side.

383. Absorption. — As we have seen (§ 256), ether waves which penetrate a body may pass through it with little interference, or they may be destroyed within the body. In the latter case the molecules of the body, which are as it were entangled in the ether, take the energy from the waves and are made to move more rapidly, the body becoming warmer ; thus radiant energy is transformed into heat, the process being called absorption.

Different substances differ very greatly in their capacity for absorption, but the absorbing power of a body equals its radiating power, so that good radiators are good absorbers. Good reflectors, however, are poor radiators and poor absorbers. These facts are expressed by *Kirchhoff's law: Every body radiates those waves which it is capable of absorbing, and in the same proportion.*

Experiment. — Make two parallel saw cuts in a board about 15 cm. apart and support in them two pieces of bright tin about 12 cm. square. Cover the inner surface of one of the pieces with lampblack by smoking it in a candle flame and attach by wax a bullet to the center of the outer surface of each square. Place a hot iron ball midway between the two squares. The wax on the outer surface of the blackened tin will soon melt and the bullet drop, but the other bullet will remain unaffected. On touching the tins with the finger, the blackened one will be found hot enough to burn the finger, while the other will still be cool. This experiment shows that the lampblack is a good absorber, but that the bright tin surface, while a good reflector, is a poor absorber.

FIG. 219. — Crookes radiometer.

384. The radiometer is an instrument invented by Sir William Crookes in 1875. It consists of a glass bulb (Fig. 219) from which all but a small part of the air has been exhausted. Within the bulb there is a little wheel having four arms or spokes, which revolves on a vertical axis. At the end of each arm is a small square vane of mica blackened on one side and silvered or polished on the other, the vanes being so arranged that the blackened side of one faces the bright side of the next one. When the instrument is placed in sunlight or near a hot body, the wheel revolves, the blackened sides of the vanes moving away from the source of radiation. When it is placed

near a cold body, the rotation is in the opposite direction.

The action of this instrument is explained by the fact that the blackened surface of the vane absorbs more energy than the bright one and becomes warmer ; hence, the molecules coming in contact with the blackened surface bound off from it with a greater velocity than those from the bright side. As each molecule bounds away from the surface it gives the vane a push in the opposite direction (third law of motion). This would not cause motion, however, if much air were present, because the rebounding molecules would come into collision almost instantly with other molecules and communicate a pressure through them around to the polished side of the vane; but when relatively few molecules are present, they bound off from the vanes in straight lines with comparatively little interference from one another. When the radiometer is placed near a cold body, the blackened surface of the vane becomes cooler than the bright side because it radiates faster; hence the rotation is in the opposite direction.

In recent years the radiometer, as modified by Professor Nichols of Columbia University, has become one of the most sensitive of instruments for detecting and measuring radiation.

385. Selective absorption. — Bodies which absorb waves of certain lengths while permitting others which are longer or shorter to pass through them are said to have the power of *selective absorption*. Lampblack, which absorbs waves of all lengths at all comparable with light waves about equally, does not exhibit this power ; but water and common glass do, because they are more or less opaque to long waves coming from bodies of comparatively low temperature, while they transmit or are transparent to shorter waves such as those contained in sunlight. This

fact explains why the glass of a greenhouse retains the heat radiated by steam pipes or the earth, while at the same time it allows the heat of the sun almost free passage.

Experiment. — Bring the light of the sun or of the lantern to a focus by a large lens. Test the heat at the focus by the hand, also by an air thermometer or a radiometer. Then place a flat bottle full of clear water between the lens and the focus and again test the heat at the focus. In the first place the heat at the focus will be quite intense, but in the second case very little heat will be perceived. Again, fill a bottle with a dark solution of iodine in carbon bisulphide and place it between the lens and the focus. The heat at the focus will be found nearly as intense as in the first case, although little light passes through. The water transmits the light waves but not the longer so-called "*heat waves*," and the iodine solution does the reverse of this. It is thought by some that the water will be made more opaque to the heat waves by dissolving alum in it; but this is erroneous.

Although the color of a body is closely related to its power of absorption, yet contrary to the prevalent belief, the color of clothing has little to do with its warmth.

VII. CHANGE OF STATE

386. Changes of state, as from a solid to a liquid or liquid to a gas, are always associated with thermal changes which cause many interesting phenomena, and the principles involved find many practical applications in the daily life of all.

387. **Melting or fusion**. — The change of a solid to a liquid by means of heat is called *melting* or *fusion*, and the temperature at which it occurs is termed the *melting point* of a substance.

Many substances do not have a well-defined melting point because they begin to soften as they approach the liquid form and pass gradually into that state. It is this plastic condition preceding liquefaction which permits sealing wax and glass to be molded and wrought iron to be welded.

A crystalline substance, like ice, has a sharply defined melting point and does not soften upon approaching it. It is for this reason that cast iron, which is crystalline, cannot be worked and welded by a blacksmith.

Freezing, the converse of melting, is the changing of a liquid to a solid by the abstraction of heat. By custom we associate the term with water and liquids that solidify at low temperatures, but it also properly applies to solidification at high temperatures. For instance, iron freezes at about 1500° C.

The temperature at which a substance freezes is the same as that at which it melts. A mixture of ice and water has a temperature of 0° C. Some of the ice may be melted by the addition of heat to the mixture, or some of the water may be frozen by the withdrawal of heat from it; but the temperature of the mixture remains at 0° during either process. Water freezes and ice melts at 0° C.

388. Change of volume at freezing. — It is well known that water when it freezes expands with great force. The bursting of water pipes in winter is sufficient proof of this. Such metals as bismuth and antimony, which are markedly crystalline, also expand on solidifying.

Usually, however, a substance contracts when it solidifies and expands when it melts. This may be illustrated by melting paraffin in a test tube. After it is partly melted, the unmelted portion sinks to the bottom instead of rising to the surface as ice does. This shows that the solid paraffin is denser than the liquid and that it expands on melting.

A change of pressure has a slight influence on the melting point of a substance. If a substance expands on freezing, increased pressure lowers the melting point because it hinders the expansion; otherwise the melting point is raised by increase of pressure.

Experiment. — Let a fine wire be passed over a cake of ice and a heavy weight be attached to it (Fig. 220). In an hour or so the wire will melt its way through the ice, but the cake will be unbroken. This happens because the ice melts under the wire on account of the increased pressure; but it immediately freezes again above the wire where the pressure is restored to its normal amount.

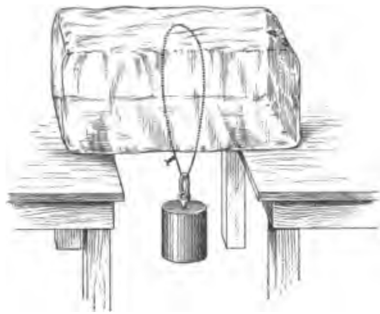

FIG. 220. = Experiment showing that ice melted by pressure freezes again when the pressure is removed.

389. Heat of fusion. — A large amount of heat always disappears when a solid is melted. We know, for example, that heat must be given to ice to melt it; and yet, as it melts, it changes from ice at 0° to water at 0°. The heat that is added to it does not make it any warmer.

The same amount of heat, however, is always produced when a liquid freezes as disappears when the solid melts. *The heat that disappears when a solid melts, or reappears when the liquid solidifies, is called heat of fusion.* It is commonly called *latent heat*, but it is really not heat at all after it becomes latent, because the heat energy in melting a body is used in doing interior work among its molecules in pulling them apart in opposition to molecular forces, being transformed thereby into potential energy within the body. The kinetic energy of heat is changed into a form of potential energy.

The common method of freezing cream is a proof of this. When ice and salt are placed together, a mixture is formed which is naturally a liquid at 0° C. The ice accordingly melts and the salt dissolves and the solution becomes very cold, because the ice and the salt use up their own heat

energy in the change of state. A mixture of 33 parts by weight of salt and 100 parts of ice produces a temperature of $-21.3°$ C.

Water is sometimes placed in cellars to keep the vegetables from freezing; it does this because, as it itself freezes, it gives out heat, potential energy in the water becoming heat energy when the water solidifies.

390. The undercooling of water. — If a bottle of boiled water is placed outside on a still cold night, it may be found in the morning unfrozen although several degrees below freezing point. When it is disturbed, it suddenly freezes and its temperature rises at once to 0° C.

The phenomenon may be illustrated by placing a test tube which contains some pure water free from air in a freezing mixture, such as salt and ice. The test tube should be closed with a stopper through which a thermometer extends into the water. The water may be cooled to $-5°$ C. without freezing it, but when the tube is shaken vigorously, the water suddenly turns to ice and its temperature at once rises to 0°. This shows that freezing is a warming process.

A similar experiment may be made with acetic acid.

Experiment. — Fill a six-inch ignition tube (a test tube is not quite safe) about half full with pure glacial acetic acid, and close the tube with a stopper through which a thermometer extends into the acid. Melt the acid, if it is not already a liquid, by setting the tube in some water, at about 30°; its melting point is about 15°. Then place the tube in some ice water and cool the acid down nearly to 0°. If now the inside of the tube is gently tapped with the lower end of the thermometer, a crystal will form and the whole mass solidify, the temperature rising quickly to the melting point of the acid.

391. Measurement of heat of fusion of ice. — This may be done by the "method of mixtures." A quantity of warm water is weighed and its temperature taken. Then small lumps of ice are added to the water and the mixture stirred till the ice is all melted, when the temperature is again taken. The mass of the ice added is found by

weighing the mixture and subtracting from this the mass of the water previously determined.

The warm water is cooled by the ice and the amount of heat it gives out equals *its mass* × *its change of temperature.* All of the heat, however, taken from the water is not used in melting the ice; some of it is used in raising the temperature of the ice water made from the melted ice from 0° C. up to the final temperature of the mixture. The latter amount, which equals the *mass of the ice* × *the change of temperature*, is subtracted from the heat given out by the warm water and the remainder is the amount of heat used in melting the ice. Dividing this remainder by the number of grams of ice melted gives the number of calories of heat required to melt one gram of ice.

The heat of fusion of ice, called the *latent heat* of water, is 80 calories per gram. This means that it requires 80 times as much heat to melt a gram of ice as it does to warm a gram of water 1° C., or eighty times as much heat to melt a ton of ice as it does to warm a ton of water 1° C. The fact that the heat of fusion of ice is so large explains why the freezing of large bodies of water retards the coming of winter and the thawing of large quantities of ice and snow retards warm weather in spring. If it were not large, the snows on the hills and mountains would melt so rapidly on a warm spring day that enormously destructive freshets would result.

Problems

1. 50 g. of ice were added to 400 g. of water at 20° C. When the ice was all melted the temperature of the water was 8.9° C. What is the heat of fusion of the ice?

2. 40 g. of ice were added to 487 g. of water contained in an aluminum cup (specific heat .215) weighing 60 g. The temperature of the water and the cup was at first 24° C.; and after the ice was all melted, it was 16.3° C. What is the latent heat of water?

3. If 10 lb. of water at 0° C. should freeze in a cellar which contains 3000 cu. ft. of air, how much would the air be warmed, the specific heat of air being .24 and the weight of 1 cu. ft. of air being .08 lbs.? *Ans.* Nearly 14° C.

4. How much would the air of a room 12′ × 20′ × 8′ be cooled by the melting of 5 lb. of ice in it? *Ans.* 10.8° C.

5. How much heat is required to change 60 g. of ice at – 20° C. to water at 70° C., the specific heat of ice being .5? *Ans.* 9600 calories.

6. A refrigerator contained 40 cu. ft. of air. When the ice was placed in it, the temperature was 85° F. After three hours 1 lb. of water had collected in the drip-pan underneath. What was the temperature in the refrigerator, provided the walls of the refrigerator and the outside air had contributed to it in this time 105.6 B.T.U's of heat? *Ans.* 35° F.

NOTE. — The heat of fusion of ice on the basis of the Fahrenheit degree is $\frac{9}{5}$ of 80, or 144.

7. 20 g. of ice were placed in 360 g. of water at 30° C. What was the temperature of the water after the ice was all melted?

8. 180 g. of ice were placed in 1200 g. of water at 19° C. What was the temperature of the water after the ice had melted, provided the water equivalent of the pitcher containing it was 75 calories and the water took up from the air while the ice was melting 96 calories of heat?

9. How many cubic feet of air would be cooled 1° C. by the melting of 10 tons of snow? *Ans.* 83,333,333$\frac{1}{3}$ cu. ft.

10. What is the thermal capacity of 1 cu. ft. of air in B.T.U's? in calories?

392. Cold by solution. — Another method of changing a solid into a liquid is that of dissolving it in a liquid. Heat is also usually transformed in this process as well as in the process of fusion and for the same reason, because heat energy is changed into potential energy. When sugar or common salt, for example, is dissolved in water, the water is cooled by the operation.

Experiment. — Make a mixture of pulverized ammonium nitrate and ammonium chloride, about 3 parts by weight of the former to 1 of

the latter. Place about half a tumbler full of this mixture in a tumbler two thirds full of ice water. Stir the solution with a small test tube full of water; the water in the tube will soon be frozen. Test the solution with a thermometer. Its temperature will be found to be about $-10°$ C. Observe the outside of the tumbler and account for the frost found upon it.

393. Vaporization, or the change from the liquid to the gaseous state, comprises *evaporation* and *ebullition* or *boiling*. Evaporation is vaporization that occurs at the surface of the liquid, slowly, quietly, and through a wide range of temperature. Ebullition, or boiling, is vaporization that occurs within a liquid, rapidly, with disturbance, and at some definite temperature. The disturbance is caused by the forming of bubbles of vapor within the liquid and their rising to the surface. Boiling cannot begin until a temperature is reached which will give to the vapor formed an elastic tension equal to the pressure upon the liquid.

A solid sometimes evaporates, passing directly from the solid state to the gaseous state. Such vaporization is called *sublimation.* Ice, for instance, evaporates or sublimes even in the coldest weather, and camphor and iodine sublime quite rapidly at ordinary temperatures.

394. Laws of ebullition. —

(1) *Every liquid has its own boiling point, which is invariable for that liquid under the same conditions.*

(2) *The boiling point of a liquid is raised by salts and lowered by gases dissolved in it.*

The presence of some foreign particles in a liquid seems to be necessary as a starting point for the formation of a bubble of vapor. The presence of dissolved air greatly aids the beginning of ebullition. When the liquid is very pure and all air particles are removed, it may be heated above its ordinary boiling point, and when the bubbles do form, they form explosively, and the liquid is said to *bump.*

Rough points on the surface of the containing vessel have the same influence as the foreign particles.

(3) *An increase of pressure raises the boiling point of a liquid and a decrease of pressure lowers it.*

395. Pressure and the boiling point. — Water boils at 100° C. when the barometric pressure is 76 cm., but it may be boiled at any temperature between 100° and 0°, if the pressure is sufficiently reduced. Water boils under a pressure of two atmospheres at 120.6° C. For pressures not differing greatly from 76 cm., the boiling point of water may be calculated by the method given in § 343.

Since atmospheric pressure decreases with elevation above sea level, the boiling point is lowered as the elevation increases. Mr. Tyndall, at one time, found the boiling point of water on the summit of Mt. Blanc to be 184.95° F. or about 85° C. The change in the boiling point is roughly 1° C. for a change in elevation of 295 meters or 968 feet. On this basis, the calculated boiling point at the summit of Mt. Blanc (15,779 ft.) would be 182.7° F., and at the summit of Pike's Peak (14,147 ft.) 185.7° F. or 85.4° C. At such elevations many cooking operations are impossible in open vessels.

An application of the principle that boiling occurs at lower temperatures under diminished pressure is found in the vacuum pan in which sugar syrup is evaporated at temperatures below that at which it chars or burns. Milk is also boiled down or condensed in a vacuum chamber at a temperature of about 90° F.

Experiment. — Fill a round-bottomed flask half full of water and boil it vigorously upon a sand bath. While the water is boiling, close the flask air-tight with a solid rubber stopper, *removing it at once from the sand bath.* If closed in this way, the space above the water in the flask will contain very little air, the steam having expelled it; but it will be filled with invisible water vapor. Invert the flask and support

it in a triangle of wire placed over a battery jar (Fig. 221). Now pour ice water over the flask or rub the bottom of it with a lump of ice, and the water will begin to boil again vigorously. The boiling may be continued in this way for several minutes or until the temperature of the water in the flask is as low as 10° C. Take the temperature of the water after the boiling has been continued as long as possible.

The water boils because the cold water or ice diminishes the elastic tension of the water vapor in the flask, the pressure on the water being thereby diminished. It is the heat still left in the water, however, that boils it.

Fig. 221. — **Franklin's experiment.** Warm water in the flask boils when the pressure is reduced by means of ice water.

396. Heat of vaporization. — Whenever a liquid changes into the gaseous state, a large amount of heat disappears; and conversely, when a vapor is condensed into a liquid, a large amount of heat is produced. The heat that disappears when a liquid vaporizes, or reappears when a gas liquefies, is called the *heat of vaporization.* It was formerly and is still called *latent heat;* but although called heat, it is really not heat after it becomes latent, because the heat energy in vaporizing the liquid is used in doing interior work among its molecules and is transformed into a form of potential energy. This same potential energy is changed back again into heat when the vapor liquefies.

The latent heat of steam is very large, 536 calories being required to change one gram of water at 100° C. to steam at 100° C.; that is, 536 times as much heat is required to

vaporize any mass of water as is required to warm the same mass of water 1° C.

Problem. — 10 g. of steam at 100° were added to 500 g. of water at 20°, increasing its temperature to 32.1°. What is the latent heat of steam?

SOLUTION. — The heat produced by the condensation of the steam plus the heat given up by it after condensation in cooling from 100° to 32.1° equals the heat the water receives. Let L = the latent heat of 1 g. of steam. Then, $10\,L + 10 \times (100 - 32.1) = 500 \times (32.1 - 20)$.

Ans. $L = 537.1$.

397. Cold by vaporization. — There are many interesting phenomena illustrating the fact that vaporization is a cooling or heat-destroying process. A few drops of ether on the bulb of an air thermometer evaporate rapidly and quickly cool the thermometer.

If a few drops of cold water are placed on a board under a watch crystal which is filled with ether, and the ether evaporated rapidly by fanning it, the water under the crystal may be frozen.

Experiment. — Place side by side a porous battery cup and a tin cup of the same shape and size and fill them with water of the same temperature. After a time the water in the porous jar will be found to be a few degrees colder than that in the tin cup. This is caused by the rapid evaporation of the water which passes through the pores of the cup to its outer surface.

The evaporation of perspiration keeps the temperature of the body down, and the temperature of a person suffering with fever is often reduced by sponging the patient with alcohol and allowing it to evaporate.

398. Boiling a cooling process. — Since vaporization is a cooling process, boiling, which is rapid vaporization, is a rapid cooling process. This is shown by the fact that after water begins to boil, its temperature ceases to rise, no matter how hot the fire may be. Heat is continually added

to the water, but the boiling destroys it, transforming it into another form of energy as fast as it is added.

Experiment. — Fill two beakers of the same size with water at about 50° C. Place one of the beakers under a small receiver of an air pump and exhaust the air. The water will soon boil. Boil it in this way two or three minutes and then compare the temperature of the water in the two beakers. The water that has been boiling will be found perhaps 10° or 15° cooler than that which has not boiled.

Experiment. — Place a shallow dish *S* (Fig. 222) containing concentrated sulphuric acid upon the air-pump plate, and over this a wire frame supporting a tiny shallow dish *D* holding about 5 cc. of ice-cold water. Cover this with a small receiver and exhaust the air. The water will boil and soon freeze, the water often boiling under the ice formed. The acid absorbs the aqueous vapor and the rapid vaporization of the water causes it to freeze.

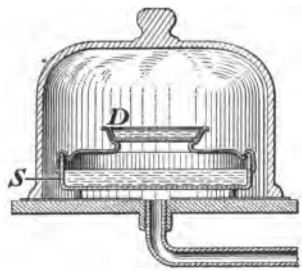

FIG. 222. — Experiment in which water freezes as it boils.

NOTE. — The interior of the pump must be dry or this experiment will not succeed. The same air pump cannot be used for this experiment immediately after being used for the preceding one.

399. Cold by expansion. — We have learned that heat is transformed or cold produced by melting, by solution, and by vaporization. Cold may also be produced by the expansion of a gas. When a gas expands, it has to do work in pushing aside the surrounding air. The energy necessary to do this work is found in the gas itself in the form of heat. This heat energy in the gas, when this work is done, is transformed into mechanical energy, and thus the gas is cooled.

There are many interesting examples of this action. This principle is used in making liquid air, which requires a very low temperature. The air is first very greatly com-

pressed. During the compression the air becomes very warm because work is done upon it. After it is again cooled, it is allowed to escape through a small opening, and it is cooled so much by its own expansion that some of it is liquefied (Fig. 223).

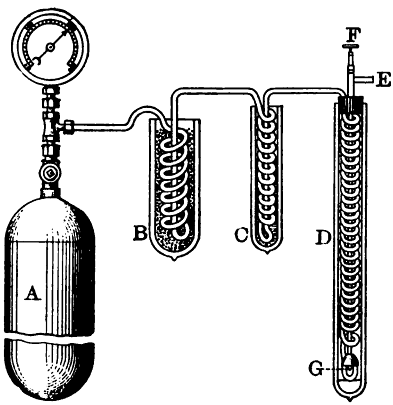

FIG. 223. — Apparatus of Dewar for liquefying gases. *A* is a cylinder containing the gas under high pressure; *B* is a Dewar flask containing a freezing mixture such as solid carbonic dioxide and ether; *C* is a second flask containing some liquid which is made to evaporate rapidly, *e.g.* ethylene or liquid air; *D* is a third flask. Coils bring the gas through *B*, *C*, and *D*, and it is allowed to escape through a fine opening, *G*, whose size is regulated by the rod *F*. As the gas escapes, it is drawn out through the opening *E*, so that as it rises it cools the coils in the tube *D*.

Air under a pressure of nearly 300 pounds per square inch is used to operate the brakes of street cars. A jet of this air striking a motorman's wet glove will freeze it almost instantly.

At the first stroke of an air pump, a slight cloud is often seen in the receiver; this is caused by the cold produced by the sudden expansion of the air. A cloud of condensed vapor is for the same reason often seen at the mouth of a pop bottle when it is opened.

Experiment. — Place a few cubic centimeters of alcohol in a large bottle of clear glass and close the bottle with a rubber stopper having two holes through it. Connect the bottle by a rubber tube to a bicycle pump. Close the other hole in the stopper with the finger and then force some air into the bottle with the pump.

When the air is allowed to escape suddenly from the bottle, it is cooled by its own expansion to such a degree that the vapor of alcohol in the bottle is condensed to a fog or mist.

The heat produced by compression is also made evident by forcing air again into the bottle. The fog at once disappears.

One can compress the air in the bottle by the lungs sufficiently to produce the same effects.

400. The manufacture of ice depends on the principles just explained. Ammonia gas or carbon dioxide gas is condensed into a liquid by means of a powerful pump (Fig. 224) driven by steam or other power. The ammonia, which is heated by this compression (Why?), is cooled by

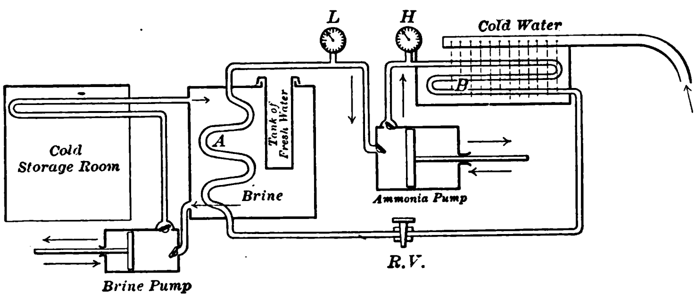

FIG. 224. — Diagram of an ice-making plant.

passing through pipes over which cold water flows. This liquid ammonia is now allowed to vaporize and expand by passing through an expansion valve *R.V.* into *A* in which there is a lower pressure. Intense cold is thus produced in this pipe, which is surrounded with brine, the latter being cooled below the freezing point of water. The water to be frozen is placed in vessels which are set in this cold brine. This brine is also used for refrigerating purposes in cold-storage buildings by being circulated through pipes in the building. The ammonia gas after expansion passes again through the condensing pump and is thus used over and over again. The pump serves not

only to condense the ammonia in the condensing coils, but also to exhaust the gas from the evaporating coils. A pressure of about 10 atmospheres is required to liquefy the ammonia.

401. Saturated vapor. — A small amount of water or other liquid may be introduced into the Torricellian vacuum of a barometer tube by means of a medicine dropper placed under the end of it. By tapping the tube the liquid works its way up through to the top of the mercury column, where some of the liquid evaporates and fills the space above the mercury with its invisible vapor. This vapor exerts an elastic tension or pressure which forces the mercury down the tube a short distance. The amount of this pressure is measured by the difference between the height of the mercury in the tube before the liquid was introduced and the height after it is introduced (Fig. 225). At 20° C. this pressure is 17.5 mm. of mercury for water vapor and 44 mm. for alcohol.

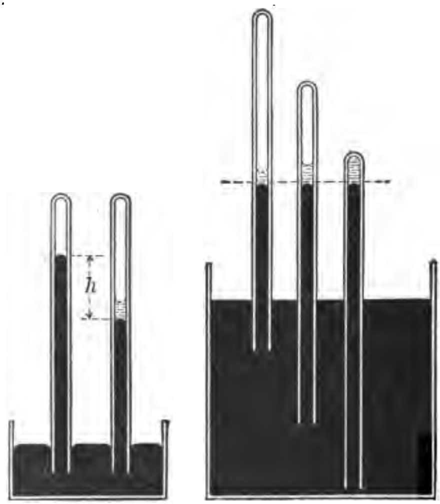

Fig. 225. — Experiments illustrating the laws of pressure of saturated vapor; a small amount of water is introduced above the mercury in a barometer tube.

We know that when a true gas is crowded into a smaller and smaller volume, it exerts a greater and greater elastic tension or pressure outward; with a vapor

it is quite different. When a vapor is compressed its pressure may increase for a time, but for a given temperature a limit is soon reached beyond which the pressure cannot increase. If the vapor is further compressed, some of it condenses into a liquid. Such a vapor is called a *saturated vapor* and its pressure the *saturation pressure* or *vapor tension*, because no more of the vapor can exist in a given space at the given temperature. When a saturated vapor is either cooled or compressed, some of it must change to a liquid.

The space above the mercury in the tube contains saturated water vapor; if this space is made smaller by thrusting the tube deeper into the cistern or by inclining it, the pressure does not change but the vapor condenses. At 10° the saturation pressure for water or aqueous vapor is 9.2 mm.; at 30°, 31.5; and at 100°, 760.

402. Aqueous vapor in air. — A cubic meter of space can contain 12.7 g. of saturated water vapor at 15° C. The presence of air in the space does not change the amount of vapor that can exist in that space. A cubic meter of space can hold 12.7 g. of aqueous vapor at 15° whether it is filled with air at the same time or not. It is customary to say that the air contains the vapor; really the space contains both the air and the vapor mixed together. The air does have, however, a great influence upon the rapidity with which a liquid may vaporize and fill a given space. The presence of air greatly retards the evaporation.

403. Humidity and dew-point. — Humidity is a term relating to the quantity of aqueous vapor in the air. *Absolute humidity* means the actual quantity of aqueous vapor present in a unit volume of space; *relative humidity* is the ratio of the amount of vapor present to the amount that would be present if it were saturated. For example, suppose a cubic meter of air, which at 20° C. can hold 17.1 g.

of water vapor, contains only 4.3 g; the relative humidity is $4.3 \div 17.1 = .25$ or 25%. If the air contains 4 g. of vapor per cu. m., is the humidity high or low? is the air dry or not? That depends on the relative humidity, which, in turn, depends on the temperature. If the temperature is 35° C., the air is very dry, because at that temperature it could contain about 40 g., the relative humidity being ($4 \div 40$) 10%; but if the temperature is 3° the air is moist, the relative humidity being ($4 \div 6$) $66\frac{2}{3}$%, since a cubic meter of air at 3° C. can hold 6 g. of aqueous vapor.

The *dew-point* is that temperature at which the relative humidity is 100%, or it is the temperature of saturation. If the temperature falls below that point, some of the vapor in the air must condense. If this condensation occurs in the air above the ground, a fog or cloud is formed; if upon the surface of the ground, dew is formed; and if the condensation takes place below the freezing point of water frost is formed.

A hot day becomes oppressive if the relative humidity is high, because the vapor tension, being near the saturation point, prevents free evaporation from our bodies.

Problems

1. If 15 g. of steam at 100° are added to 400 g. of water at 12°, what is the resulting temperature? *Ans.* 34.6° C.

2. How much heat will be required to change 2 Kg. of water at 10° to steam at 100°?

3. An aluminum cup (sp. heat .21) weighing 80 g. contained 483.2 g. of water at 18°. 16 g. of steam at 100° being added to this, its temperature became 37°. Calculate the latent heat of steam.

4. How much steam is necessary to melt 7.95 g. of ice?

5. If 2 lb. of steam condenses in a radiator and imparts its heat to the air of a room $15 \times 12 \times 7$ ft., how many degrees C. will the air be warmed? (1 cu. ft. of air weighs .08 lb. Sp. heat of air is .24.)

6. How much heat is necessary to change 50 g. of ice at 0° to steam at 100°? *Ans.* 35,800 calories.

7. What is the boiling point of water when the barometer stands at 74.5 cm.? 73.8 cm.? 76.5 cm.? 70.7 cm.? 68.2 cm.? 73.3 cm.?

8. If water boils at 98.1°, what is the barometric pressure?

9. Water boiled in a closed vessel at 103°. What was the pressure of the steam formed?

10. Find the error of a thermometer which gives the boiling point of water as 98.5° when the barometer stands at 75 cm.

11. If 1 Kg. of coal in burning yields 7500 kilogram calories of heat, how many kilograms of ice at 0° can be melted and raised to a temperature of 15° by burning 5 Kg. of coal?

12. How many Kg. of water at 15° can be changed to steam at 100° by the heat produced by burning 10 Kg. of coal?

13. 100 g. of steam at 100° were passed into a 500-g. mixture of ice and water at 0°. The final temperature of the mixture was 60°. How much ice was there in the mixture at first?

14. How much water at 16° could be boiled away, pressure being 76 cm., by the heat of 100 Kg. of coal, if no heat were wasted?

15. Saturated air at 5°, 8°, 14°, 18°, 23°, 26°, and 28° contains respectively 6.8, 8.2, 12.0, 15.2, 20.4, 24.1, and 27.0 g. of water vapor per cu. m. Find the relative humidity for each of the above temperatures except 5° when the absolute humidity is 8.2 g. per cu. m. What is the dew-point? What would be the state of affairs if the temperature of the air fell to 5°?

16. How much heat is evolved in changing 10 g. of steam at 125° C. to ice at $-15°$ C.?

17. What is the volume of a gas at 77° C. and pressure of 74 cm. which has a volume of 370 cc. at 7° C. and pressure of 70 cm.?

18. How much heat is developed by stopping a mass of 20.95 Kg. having a velocity of 2 Km. per second?

19. If 14 lb. of water are warmed 50° F. by 1 oz. of coal, what is the value of 1 lb. of coal in B.T.U.'s?

20. What is the thermal capacity of 5 lb. of iron in B.T.U.'s? What is the thermal capacity of 5 g. of iron in gram calories?

21. If 5 kilogram calories are changed into mechanical energy, how far could a man weighing 61 Kg. be lifted by it? *Ans.* 35 meters.

22. How many B.T.U's of heat would be generated by stopping a cannon ball weighing 100 lb. and having a velocity of 2000 ft. per second?

23. If the volume of a gas at 87° C. and pressure of 72 cm. is 150 cc., at what temperature will its volume be 180 cc., the pressure being 50 cm.?

24. An aluminum cup weighing 50 g. held 421.5 g. of water at 36° C. 90 g. of ice were melted in it and cooled it to 16° C. What is the heat of fusion of ice?

25. A block of copper weighing 15 Kg. was placed upon a block of iron weighing 21 Kg. The temperature of the copper was 60° and that of the iron 10° at first. What was the resulting temperature if no heat was lost to other bodies?

26. 15 g. of steam at 100° were added to 444 g. of water at 10° C. and warmed it to 30° C. The water was in an aluminum cup weighing 50 g. Find latent heat of steam.

27. At what pressure does water boil at 97° C.?

28. What is the boiling point of water when the barometer stands at 80 cm.?

29. If a 500-g. ball of copper at a temperature of 300° C. is dropped into 800 g. of water at 15° C., what is the resulting temperature?

30. How much snow at −10° C. could be melted by the heat produced by burning 50 lb. of such coal as that mentioned in problem 4, page 317?

31. Why can you "see your breath" on a cold day?

32. How many pounds of water at 15° C. will be required to condense 20 pounds of steam into water at 100° C.?

33. Explain the statement, "the relative humidity is 75%."

34. If the amount of water actually present in the air remains constant but the temperature is raised, how is the relative humidity altered, if at all?

CHAPTER V

ELECTRICITY AND MAGNETISM

404. Electricity is a word familiar to all; but what electricity really is, is unknown. Yet much is known *about* electricity; electrical quantities can be measured with great exactness, there are many laws and principles relating to it definitely established, and it can be controlled and used to a great degree.

For convenience we may divide this branch of science into three parts: Electrostatics, which treats of electricity at rest; Magnetism; and Current Electricity.

PART I. ELECTROSTATICS

I. ELECTRICAL CHARGES

405. Electrical force. — Experiment. — Suspend a pith ball by a silk thread from a glass tube and rub a dry glass rod with silk and bring it near the pith ball. The rod will at first attract the ball, but after the latter has been in contact with the rod it will be repelled (Fig. 226). The glass rod will act in the same way upon bits of paper and other light objects placed upon the table.

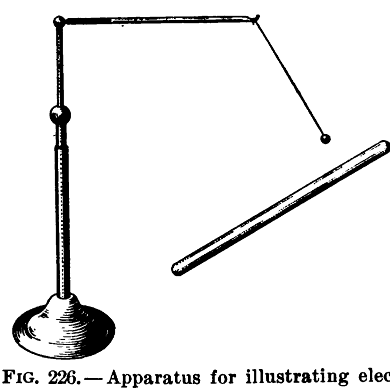

FIG. 226. — Apparatus for illustrating electrical force.

The force which the rod exerts is called an

electrical force and the body exerting the force is said to be *electrified.*

406. Electricity. — A metal ball or cylinder, as B (Fig. 227), supported upon a glass stem S will be found to be electrified after it has been in contact with the electrified glass rod, and it will repel the pith ball after the latter has been in contact with the rod.

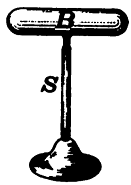

FIG. 227.—An insulated conductor.

The opinion prevails that the electrical force acts between the *particles* of an *agent*, either in or upon the body, known as *electricity.* In accordance with this belief we may say that the electrified body is loaded or *charged* with electricity, and that the glass rod imparted some of its *charge* to the body B and to the pith ball when it touched them.

407. Conductors and insulators. — If a charged body B (Fig. 227) is touched with a rod of sealing wax or with paraffin, it still remains electrified or charged; but if it is touched with a metal rod held in the hand, it loses its charge. The former bodies are called non-conductors or good *insulators* because the charge does not pass from the electrified body through them, but the metal rod is called a good *conductor.* Metals, carbon, and solutions of some salts and acids are good conductors. Moist earth and the human body are also conductors. Among the best insulators are paraffin, silk, sealing wax, dry glass, porcelain, mica, shellac, hard rubber, gutta-percha, and air either dry or moist.

A metallic body, as B (Fig. 227), upon an insulating support such as glass or hard rubber is called an *insulated conductor.* A conductor can be electrified only when it is insulated or surrounded on all sides by a non-conductor.

408. Two kinds of electricity. — A rod of sealing wax

rubbed with dry flannel will serve for the preceding experiments as well as the glass rod and silk, for it will act in the same way; but a pith ball which has been charged by being in contact with the glass rod and is repelled by it is attracted by the electrified sealing wax.

Experiment. — Tie a silk cord around the center of a glass rod so that it will balance horizontally when suspended and cement the cord to the glass so that it will not slip out of place. Suspend the rod by the cord (Fig. 228) and electrify one end of it by rubbing it with silk. Bring near to this another electrified glass rod and also a rod of sealing wax rubbed with flannel. There will be repulsion between the two glass rods, but attraction between the glass rod and the sealing wax.

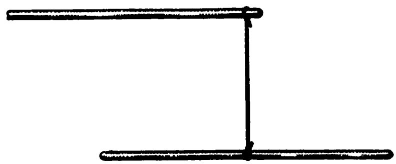

FIG. 228 — Glass rod suspended by a silk cord.

Experiments show that all electrified bodies will behave toward the electrified glass rod either as the second glass rod does or as the rod of sealing wax does. Hence, there are two kinds of electricity, called *positive* and *negative*. Electricity like that on glass when rubbed with silk is termed positive; and electricity like that on sealing wax when rubbed with flannel, negative.

The above experiment illustrates the following fundamental law: *Electricities of the same kind repel each other, and those of the opposite kind attract each other.*

409. Electrical influence. — Two small metal balls are charged equally, one positively and the other negatively, if they act with equal and contrary electric force upon a small electrified body at the same distance from each of them. Two such bodies, one charged with $+e$ and the other with $-e$, are found to be unelectrified after being brought in contact with each other; that is, equal positive

and negative quantities of electricity neutralize each other. Likewise we may regard the atoms of a body which is unelectrified as charged equally with $+e$ and $-e$. If an atom loses some of its negative electricity, it is then positively electrified; or if it loses some of its positive electricity, it is negatively electrified. Thus the electrical condition of a body is determined by that of the atoms which compose it.

Let a negatively electrified rod of sealing wax C (Fig. 229) be brought near to an insulated conductor AB. According to the above discussion we may think of each point of the conductor as having both $+e$ and $-e$ at the same time. The $+e$ will be attracted by the sealing wax and the $-e$ will be repelled by it. The positive electricity of the conductor will therefore move toward A and the negative toward B, and AB will be electrified positively at A and negatively at B. If the sealing wax is removed, the separated electricities reunite and the conductor is again unelectrified. It is said to have been electrified by *influence* or *induction* under the action of the electrified sealing wax placed near it, and the two electricities are sometimes called influence-electricities. Electrical influence is often called *electrostatic induction*.

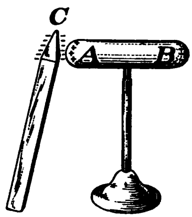

FIG. 229. — Diagram illustrating electrical influence.

410. The electroscope is an instrument for detecting electrical charges and for determining the kind of charge upon a body. The most common form, the *gold-leaf* electroscope, consists of a metal rod (Fig. 230) which has a ball or a disk at the top and which extends through an insulating cap or stopper into a flask. The rod has suspended from its lower end two leaves of gold


FIG. 230. — Electroscope.

or aluminum foil which ordinarily hang down side by side by their own weight. If a charge, either positive or negative, is given to the electroscope, the leaves stand apart as shown in the figure. This divergence of the leaves is caused by repulsion between them because both are charged with the same kind of electricity.

A proof plane (Fig. 231) is often used to convey small amounts of electricity from a charged body to an electroscope. It consists of a small metal disk with a handle of sealing wax or other insulating material.

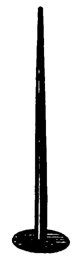

FIG. 231.—Proof plane.

411. To test the kind of a charge upon a body. —First, charge the electroscope with some of the unknown electricity. This may be done by touching the electrified body with the disk of the proof plane and then placing the disk upon the knob of the electroscope. Then transfer to the electroscope by the proof plane a charge of a known kind, either from a glass rod or a rod of sealing wax. If the unknown charge is like the known charge used, the leaves will diverge farther than at first, but if they are of opposite kinds the leaves will collapse. Or, first give the electroscope a charge of a known kind and then bring the body to be tested *near* the electroscope. As it approaches it will act by electrical influence, repelling the charge in the electroscope, if it is like itself, into the leaves and causing greater divergence; but if the charges are opposite in kind, the charge in the electroscope will be attracted from the leaves to the knob and the leaves will collapse. To illustrate this, transfer a small charge from an electrified glass rod to the electroscope and then bring near it in turn the electrified glass rod and a rod of sealing wax rubbed with flannel.

II. INDUCED CHARGES

412. Charging a body by influence.—Experiment.—Place the insulated conductors *A* and *B* (Fig. 232) in contact or connect them by laying a wire across them. (Two eggshells covered with aluminum paint and supported on blocks of paraffin may be used for this and similar experiments.) Bring near to one of them an electrified glass rod which will act upon them by influence or induction as described in § 409. While the rod is still present, separate the two conductors or remove the wire by means of an insulating handle (the wire and conductors must not be touched by the fingers) and then remove the rod.

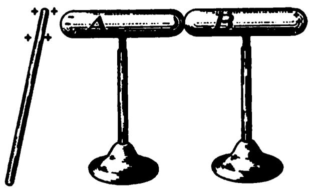

FIG. 232.—Charging by induction or influence.

Since *A* and *B* are disconnected while under the influence of the rod, there is no opportunity for the two electricities to unite after the rod is removed, and hence both *A* and *B* remain charged as represented in Figure 233. If the rod is of sealing wax and negative, *A* will be positive and *B* negative.

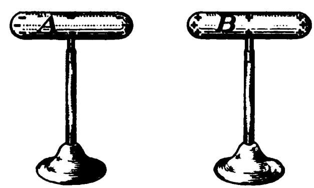

FIG. 233.—Conductors with opposite charges.

The charges on *A* and *B* may be tested by the electroscope. If *A* and *B* are brought together very carefully, a spark which may be heard by the whole class will pass between them just before they touch. The charges on *A* and *B* are evidently equal because they exactly neutralize each other. When two opposite charges are neutralized in this way, the bodies holding the charges are said to be *discharged.*

413. To charge the electroscope by influence, place an electrified glass rod near the knob and while it is there touch the knob with the finger. Remove first the finger and then the glass rod. The electroscope will then be negatively charged. In this case the electroscope answers to *A* in the preceding experiment, the arm and body play

the part of the wire, the positive electricity being repelled through the body to the floor or the earth, which takes the part of *B*.

414. The dielectric. — The charged body which exerts the electrical influence must be separated from the conductor upon which it acts by an insulating substance, — the air in the above experiments. The name *dielectric* is given to an insulator which, while preventing the passage of electricity to the conductor, permits electrical influence to act through itself. Substances differ in their dielectric properties, paraffin, for example, being 2.3 times as good a dielectric as air.

415. Opposite charges always produced. — If the silk used to electrify a glass rod is tested, it will be found to be negatively charged; and the flannel used upon sealing wax, positively charged. In fact experiments have proved that *one kind of charge is never produced without the production of the opposite kind at the same time*, and further that *the two charges are always equal.*

The following experiment will illustrate the above law, the charges being produced by friction:

Experiment. — Cement a stick of sealing wax for an insulating handle to the center of a piece of plate glass about 10 cm. across (Fig. 234), and attach a similar handle to a plate of hard rubber of the same size.

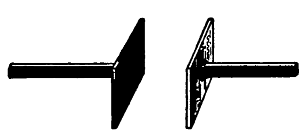

FIG. 234. — Apparatus for showing that charges are opposite and equal.

First make sure that neither plate is charged. If they are charged, the charge can be removed by passing them rapidly through a Bunsen flame. Place the two plates together, face to face, and rub one with the other, and then test each separately by bringing it near the electroscope which has a small charge of positive electricity. One will be found charged positively, the other negatively. Place the plates together again and bring them near the electroscope; no effect will be produced. Thus the two charges are shown to be equal. They may

also be shown to be equal by placing the plates one on each side of the knob of the electroscope, opposite to each other, and equally distant from it, when they will not change the divergence of the leaves.

416. Faraday's ice-pail experiment. — Place a tall, narrow metallic cup upon an insulating support and connect it by a wire (Fig. 235) to an electroscope. Charge a metal ball suspended by a silk thread positively and lower it into the cup, not allowing it to touch the cup. It will act by electrical influence on the cup as shown in the figure.

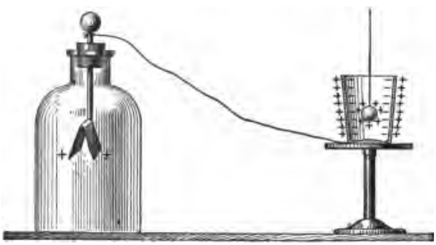

FIG. 235. — Faraday's ice-pail experiment.

Remove the ball, and the leaves of the electroscope collapse because the two influence-electricities are equal. Place the ball again in the cup, note the divergence of the leaves, and then allow the ball to touch the cup. No change in the divergence occurs. This shows that the negative influence-electricity on the inside of the cup is just equal to the positive on the ball, and consequently that on the ball was equal to the positive charge on the outside of the cup. The ball when taken from the cup is now found to be without a charge.

Again, discharge the electroscope and cup, and lower the ball, charged positively, into the cup; touch the cup with the finger, and the leaves collapse. Remove first the finger and then the ball, and the leaves diverge again just as much as before, the cup and electroscope now being negative. If the ball now touches the cup, the change upon it will exactly neutralize this charge, again showing the induced negative charge to be equal to the inducing positive charge on the ball.

417. The electrophorus is an instrument for obtaining electrical charges by electrical influence. It consists of a smooth disk of resin or vulcanite, — the *cake* A (Fig. 236) upon a metal base, or *sole*, C, and a metal disk, or *cover*, B, with an insulating handle H. To use the instrument, the cake is first electrified negatively by rubbing it with cat's fur; the cover is then placed upon it and touched for an instant with the finger, when on being lifted by the handle it will be found positively charged. If the knuckle is presented to it, a spark may be drawn from it possibly a centimeter long, and the cover is discharged. The cover may in this way be charged and discharged an indefinite number of times without any renewal of the charge on the cake.

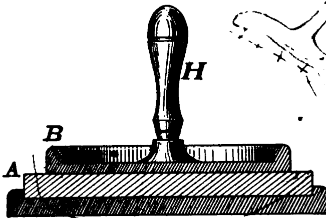

FIG. 236. — Electrophorus.

Its action may be explained thus: since the cover touches the cake, which is non-conducting, in a few points only, the charge of the cake is not removed by it. The two are separated by a very thin layer of air, a dielectric, and the cover is electrified by influence, + on the lower face and – on the upper. When it is touched by the finger, the negative influence-electricity passes to the earth, while the positive is held or *bound* by the attraction of the negative charge on the cake. When the cover is lifted, the charge becomes *free*. This charge on the cover represents a definite quantity of potential energy, which at the discharge appears as heat. This energy is derived from the work done in lifting the cover from the cake in opposition to the electric attraction between them. Thus mechanical energy is transformed into electrical potential energy.

418. Electrical influence machines. — The methods so far described for obtaining electrical charges produce

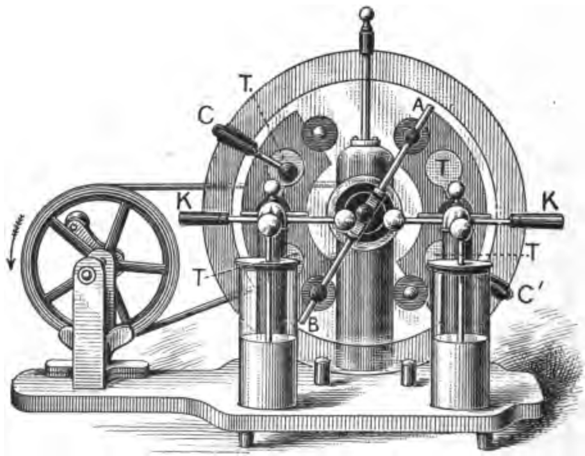

Fig. 237. — Toepler-Holtz machine.

only very small charges and do it very slowly. At first friction machines were used to produce larger charges. These consisted of a glass cylinder or a glass disk mounted on an axle and turned by a crank. Pads or rubbers pressed upon the disk so that when the latter revolved it became electrified positively by friction. This $+e$ passed to a large insulated conductor, which formed a part of the machine, by sharp pointed combs placed very close to the revolving disk. Such machines were not at all satisfactory, and have given

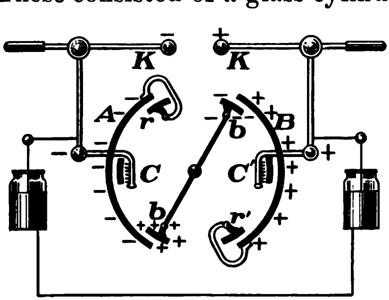

Fig. 238. — Diagram of the Toepler-Holtz machine.

place to those which depend on electrical influence for their action, doing very rapidly what is done by an electrophorus slowly and awkwardly. The best-known influence machines are the Holtz (1865) and the Wimshurst (1883). Figures 237 and 238 illustrate a modified form of the Holtz machine.[1]

III. ELECTRICAL DISTRIBUTION

419. Electrical surface density. — An electric charge is not usually distributed evenly over the surface of a conductor. The distribution is affected by the influence of neighboring bodies and by the shape of the conductor itself. *Electrical density means the quantity of electricity per unit of area of the conductor.* On an egg-shaped conductor more of the charge is found at the small end than elsewhere, and on a conductor with sharp edges or angles the density is very much greater at those edges and angles than on other parts of the conductor. This is easily shown by touching a charged conductor at different places with the proof plane and testing the charge taken away by it with the electroscope. It will always carry away a larger charge from a sharp edge or point than from a surface of less curvature. In making such tests place the proof plane disk flat against the surface to be tested.

420. Action of points. — At sharp points of a charged conductor the density becomes so great and the forces so strong that the electrical particles escape in a stream from the point with such velocity that they impart motion to the air about them. This gives rise to currents of air directed from the points, which are known as the "electrical wind."

Experiment. — Cement to the middle of a needle a stick of sealing wax for a handle. Place a positively charged body near the electro-

[1] See Appendix, page 477.

scope, and hold the needle against the knob of the latter with its point toward the charged conductor. The negative influence-electricity will pass off at the point of the needle to the conductor. Remove the needle and conductor; the electroscope will be found positively charged. Repeat the experiment, holding the needle so that it points away from the conductor; the positive influence-electricity will now pass off, leaving the electroscope negative.

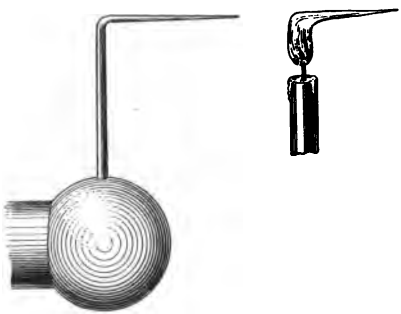

Fig. 239. — Flame blown by electrical wind.

Experiment. — Attach a sharp-pointed wire to the conductor of an electric machine, and while the machine is working, hold a lighted candle in front of the point (Fig. 239). The electrical wind will be shown by its action on the candle flame.

Experiment.—Support the electrical whirl (Fig. 240) on an insulated stand and connect it with the electrical machine. It rests on a pivot at its center and will revolve rapidly when the machine is in action. This is due to the reaction (Newton's third law of motion) of the electrical wind passing off from the points.

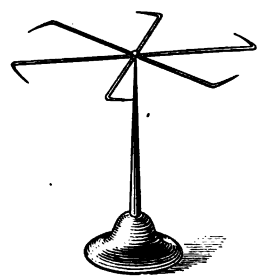

Fig. 240. — Electrical whirl illustrating the discharge of electricity from sharp points.

421. The charge resides on the outside of conductor. — Faraday constructed a cage large enough for him to enter and take his instruments inside with him. Although the cage was strongly electrified, there was no effect on the electroscope within, even when it was connected to the cage by a wire. In the ice-pail experiment, after the ball touches the cup there is no charge within the cup, and the ball, if tested, will be found to be without any charge.

Experiment.—Place a long brass chain in the cup, arranged as in the ice-pail experiment (§ 416), and electrify the cup. By means of a bent pin, having a stick of sealing wax for a handle, lift some of the chain out of the cup. As the chain is raised, the leaves of the electroscope gradually fall. This happens because the chain while inside the cup has no charge upon it; but when it is lifted out, it becomes a part of the exterior of the conductor and shares in its charge. Lower the chain into the cup again and the leaves diverge again as before.

422. Quantity of electricity.—In order to measure electricity it is necessary to have a unit. The unit of quantity that has been adopted is the charge which at the distance of one centimeter in air repels an equal charge of the same kind with the force of one dyne.

Coulomb proved (1777) that the force exerted between two charges varies directly as their product and inversely as the square of the distance between them, that is,

$$f = \frac{qq'}{d^2},$$

f being the force in dynes, d the distance between them in centimeters, and q and q' the two charges.

Problems

1. Two equal charges of 6 units each are 3 cm. apart. What is the force acting between them? *Ans.* 4 dynes.

2. Two equal charges repel each other at a distance of 10 cm. with a force of 64 dynes. How great are the charges? *Ans.* 80 units each.

3. A charge of 20 units repels another at a distance of 8 cm. with a force of 60 dynes. How large is the other charge? *Ans.* 192 units.

IV. POTENTIAL

423. Electrical potential.—A very full and exact discussion of the term *potential* would be out of place here, but as we shall need to use the term, let us attempt to get some notion of its meaning.

If a charged conductor touches another not charged, some electricity will pass from the former to the latter; or if two charged conductors are brought together or are connected by a wire, electricity may pass from one to the other. It may pass from the one having the greater quantity of electricity upon it to the one having the less, but often it passes from the one having the less to the one having the greater quantity. Evidently the quantity of electricity alone does not determine or regulate the flow of electricity from one body to another. What does determine it? Some comparisons will help us in answering this question.

Suppose two tanks, *A* and *B* (Fig. 241), to contain water and to be connected by a pipe *C*, the level being higher in *A* than in *B*. When the stopcock is opened, water will flow from *A* to *B*, although *A* contains the greater quantity of water. It is the difference of level or, what amounts to the same thing, the difference of pressure at the ends of the pipe *C* that determines the flow.

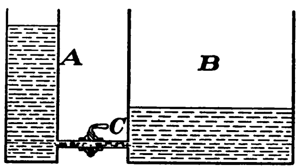

FIG. 241. — Diagram illustrating difference of potential by difference of level or pressure in water.

Again, if two gas bags are filled with compressed air and are connected by a tube, air will flow from the bag of higher pressure to the one of lower pressure, but if the pressure is the same in each there will be no flow. It is difference of pressure that determines the flow.

Or, again, when two hot bodies are in thermal connection, heat will pass from the body of higher temperature to the one of lower temperature, the temperature, not directly the quantity of heat in the bodies, determining the flow of the heat.

Corresponding to *pressure* in the case of the water and

also in the case of the compressed air, and to *temperature* in the case of heat, we have in electricity *electric pressure*, or *electric potential*, to which the flow of electricity along a conductor is ascribed.

Just as water or compressed air flows from points of higher pressure to points of lower pressure, so we say electricity flows from points of higher to points of lower potential; or, just as heat flows from a body of higher temperature to one of lower temperature, so electricity flows from a body of higher to one of lower potential.

Electric potential measures the condition of a body which determines its power of communicating electricity to or receiving it from other bodies.

424. Difference of potential. — As in the case of water, it is not simply pressure, but difference of pressure, that causes it to flow from one tank to the other, so with electricity, it is not potential but difference of potential (usually denoted by the letters P.D.), that causes the flow. If we wish to maintain a continuous flow of water through C (Fig. 241), we must maintain the difference of pressure in A and B by adding water to A and taking it out of B. Likewise, if we wish to have a continuous flow of electricity through a conductor, the P.D. between its ends must be maintained; if there is no P.D., there is no flow. It follows from this that all points of a conductor containing a *static* charge are at the same potential.

425. Zero potential. — Just as the level of the sea is taken as the zero from which the height of mountains is measured, so the potential of the earth is taken as the zero from which the potential of other bodies is measured. This can be done because the earth is so large that any charge we can give it does not perceptibly affect its potential, just as the level of the ocean is not sensibly changed by the rain or the water from the rivers. If

positive electricity flows from a body to the earth, the body is said to have a positive potential; but if the flow is from the earth to the body, its potential is negative.

426. Unit of potential difference. — In general we say that electricity flows from a higher to a lower potential; but just as water or air can be made to flow from a place of lower pressure to one of higher pressure when work is done upon it, so electricity is caused to flow from lower to higher potential, *if energy is expended and work done upon it.* We have an illustration of this in an electric railway system. At the power house large steam engines are constantly at work causing the electricity to flow from the earth to the trolley wires, maintaining them at a potential about 600 units higher than that of the earth; when the electricity flows back again to the earth through the car motor, it expends the energy gained at the power house and does work in moving the car.

Difference of potential between two bodies is measured by the work done in transferring a unit quantity of electricity from one to the other. If the work done is one erg, then the P.D. is one electrostatic unit of P.D. This unit is inconveniently large, and for practical purposes a unit $\frac{1}{300}$ as large has been taken. This practical unit is called a *volt* from Volta, an Italian physicist (1745–1827). A volt is the P.D. existing between two bodies when $\frac{1}{300}$ of an erg of work must be done to transfer a unit quantity of electricity from one to the other.

427. Electrical capacity. — When heat is added to a body, its temperature rises; but, because bodies differ in their thermal capacity, it takes different quantities of heat to raise their temperature 1°. So also when $+e$ is added to a body, its potential is raised, the potential rising as long as the charge is increased. Different conductors, however, vary in the quantity of electricity required to

raise their potential one unit. *The quantity of electricity necessary to raise the potential of a body from zero to unit potential is termed its electrical capacity.* The electrical capacity of a body depends in part on its size; the larger the surface of a body, the greater its capacity.

If q is the charge on a body, v its potential, and c its capacity, then $c = \frac{q}{v}$, or $q = cv$, or $v = \frac{q}{c}$.

Problems

1. If 200 units of $+e$ are added to a conductor whose capacity is 25 units, how is its potential affected?

2. How much electricity will be required to raise the potential of a body 75 units, if the capacity of the body is 30 units?

3. What is the electrical capacity of a conductor which requires 2 units of $+e$ to raise its potential 100 units?

V. ELECTRICAL CONDENSERS

428. Condensers.—Experiment.—Let a metal plate, such as a sheet of tin, A (Fig. 242), be suspended in a vertical position by a silk cord and joined to an electroscope. Charge it strongly with $+e$ so that the leaves of the electroscope diverge widely. Then let a second metal plate B of the same size and having a metal handle be brought near A. The plate B being held by the hand is connected with the earth; it should not be allowed to touch A. As B approaches A, the divergence of the electroscope leaves decreases; but when B is removed, the leaves separate again as at first. This shows that the potential of A is lowered by the nearness of B. However, no electricity is removed from A by the operation. Plates of glass with sheets of tin foil pasted on them serve as well for this experiment as metal plates. The tin foil should be a little smaller than the glass plate.

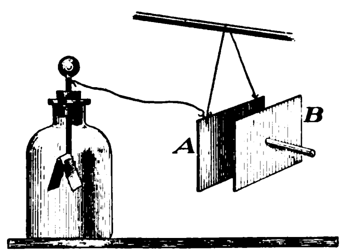

FIG. 242.—A simple condenser.

Evidently the capacity of A is greatly increased by the presence of

B; because when B is near A, a much larger charge must be given to the latter to cause the leaves to diverge as much as at first and to give it the same potential as before. The two metal plates and the air between them form a *condenser* which is defined as *two conductors separated by a dielectric.*

We have shown that the capacity of a body depends on its size, but this experiment shows that it also depends upon the presence of other bodies near it.

429. Action of the condenser explained. — Suppose A to have a positive charge. When B is brought near, this $+e$ acts by influence through the dielectric, attracting $-e$ to the nearer surface of B and repelling $+e$ to the earth. The $-e$ on B in turn attracts the $+e$ from the leaves of the electroscope to that surface of A nearest to B, as it were, making more room for additional charges. The two opposite charges attract each other through the air or glass and are called *bound* charges.

430. The Leyden jar is perhaps the oldest form of condenser. It is a glass jar (Fig. 243), coated inside and out about two thirds of the way up with tin foil. A brass rod with a ball at its top extends through the cover of the jar and has a brass chain hanging from its lower end to connect it with the inside coating.

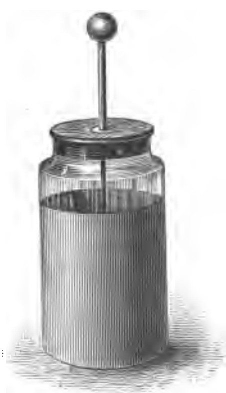

Fig. 243. — Leyden jar.

Condensers used for electrical measurements are made of many alternate layers of tin foil and mica or paraffined paper. Half of the tin-foil sheets, or every other one of them, are connected together on one side to form one conductor, and half on the other side for the other conductor. Thus two conductors of large area are formed which are separated by a very thin dielectric.

Two thunder clouds, or a cloud and the earth beneath, with the air between them as the dielectric, often form a condenser of huge dimensions.

431. Charging a condenser. — A condenser is charged by connecting one of its conductors with the earth and the other with some source of electricity, as the conductor of an electric machine; or by connecting its two conductors to the opposite terminals of an influence machine. In this way large and even dangerous charges may be obtained.

432. The discharge of a condenser is brought about by connecting its two metal surfaces by a conductor. To discharge a Leyden jar a *discharger* is used which consists of a V-shaped wire terminating in metal balls k_1 and k_2 (Fig. 244), and having an insulated handle H. The ball k_2 is first placed against the outside coating, and then the other ball k_1 is brought near the ball of the jar K; as they approach each other, a spark will pass between them which almost entirely discharges the jar and which is loud and bright in proportion as the quantity of the electricity discharged is great.

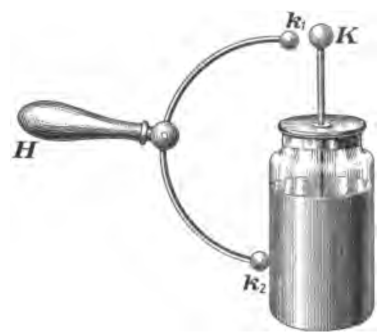

Fig. 244. — Discharger.

Hold a card between the two balls, and the spark will pierce it. The jar may be discharged through the human body or through a long row of persons, but great care must be observed in doing this, for the *shock* of too large a charge is very dangerous.

A Leyden jar may be charged so heavily that it will

discharge itself, either up over the edge of the glass, or through the glass itself, breaking the jar.

433. Lightning and lightning rods. — Franklin first proved the identity of lightning with the electric spark by his famous kite experiment in 1752. The kite, which had a pointed wire at its top, was raised during a thunderstorm. As soon as the hempen kite string became wet, electric sparks were drawn from a key attached to it, Leyden jars were charged, and other electric effects produced.

The lightning flash is due to the discharge of a cloud condenser; in this case the potential of the cloud becomes so enormous that the air can no longer withstand the strain, and a charge passes through it to the earth or to another cloud, a spark of great length and power being produced.

Franklin also invented the lightning rod, a metallic conductor which should terminate in sharp bright points above the building to which it is attached and extend down into the ground until earth always moist is reached. Heavily charged clouds acting by induction sometimes cause large charges to collect in the earth beneath them. Lightning rods afford protection because they permit a rapid and quiet discharge of electricity from their points, and thus prevent the accumulation of a large charge in the neighborhood of the building.

434. Heat of the discharge. — If the discharge of a Leyden jar takes place through a very fine platinum wire, it may heat the wire red-hot. The spark itself is due to the high temperature to which the small quantity of air along its path is raised. Spectrum analysis shows that even minute particles of the metal terminals between which the spark passes are vaporized.

The noise of the spark is caused by the sudden expansion produced by the heat of the spark, and thunder is only the report from a very large electrical discharge or spark.

Experiment. — Connect the outer coating of a charged Leyden jar with a Bunsen burner. Turn on the gas and then place one ball of a discharger, having an insulating handle, on the ball of the jar and bring the other down near the top of the burner so that the spark will pass through the escaping gas. The heat of the spark will ignite the gas.

Ether may also be ignited by passing the spark through some of it placed in a spoon, and powder may be exploded by the spark if placed in a hole in a block of paraffin.

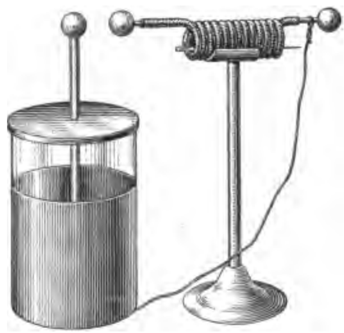

FIG. 245.—Apparatus for illustrating the magnetizing effect of the discharge of a Leyden jar.

435. Magnetic effect of the discharge. — Experiment. — Let a piece of flexible lamp cord be wound several times around a piece of small glass tubing (Fig. 245), and a large sewing needle be placed within the tube so that it shall be surrounded by wire. If the discharge of a Leyden jar is now passed through the coil, the needle will be magnetized. Thus the discharge is proved to have magnetic properties similar to those of current electricity, which we shall study later.

PART II. MAGNETISM

I. MAGNETS

436. Magnetism and electricity are so intimately related that it is probable when the true nature of one becomes known the nature of the other will also be understood. Magnets are especially characterized by their power of attracting iron, and the unknown cause of this power is called *magnetism.*

437. Natural magnets. — It was known to the ancients that certain black stones possessed the power of attracting

to themselves small pieces of iron. These stones — an iron ore called *magnetite* — are composed of an oxide of iron, having the chemical composition Fe_3O_4. They were commonly found in Magnesia in Asia Minor, from the name of which the terms magnet and magnetism are derived. They are now found in many parts of the world.

At a later period it was discovered that these stones when freely suspended by a thread always placed themselves lengthwise in a north and south line, and for this reason the name *loadstone*, or leading stone, was given to them.

438. Artificial magnets. — Permanent artificial magnets are made of hardened steel. If a piece of such steel is stroked from end to end, always in the same direction, by one end of a magnet, it becomes a permanent magnet. There are other methods of stroking a piece of steel with magnets to magnetize it, but it can be done better by the aid of an electric current in a way which we shall study later. Artificial magnets may be made in any desired shape, but the two most common forms are the *horseshoe* (Fig. 246) and the *bar* magnets (Fig. 247). A thin strip of steel, usually in the form of an elongated lozenge (Fig. 248), supported so as to turn freely about an axis, is called a *magnetic needle*, or compass needle.

FIG. 246. — Horse-shoe magnet.

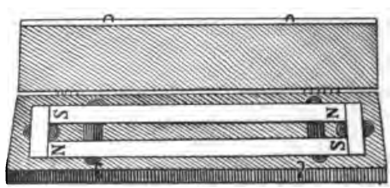

FIG. 247. — Bar magnets with armatures.

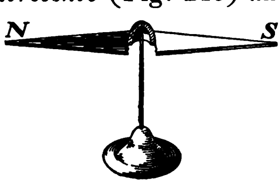

FIG. 248. — Magnet needle or compass.

439. Magnetic poles. — The power to attract is not distributed uniformly in a magnet,

but it is largely concentrated in certain parts of it, usually in two places at opposite ends of the magnet. This concentration of the magnetic attraction at particular places in the magnet is called *polarity*, and these places of greatest attraction are called *poles*. A straight line joining the two poles is the *magnetic axis* of the magnet. Between the two poles there is a place where the magnet has no power of attraction at all.

Experiment. — To illustrate polarity roll a piece of loadstone and also a magnetized darning needle or any bar magnet in iron filings. These (Fig. 249) will cling to the magnets in bunches largest at the poles, but diminishing in size to zero a short distance away.

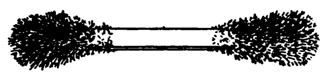

FIG. 249. — Bar magnet and iron filings illustrating polarity.

440. Poles named. — There are two kinds of poles in a magnet. The following experiment will enable one to show this and to designate the poles:

Experiment. — Magnetize a darning needle and float it upon a flat cork in a common saucer filled with water. It will come to rest pointing north and south, and if turned out of this line will return to it again, the same end always toward the north.

The pole of the magnet toward the north is named the *north-seeking* pole, and the one toward the south, the *south-seeking* pole, of the magnet, although it is customary to speak of them simply as north and south poles.

441. Law of magnets. — *Like magnetic poles repel and unlike attract.* The following experiment will enable one to prove this law:

Experiment. — Magnetize two darning needles by stroking each of them from eye to point with the same pole of a bar magnet, and then determine the poles of each of them as in the preceding experiment. Place one of them on the cork in the saucer, and taking the other in the hand, present its north pole first to the north and then to the

south pole of the floating magnet. Repeat the operation, presenting the south pole of the needle in the hand to the two poles of the floating needle. It will be found that two north or two south poles will repel each other, but that a north and a south pole will attract each other. By means of this law the poles of a magnet are always easily determined.

442. Substances classified with reference to magnetism. — A *magnetic* substance is one that is attracted by a magnet and can be magnetized. Iron with its compounds is preëminently the magnetic substance; the metals cobalt and nickel and a few other substances are somewhat magnetic. Substances that are repelled by a magnet are called *diamagnetic*. The two metals antimony and bismuth are the chief diamagnetic substances, but even with them the force of repulsion is small.

Strictly speaking, it is probable that all substances belong to one or other of these two classes, but for all practical purposes the vast majority of substances may be classed as *nonmagnetic*, that is, neither magnetic nor diamagnetic, because the influence of a magnet upon them, being extremely small, is not easily detected.

443. Magnetic transparency. — A magnet placed inside of a thick shell of soft iron exerts no force outside the shell, and is unaffected by external magnets; but if it is surrounded by a substance that is not magnetic, there is no interference with the magnetic forces, which act through such substances with perfect freedom.

Experiment. — Support a bar magnet with its south pole near the north pole of a large compass needle. Turn the needle aside for a moment; it will vibrate for some time, finally coming to rest. If the bar magnet is placed closer, the rate of vibration is faster because the attraction is greater. Make the distance about 3 cm. Set the needle vibrating and place a book or a board between the needle and the magnet. The rate of vibration of the needle will not be changed by the presence of the book. This shows that the magnetic force acts through the book without hindrance. Try also glass, copper, and

lastly a piece of sheet iron. Only the iron will affect the force, which will be shown by the slower vibration of the needle as soon as the iron is interposed.

II. TERRESTRIAL MAGNETISM

444. The earth's magnetism. — The earth itself is a great magnet and has magnetic poles as well as geographic poles. These poles, however, do not coincide, the north magnetic pole being more than a thousand miles south of the geographic pole, in latitude about 70° and longitude 97° W. It was discovered in 1831 by Sir James Ross, and again, 1904–05, by Dr. Amundsen in King William Land, not far from the earlier location.

Since unlike poles attract each other, the north magnetic pole of the earth must be unlike the north-seeking pole of a magnet; in reality the north magnetic pole of the earth is a south-seeking pole. This leads to confusion sometimes, and for this reason the word *seeking* is added to the names of the magnetic poles.

445. Declination. — We say that a compass needle always points north and south, but this is not true. Indeed, the variation is often considerable. This variation of a compass needle from a true north and south line is called *declination*, and the angle between the direction in which the needle points at any place and the true north and south line is the *angle of declination* for that place. The direction in which the needle points determines the *magnetic meridian* of the place.

446. Isogonic lines. — Lines upon the earth through places having the same angle of declination are called *isogonic lines*. The *agonic* line is that particular isogonic line which passes through places where the angle of declination is zero. Along this line the compass points exactly north and south.

In 1800 the agonic line passed near Cape Hatteras and near the cities of Washington and Buffalo; in 1900 it passed near Charleston, S.C., through eastern Kentucky, and a little west of Columbus, O., and Lansing, Mich. At places east of this line the declination is west, and at places west of it the declination is east. The declination increases with the distance from the agonic line.

The declination at any place varies slowly. At London it has varied from 11° 17′ east in 1580 to 24° 30′ west in 1816, and it is now about 16° west.

447. Inclination or dip. — The magnetic needle (Fig. 248) turns about a vertical axis in a horizontal plane, and is a *declination* needle; but one turning on a horizontal axis in a vertical plane (Fig. 250) is called a *dipping* needle. Such a needle, if perfectly balanced at its center of gravity, would rest in any position before it is magnetized, but after it is magnetized its north-seeking end dips downward in the northern hemisphere and its south-seeking end in the southern hemisphere. The axis of a dipping needle should be east and west.

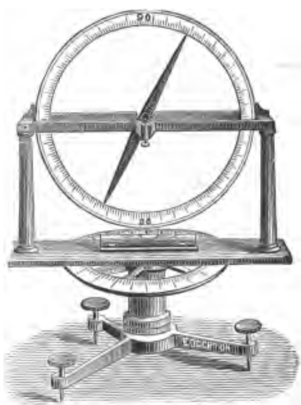

FIG. 250. — Dipping needle.

The angle a dipping needle makes with the horizontal is called the *angle of inclination* or *dip*. At the magnetic equator of the earth the needle is horizontal, but north of that line the angle of dip increases until at the magnetic pole it is 90°, that is, the needle is vertical.

Experiment. — Pass a knitting-needle through a cork, and at right angles to this place two stout sewing needles. Support the needles on the edges of two wine glasses, with the short needles east

and west. Adjust the long needle so that it will balance horizontally. After it is balanced, magnetize it, being careful not to change the position of the cork. It will now rest with its north pole inclined downward. If it is remagnetized so as to reverse the poles, the other end of the needle will point downward.

448. Isoclinic lines are lines on the earth extending through places of equal dip. They form magnetic lines of latitude, but are quite irregular. The isoclinic line of 72° runs north of the city of New York south of Cleveland, through Chicago, and near the cities of Sioux City, Butte, Spokane, and Vancouver.

Isometric charts of the United States, showing the isogonic, isoclinic, and isodynamic lines, may be obtained from the United States Coast and Geodetic Survey, Washington, D.C.

III. MAGNETIC INDUCTION

449. Magnetic induction. — When a piece of iron, such as a nail or a tack, is placed near or against a magnet, it becomes a magnet. This development of magnetism in a magnetic substance by the influence of a neighboring magnet is called *magnetic induction.*

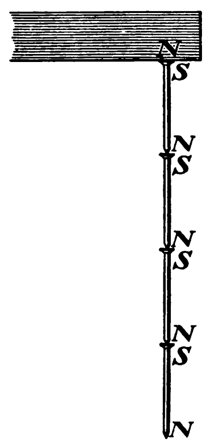

FIG. 251. — Nails made magnets by induction.

Experiment. — Dip one end of a short rod of soft iron into some iron filings. Very few, if any, filings will cling to it. Again, hold one end of the rod against a pole of a strong bar magnet and dip the other end into the filings. The filings will now cling to the rod, for it has become a magnet by induction. Remove the bar magnet and the rod loses most of its magnetism, and the filings drop off.

Experiment. — Suspend a small nail from one pole of a strong magnet (Fig. 251). This nail becomes a magnet by induction, and it will support a second nail, which in turn becomes a magnet and may support a third nail, and that possibly a fourth.

450. Temporary and permanent magnets.—If in the first experiment of the preceding article, a rod of hard steel, not previously magnetized, is used instead of soft iron, it will attract less filings than the soft iron; but after the bar magnet is removed, it will retain some magnetism and attract more filings than the iron. The iron is more susceptible to the inductive influence of the bar magnet than the hard steel, but the purer and softer the iron, the less magnetism will it retain. The soft iron forms a temporary magnet, the hard steel a permanent magnet.

The iron has greater *magnetic susceptibility* than the steel, but the latter has the greater *retentivity*, or power to retain magnetism.

451. Magnetic induction of the earth.—**Experiment.**—Procure a rod of very soft iron (Swedish iron) about 50 cm. long and 2 cm. in diameter. Hold this rod with one end pointing northward but dipping downward about 70°. While it is in this position, bring the lower end of it near a compass needle; it will repel the north end and attract the south. Test the upper end of the rod and it will be found to repel the south pole and attract the north. This will show that the rod is magnetized, its upper end being a south pole and its lower end a north pole. Reverse the rod and the poles in it will be reversed, the lower end again being north and the upper end south.

The rod becomes a magnet because it is under the inductive influence of the earth's magnetism.

It may be, if the iron is not very soft, or if it has been near a strong magnet, that its poles will not change when it is reversed. In that case a blow or two by a hammer will help to bring about the change.

Experiment.—Test by a compass the iron end of a school desk or seat. It will be found to be magnetized by induction by the earth, the top being a south pole and the bottom a north pole. A stove or an iron post will be found to be magnetized in the same way.

If we remember that the pole of the earth acting on the rod in the northern hemisphere is a south-seeking pole, we can see that these experiments illustrate the following principle:

The inducing pole develops a pole unlike itself in the part of the iron nearest it and a like pole in the part most remote.

IV. MAGNETIC FIELD

452. Magnetic field. — We have seen that a magnet exerts a magnetic force on other magnetic bodies in its

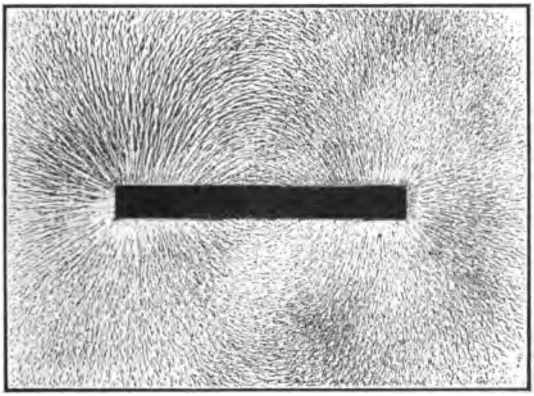

FIG. 252. — Field of a bar magnet.

neighborhood; the space around the magnet, throughout which this force is exerted, is called a *magnetic field*. At

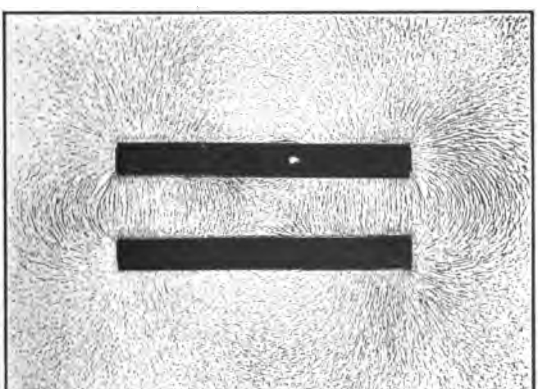

FIG. 253. — Field of two bar magnets, unlike poles adjacent.

every point in this field, the magnetic force has a definite magnitude and a definite direction. The direction of the forces at the various points in the field may be represented by lines, and hence it is thought of as pervaded by lines called *magnetic lines of force* or *lines of magnetic induction.* The magnitude of the force is not here represented by the length of the lines, but rather by the number of lines in a

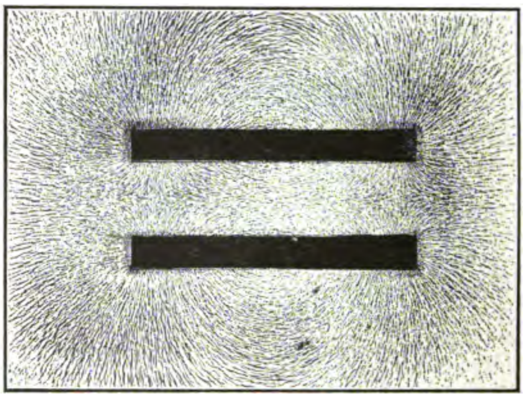

Fig. 254. — Field of two bar magnets, like poles adjacent.

given space. An accurate and beautiful map of a magnetic field may be made in the following manner:

Experiment. — Place a sheet of paper over a bar magnet and sift iron filings evenly and thinly upon the paper from a muslin bag. Then tap the paper gently. Each filing as it falls becomes a magnet by induction, and when the paper is tapped they arrange themselves along the magnetic lines of force or induction. By means of blue-print paper or other photographic paper, permanent maps of magnetic fields are easily made. Figures 252, 253, 254, and 256 are reproductions of maps made by the blue-print process.

453. Direction of lines of force. — Lines of force are to be regarded as extending in the direction in which a free

north-seeking pole is urged. For example, if the north pole of a small compass needle were at the point *a* (Fig. 255), it would be repelled by *N* with a force represented by the line *ai* and attracted by *S* by a force represented by *ae*. The resultant of these two forces, *ac*, would be the direction of the line of force in the field of the magnet at the point *a*. Lines of force leave a magnet at its north pole and, passing through the air, enter it at its south pole, going through the magnet from south to north.

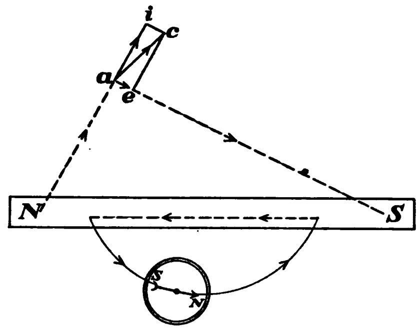

Fig. 255. — Diagram illustrating the direction of lines of force.

Every line of force is supposed to form a complete circuit without ends; it always returns to itself. In Figure 253 the lines of force extend from the north pole of one magnet to the south pole of the other. Wherever lines of force leave a piece of iron, north polarity manifests itself; and where they enter, south polarity is shown.

454. Permeability. — Lines of force seem to prefer an iron path to an air path, or it is easier for lines of force to exist in iron than in air or any other substance. The property of iron, especially of soft iron when placed in a magnetic field, of concentrating and increasing the lines of force, is called *permeability*. In Figure 256 observe how the lines are turned aside to enter and pass through the piece of soft iron placed between the poles of the magnets. A knowledge of the magnetic circuit and the permeability of iron is of very

great importance in the designing and building of electrical machinery.

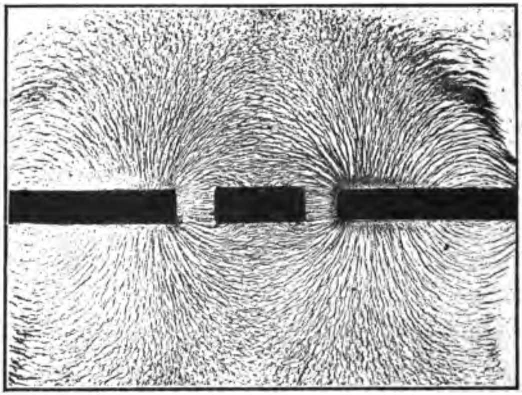

FIG. 256. — Field showing the permeability of soft iron, unlike poles adjacent with a piece of soft iron between.

455. Properties of lines of force. — Magnetic phenomena are explained by supposing a tension to exist along the lines of force combined with a force of mutual repulsion between them and at right angles to them. Whenever lines of force in a magnetic field are distorted, they tend to regain their original form; hence, they are said to be elastic. They do not cross each other nor pass each other in opposite directions, but they tend to become parallel and extend in the same direction.

PART III. CURRENT ELECTRICITY

I. THE VOLTAIC CELL

456. The electric current. — We speak of a *flow* of electricity along a conductor from one body to another. For example, when a Leyden jar is discharged, the two

coats are connected by a wire so that the electricities can pass or flow from one to the other. While this is occurring we say an *electric current* is flowing through the conductor.

In all cases so far considered this current has existed only for an instant, because the P.D., which causes the flow, is almost instantly destroyed by the discharge. To produce a continuous current, a P.D. must be constantly maintained.

The agencies used to produce continuous currents are the *voltaic cell*, often called a battery, and the *dynamo*.

457. Current strength.— This term has reference to the quantity of electricity flowing through a conductor per unit of time. The strength of the current, or simply the current, is measured by a unit called the *ampere*. A current has a strength of one ampere when one *coulomb* or unit quantity of electricity traverses the conductor per second. A ten-ampere current is one in which ten coulombs are transferred along the conductor every second. We express the size or strength of a stream of water in an analogous way by saying that it carries so many cubic feet of water per minute, but we have no special name for one cubic foot of water per minute as we have for one coulomb of electricity per second. Current strength will be represented in this book by the letter I.

458. The simple voltaic cell.—During the closing years of the eighteenth century, the work of Galvani and Volta, two Italian scientists, led to the discovery of the voltaic cell, first described by Volta in 1800. The action of the cell can best be studied by the student in the laboratory, but it will be described here.

If two plates, one of copper and another of zinc, are placed in dilute sulphuric acid (Fig. 257), a P.D. exists

between them; and if the two plates are connected by a wire, a continuous current of electricity flows through it from the copper to the zinc. The P.D. between the plates may be made evident in the following manner:

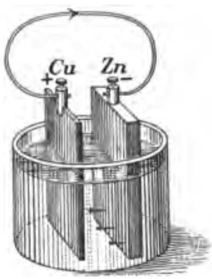

FIG. 257. — Simple voltaic cell.

The metal plates *B* and *C* (Fig. 258), about 10 cm. in diameter, being coated with shellac, form an electrophorus. Let the zinc plate be joined by a wire to *C* and the copper to *B*. After the connecting wires are removed and *B* lifted by its insulating handle, *C* will be shown to be charged, and the charge may be proved to be negative. The plates must be perfectly flat. If the divergence of the leaves is too small to be observed at a distance, it may be increased by the use of five or six dry cells in series.

When a strip of zinc is placed in the liquid, the acid acts upon it chemically, forming hydrogen gas, which collects in bubbles on the zinc and rises to the surface of the liquid. A strip of copper placed in the acid does not change the action at all until the two metals are united by a wire or are allowed to touch each other. As soon, however, as the metallic connection is made between them, the gas bubbles form on the copper as well as on the zinc. The current along the wire connecting the plates is made evident by its effect on a magnetic needle placed near it.

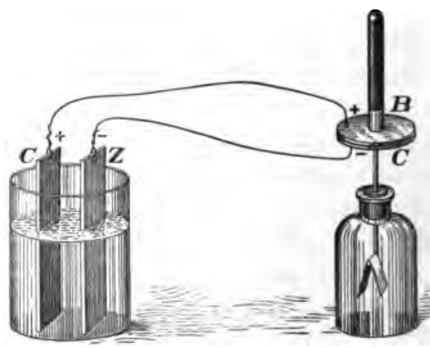

FIG. 258. — Experiment illustrating the difference of potential of two plates of a voltaic cell.

If the zinc is now amalgamated by rubbing a little mercury over its surface, the bubbles of gas cease to form on its surface; but they form on the copper, and the current flows in the wire as before. The chemical action is now apparently all on the copper; but in reality it is all on the zinc. This is evident, because by continued use the zinc strip is consumed by the acid, while the copper remains unchanged.

459. The essentials of any voltaic cell are illustrated by the one just described. Every such cell consists of two dissimilar conducting plates in a liquid which acts chemically on one of them, some element, not necessarily hydrogen, being deposited at the other. Two fillings, for example, in the teeth may with the saliva form a voltaic element and cause injurious galvanic action. Theoretically almost any two metals may be used for the plates of a cell, but practically zinc is invariably used for one of them and generally carbon or copper for the other. The liquid is a solution of some acid, salt, or base whose molecules are capable of dissociating or dividing into two parts called *ions*, which travel through the liquid, carrying electrical charges from one plate to the other.

460. Definitions. — The plates of a cell are called *poles* or *electrodes*. The one destroyed by the liquid (the zinc) is the *negative* electrode, receiving negative charges from the liquid; and the other is the *positive* electrode, receiving positive charges from the liquid.

The direction of the flow of the positive electricity is taken as the direction of the current, although the whole current is the sum of the two kinds of charges traversing the cell and the wire. The current flows from the positive pole or electrode along the wire to the negative pole and on through the liquid to the positive pole again. The entire path of the current is called the *circuit*. That part

of the path within the liquid is termed the *internal* and the remainder the *external* circuit. Interrupting the flow of the current in any way, as by lifting the plate out of the liquid or by cutting the wire, is called *breaking* or *opening* the circuit. Making the path complete by bringing the conductors together where the circuit is broken, is termed *making* or *closing* the circuit.

461. Theory of the voltaic cell. — According to the accepted theory, the liquid or electrolyte of the cell consists of a substance some of whose molecules on solution divide into two electrically charged parts called *ions*, one being positively, the other negatively charged. Let us suppose the electrolyte to be sulphuric acid. The chemist represents a molecule of it by the formula H_2SO_4, which means that each molecule of the acid is composed of two atoms of hydrogen, one of sulphur, and four of oxygen. When this acid is dissolved in water, some of its molecules separate or dissociate into ions, the two hydrogen atoms with their positive charges constituting positive ions, and the sulphur and oxygen atoms together (SO_4) with their negative charge a negative ion. The SO_4 ion is given the name *sulphion.*

When the cell is in operation, the zinc atoms from the zinc plate pass into the solution and form with the sulphions a new compound, zinc sulphate ($ZnSO_4$). In this action the zinc atoms or ions bring positive charges into the solution and the sulphions surrender their negative charges to the zinc plate. In this way the zinc plate becomes negatively charged and the solution around it becomes positive. The + hydrogen ions are driven by this positive liquid toward the copper plate, to which they surrender their charges, at the same time forming bubbles of gas. Thus the copper plate becomes positive.

The chemist expresses the chemical action of such a cell

by the expression $Zn + H_2SO_4 \longrightarrow ZnSO_4 + H_2$, reading it thus, "Zinc and sulphuric acid combine and produce zinc sulphate and hydrogen."

With other electrolytes the actions are similar. Metal and hydrogen atoms, for some unknown reason, always carry positive charges into solution and form positive ions; while such atoms as oxygen and chlorine carry negative charges and form negative ions.

462. Electro-motive force. — The P.D. between the plates of a cell when the circuit is broken is the *electro-motive force* of the cell. It is sometimes called electric pressure. Electro-motive force — usually denoted by the letters E.M.F. — may in general be defined as *the power of an agent to produce electric pressure or P.D.* Both P.D. and E.M.F. are measured by a unit called a *volt*, a name derived from Volta. The seat of the E.M.F. of a cell is at the contact surfaces between the metals and the acid where the chemical action occurs. The P.D. between zinc and sulphuric acid is about 1.8 volts and between copper and sulphuric acid 0.8 volt (Fig. 259). This latter P.D. is in opposition to that of the zinc and acid and tends to send the current in the opposite direction. The E.M.F. of such a cell is accordingly about one volt, or the difference between the E.M.F. of the zinc and acid and the *opposing E.M.F.* of the copper and acid.

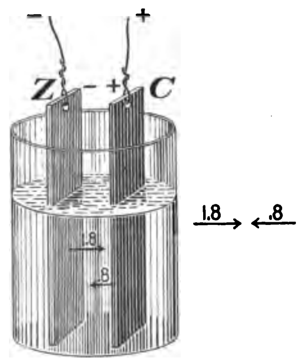

FIG. 259. — The arrows show the direct and the opposing E.M.F. of a voltaic cell.

The E.M.F. of a cell does not depend upon the size or shape of its plates nor upon their distance apart; but it does depend upon the kind of materials of which the

cell is composed. For example, the E.M.F. of the cell would be increased by substituting a carbon plate for the copper, but it would be diminished if a solution of common salt were used instead of sulphuric acid. Usually the E.M.F. of a cell decreases with a rise of temperature.

463. Local action. — We have seen that the chemical action on the zinc of a cell produces hydrogen, some of which appears at the copper or positive plate and some at the zinc plate. Only that part of the electro-chemical action which produces the hydrogen appearing at the positive plate is useful and contributes to the current flowing through the circuit; that part of the electro-chemical action producing the hydrogen which appears at the zinc plate, and which contributes no current to the main circuit of the cell, is called *local action.* Local action wastes the materials and energy of the cell and continues even when the circuit is broken. It is due to particles of impurities in the zinc, such as iron, carbon, and other elements. These impurities (Fig. 260), acting as positive poles, and the adjacent zinc particles, acting as negative poles, form, with the liquid, multitudes of little voltaic cells on the zinc. Since pure zinc is too expensive to be used in batteries, local action is prevented by amalgamating the zinc. The mercury dissolves some of the zinc but not the impurities. This forms an amalgam of zinc and mercury which covers up the impurities and presents a bright, clean surface to the liquid.

Fig. 260.—Diagram illustrating local action.

464. Polarization of cells. — The E.M.F. of the simple cell and of many other cells runs down when the current flows through the circuit, and the current becomes weaker. For instance if an electric door bell is rung continuously

for several minutes, the battery may become too weak to ring it longer. If the battery is allowed to rest, it may recover its original E.M.F. after a time. This weakening of the E.M.F. of a cell by the passage of a current through it is called *polarization*. It is caused chiefly by the coating of hydrogen on the positive plate. This hydrogen in effect converts the copper plate into a hydrogen plate and the *opposing* E.M.F. between the hydrogen and the acid is almost as great as that between the zinc and the acid, and consequently the E.M.F. of the cell is reduced.

Experiment. — Make a simple cell out of a piece of zinc, an arc light carbon and dilute sulphuric acid, placing the carbon rod in a small cup of porous earthenware which sits in the jar (Fig. 261). The porous cup should contain sulphuric acid as well as the tumbler. Join this cell to an electric bell of low resistance. It will ring the bell for a little while only, because it becomes polarized. Now add about 5 cc. of strong nitric acid to the acid in the porous cup. The cell will now be able to ring the bell for some time without polarizing.

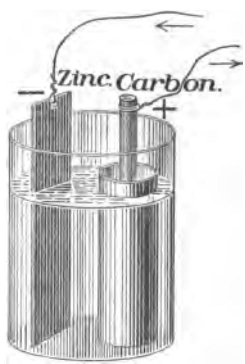

FIG. 261.—Cell with porous cup around the positive plate.

The nitric acid, being present around the carbon, oxidizes the hydrogen as fast as it appears there, the oxygen of the nitric acid and the hydrogen uniting to form water. Polarization is thus prevented. Many cells contain an extra chemical agent to act as the nitric acid does in this case, *i.e.* to prevent the accumulation of hydrogen at the positive plate. Such a chemical is called the *depolarizing* agent.

465. Cells classified.—Many modifications of the simple voltaic cell have been devised, and the great variety is largely due to the methods adopted to prevent polarization. Cells may be classified as *single-fluid* and *two-fluid* cells. In single fluid cells the depolarization agent is

mixed with the electrolyte of the cell, or, if it is a solid, it is placed in or against the positive plate. Two-fluid cells, as the name implies, have two different liquids, one the electrolyte which acts on the zinc, and the other the depolarizing agent, which is placed about the positive plate to dispose of the hydrogen. The two liquids are separated from each other by the walls of a porous earthenware cup, which allow the ions to pass through while preventing the mixing of the liquids.

466. The bichromate cell, a single-fluid cell, consists of two plates of carbon joined together to form the positive pole of the cell with a plate of zinc between them. The fluid used is dilute sulphuric acid in which the depolarizing agent, bichromate of sodium or potassium, is dissolved. The chromic acid, formed by the action of the sulphuric acid on the bichromate, furnishes oxygen which disposes of the hydrogen by uniting with it to form water. This cell has an E.M.F. of about 2 volts and gives a strong current for a long time. Since the chromic acid attacks the zinc when the cell is not in use, the cell is so made that the zinc, or all of the plates, may be withdrawn from the liquid. In the Grenet form of this cell (Fig. 262) the zinc is attached to a sliding rod by which it can be lifted. Several bichromate cells are sometimes so arranged that all the plates can be raised and lowered by turning a crank; it is then called a *plunge battery*. The fluid contains by weight 180 parts of water, 25 of sulphuric acid, and 12 parts of sodium bichromate.

Fig. 262.—Grenet cell.

467. The Leclanché cell.—This single fluid cell (Fig. 263) consists of a zinc rod in a solution of the salt, ammonium chloride (sal ammoniac), and a carbon plate placed in a porous cup which is packed full of manganese dioxide and graphite. The manganese dioxide, a solid, is the depolarizer. Many modified forms

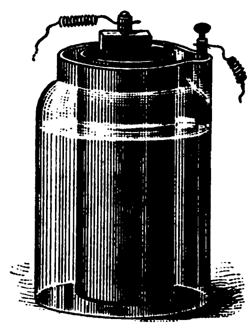

Fig. 263.—Leclanché cell.

of this cell have been in the market under various trade names. Usually the porous cup is dispensed with and the carbon is made in the form of a hollow cylinder, to give a large surface for the hydrogen to collect upon, and the manganese dioxide is embedded in the carbon or omitted altogether. The cell has an E.M.F. of about 1.3 volts. It is an excellent cell for intermittent work such as the ringing of a door bell, but it is not adapted to continuous work, as it quickly polarizes. The manganese dioxide does not furnish oxygen so rapidly as the hydrogen accumulates.

468. The dry cell is a modification of the Leclanché cell. The cup itself is made of zinc and forms the negative pole. It is not really dry, as its name implies, but the solution of ammonium chloride is held by porous material packed about the carbon plate. The cup is closed by a resinous cement. The moist paste of the cell usually consists of one part of ammonium chloride, one part of zinc oxide, one part of zinc chloride, three parts of plaster of Paris, and two parts of water. It is a cheap, portable, and very convenient cell.

469. The Daniell cell (Fig. 264) is a two-fluid cell having copper and zinc plates and either a solution of zinc sulphate or sulphuric acid for the active liquid, and a saturated solution of copper sulphate ($CuSO_4$) for the depolarizing agent. The zinc prism is placed with the zinc sulphate in a porous cup which stands in the center of a glass jar. A sheet of copper bent into the form of a cylinder nearly surrounds the porous cup and stands in the copper sulphate solution, which nearly fills the jar. To keep the solution of copper sulphate saturated, a pocket with perforated bottom is attached to the copper cylinder and is kept full of crystals of copper sulphate.

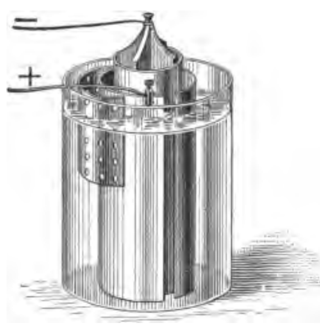

FIG. 264.—Daniell cell.

When sulphuric acid is used, the hydrogen ions travel

toward the copper as in the simple cell, but at the porous cup they meet the copper sulphate. From this point the copper ions travel on in their stead, and copper, not hydrogen, is deposited on the copper plate. Copper added to the copper plate of course does not polarize it.

When zinc sulphate is used instead of sulphuric acid, the zinc atoms are the positive ions, and they act in the same way as that just described for the hydrogen ions. In either case the SO_4 ions carry negative charges to the zinc and form zinc sulphate as in the simple cell. In this cell the copper plate becomes heavier and the zinc plate is consumed; the zinc sulphate increases in quantity and the copper sulphate is consumed.

This cell cannot furnish a large current because of its high internal resistance; but it is an excellent cell for closed circuit work, since its E.M.F. is very constant, being about 1.08 volts. It is often used as a standard of E.M.F. by which the E.M.F. of other cells is measured. The cell should not be left on open circuit when not in use, as the copper sulphate diffuses through the porous cup and makes a muddy deposit of copper oxide on the zinc, and copper is deposited in the pores of the cup. When the circuit is closed, the copper ions are kept moving toward the copper plate and this damage is prevented.

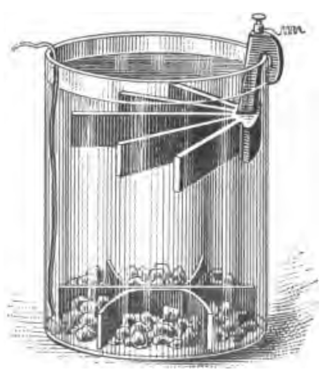
FIG. 265.—Gravity cell.

470. The gravity cell (Fig. 265) has the same materials and chemical action as the Daniell cell. The copper electrode is placed at the bottom of the jar and is surrounded by a saturated solution of copper sulphate and undissolved crystals

of copper sulphate. The zinc electrode is suspended near the top of the jar in a weak solution of zinc sulphate. There is no porous partition between the two liquids, but the greater density of the copper sulphate solution and the action of the current tend to keep them separate. The cell is not suitable for open circuit work since, when the circuit is broken, the copper sulphate diffuses to the upper part of the cell and objectionable chemical action occurs between it and the zinc plate. Many thousands of gravity cells are used in this country to operate the electric telegraph. Its E.M.F. is about 0.98 volts.

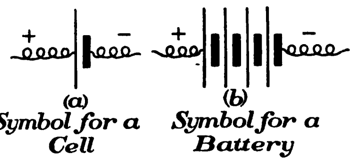

FIG. 266.

471. Symbol of a cell. — In diagrams representing electric circuits it is customary to represent a cell by the symbol shown in Figure 266 (*a*). The long thin line represents the positive pole and the short thick line the negative pole of a cell. Figure 266 (*b*) represents several cells joined to form a battery.

II. MAGNETIC EFFECTS OF THE CURRENT

472. Deflection. — **Experiment.** — Let one end of a wire be fastened to one of the poles of a voltaic cell. Stretch the middle of this wire above a compass needle (Fig. 267), holding it in a north and south line parallel to the needle. When the circuit is completed by joining the free end of the wire to the other pole of the cell, the needle will be turned aside or *deflected* by the current in the wire. The current strives to place the needle east and west, or at right angles with the wire, but the attraction of the earth opposes it, and it takes up an intermediate position between east and west and north and south.

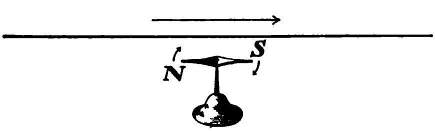

FIG. 267. — Diagram illustrating deflection produced by an electric current.

The turning of a magnetic needle by an electric current

is called *deflection.* The deflection is said to be *east* or *west* according as the north pole turns toward the east or west. This phenomenon, which was discovered by Oersted in 1819, shows that an electric current has magnetic properties and is surrounded by a magnetic field.

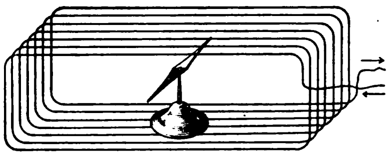

FIG. 268.—Diagram of a needle within a coil of wire.

When the current is below the needle instead of above, the deflection is reversed; it is also reversed when the current in the wire is reversed. The deflection is increased by bending the wire so that the current passes above the needle and back underneath it, and it is still more increased by passing the wire about the needle a number of times (Fig. 268).

Deflection affords a simple means for detecting and measuring electric currents, and also for determining the direction of a current. The following rule will assist the memory in determining either the deflection or direction of the current: *Grasp the wire with the right hand with the thumb outstretched; if the thumb points in the direction of the current, the fingers on the side of the wire nearest the needle indicate the direction of the deflection. Conversely, if the fingers correspond with the deflection, the thumb indicates the direction of the current.*

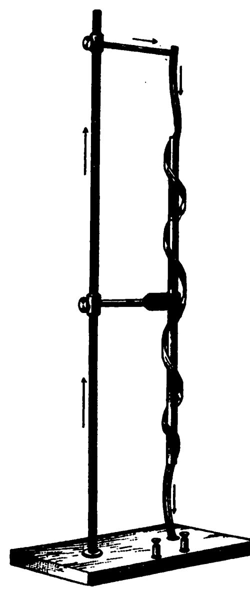

FIG. 269.—Flexible conductor coiling around a magnet under the influence of a current.

Experiment. — The tendency of the current and the magnet to assume positions at right angles to each other is beautifully illustrated as follows: Let a long slender bar magnet be supported in a vertical position (Fig. 269), and let a light flexible conductor, made of about three strands of copper tinsel, be suspended by the side of it. Let a battery of about three dry cells be connected to the tinsel. When the circuit is closed, the tinsel will coil itself around the magnet, as shown in the figure; if the current is reversed by a commutator, it will uncoil and rewind itself about the magnet in the opposite direction.

473. Magnetic field about a current. — **Experiment.** — Support a piece of cardboard in a horizontal position and pass a thick wire vertically up through its center (Fig. 270). Connect the wire to a battery of bichromate or dry cells so as to send a strong current through it and then sift iron filings on the cardboard about the wire. Tap the cardboard with a pencil, and the filings will arrange themselves about the wire in concentric circles, as shown in the figure. The direction of the lines of force about the wire may be determined by placing a small compass on the card near the wire. They will be found to agree with the following rule:

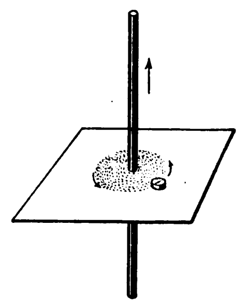

Fig. 270. — Diagram of a magnetic field about a conductor when a current is passing through it.

Grasp the wire with the right hand with the outstretched thumb in the direction of the current; the fingers will indicate the direction of the lines of force. This field exists all along the wire and often extends many feet in all directions from it.

474. The magnetic field about two parallel currents. — **Experiment.** — Let two wires be passed through a piece of cardboard, as in the last experiment, the wires being parallel and about 2 cm. apart. Connect these wires to a battery, so that the current shall flow up one and down the other. Then sift iron filings about them. Figure 271 represents a magnetic field mapped in this way.

Experiment. — Connect the two wires of the last experiment in such a way that the current shall flow in the same direction in both

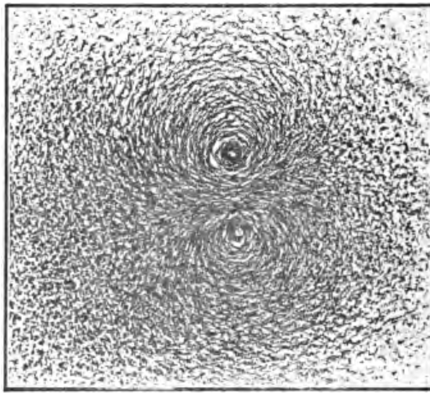

FIG. 271. — Magnetic field about two parallel currents flowing in opposite directions.

of them, either up or down both of them. Let the field be mapped as before. Figure 272 represents such a field.

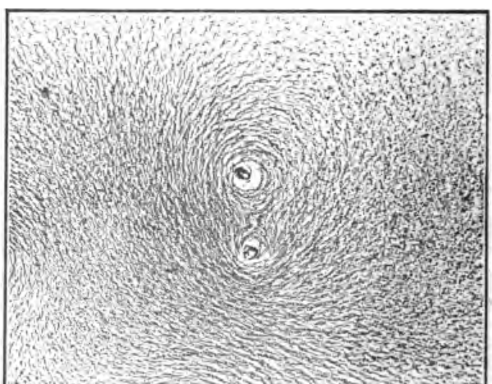

FIG. 272. — Magnetic field about two parallel currents flowing in the same direction.

This field may be explained by Figure 273, in which *a* and *b* are sections of the two wires. Suppose the current is away from the observer in both of them. Then if either of the wires were present alone, lines of force would extend around it, as shown by the dotted lines; but when both currents are present, the lines of force of wire *a* meet those of *b* between the two wires coming in the opposite direction. For this reason very few, if any, lines pass between the wires, but the two sets of lines unite and extend around both of them.

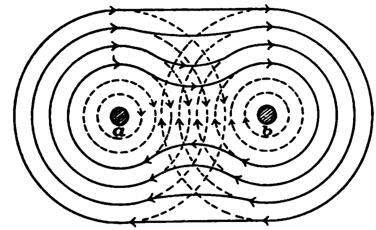

Fig. 273. — Diagram of the magnetic field about two parallel currents flowing away from the observer.

In fact, the field about a bundle of wires in all of which the current is in the same direction has the same general form as that about a single wire, for the lines of force encircle all of them. When, however, the currents are opposite in two wires, the lines of force pass between the wires and about each separately.

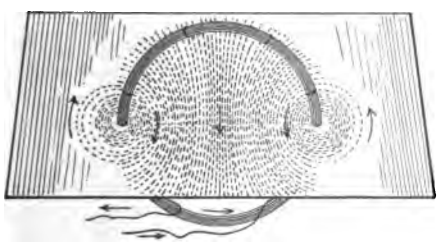
Fig. 274. — Diagram of the magnetic field of a current flowing in a circle, in a plane passing through the center of the circle, and at right angles to it.

475. The magnetic field of a coil of wire. — Let a coil of wire of about 30 turns be placed half above and half below a sheet of cardboard, as shown in Figure 274. While a strong current is passing through the coil let its field be mapped with

filings as before. The figure shows the field as mapped out in this way. This is the field in a horizontal plane; but the field would be the same in a vertical or any oblique plane perpendicular to the coil. This field indicates that the coil, when traversed by a current, is a magnet. The magnetic lines of force go through such a coil or ring and return on the outside of it, each line of force forming a complete circuit. The face of the coil from which the lines come is the north pole of a magnet and the face which they enter is the south pole.

A careful study of these magnetic fields about currents will greatly aid the student in his further study of electricity.

476. "Apparatus *M*."—*R* (Fig. 275) represents a light rectangular frame of wood about 12 × 24 cm., around which about 250 turns of No. 26 copper wire are wound. One end of the wire is soldered to an iron rod *d* and the other end to a rod *c*. These two rods act as axles upon which the rectangle revolves. The lower one is pointed and rests in an iron cup *o* in the square brass rod *B*, the upper one *d* passing through a hole in the brass rod *A*. The wires from a battery are joined to the rods *A* and *B* by the binding posts *P* and *P*, and the current can pass along the rod *A* to *d*, thence around the coil to *c*, and then to the rod *B* and back to the battery *y*. In order to make the electrical contact perfect between *A* and *d* the latter carries a little circular iron cup *i* filled with mercury and an iron latch from *A* dips into the mercury. The cavity *o* is also filled with mercury to connect *c* with *B*. The coil is thus connected to the battery while it is free to revolve. As we wish to use this apparatus frequently in the future, we will designate it as "Apparatus *M*."

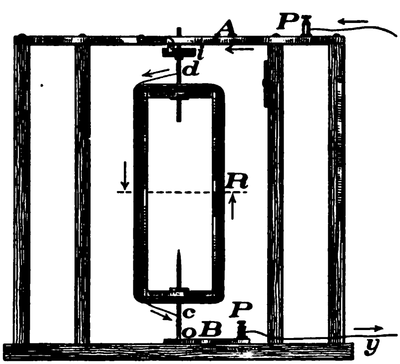

FIG. 275.—Diagram of "Apparatus *M*."

477. To prove that a coil is a magnet if a current is passing through it, pass a current through coil *R* of Apparatus *M* and present the north pole of a bar magnet to it. One face of it will be repelled by the magnet and thus be shown to be a north pole, and the other face will be attracted by the magnet, and thus be shown to be a south pole. The coil will behave toward the magnet just as a compass needle does, and in fact, if it turns easily, and the current is very strong, it will face north and south in obedience to the earth's attraction. If a map of the magnetic field in a horizontal plane at the dotted line (Fig. 275) were made, it would be exactly similar to that shown in Figure 274.

478. The magnetic field of a spiral. — Let a cylindrical spiral of wire be so placed that half of it is above and half

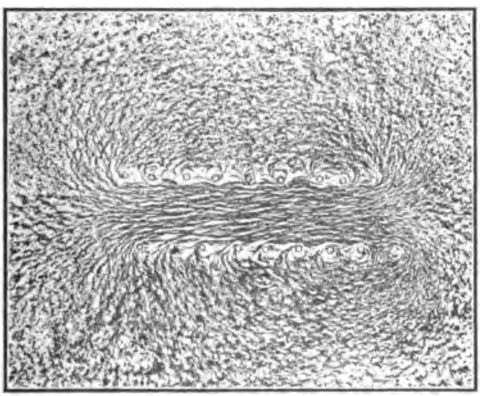

Fig. 276. — Magnetic field in a plane through the axis of a spiral current without an iron core.

below the surface of a horizontal cardboard, as shown in Figure 276. Let a current be passed through the spiral and the magnetic field be mapped with iron filings. If this experiment is repeated with a rod of soft iron in the

spiral under the cardboard, the increase in the strength of the magnetic field will be made evident by the greater

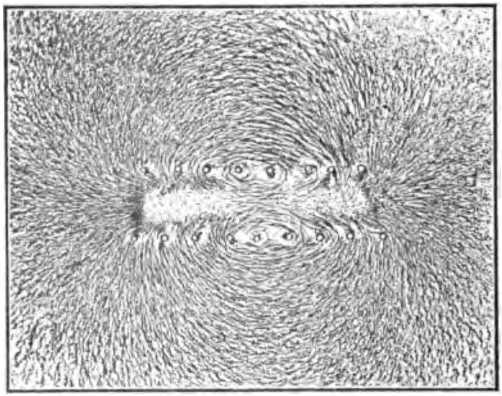

FIG. 277. — Magnetic field in a plane through the axis of a spiral current with an iron core.

definiteness of the lines formed by the filings on the outside of the spiral (Fig. 277).

In this experiment the lines of force are seen to extend through the interior of the spiral. The exterior field resembles that of a bar magnet.

479. The electro-magnet. — Let four or five layers of No. 16 insulated copper wire be wound closely upon a paper tube about 1 cm. in diameter. To fix the wire in position it may be covered with hot sealing wax. When a current is passed through this coil, its ends will attract and repel a compass needle exactly as a bar magnet does. If one end of this coil is placed in a

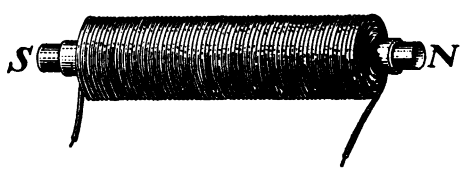

FIG. 278. — Electro-magnet formed of a soft iron rod in a paper tube around which are wound layers of insulated copper wire.

box of small nails, it will pick them up. A rod of soft iron if presented to the coil will be drawn into it. This rod, with the coil around it (Fig. 278), will now be found to be a strong magnet, and it will support quite heavy pieces of iron which will fall as soon as the current in the coil ceases. The iron rod increases the lines of force through the coil enormously.

Such a coil of wire is called a *helix* and the rod of iron within it is called the *core*. The two together constitute an *electro-magnet*.

480. Polarity of the helix. — If the direction of the current in the helix just described is traced, and its polarity determined, the relation between the two will be found to conform to the following rule: *Grasp the helix with the right hand so that the fingers point in the direction of the current around the core; then the extended thumb will point toward the north pole of the magnet.*

481. The magnetic circuit. — The student should observe that the lines of force are in general at right angles with the direction of the current, also that in the electro-magnet the current does not pass into the iron core, but goes around it, being carefully insulated from it. As the entire path of the current is called the electric circuit, so the path of the lines of force is called the *magnetic circuit*. Every magnetic line of force is a *closed circuit*, returning on itself, so that it is continuous. In the magnet described, the lines of force extend through the iron core, coming out at the north pole and returning through the air to the south pole. The magnetic path or circuit is partly through iron and partly through air. The core and helix of a magnet may be bent into the horseshoe form (Fig. 279), in

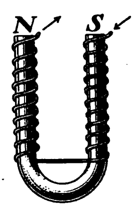

FIG. 279. — Diagram showing a bar electro-magnet bent into the horseshoe form.

which the air path is greatly shortened. Usually, however, horseshoe electro-magnets are made as shown in Figure 280, the iron being in three pieces, and the wire on two spools and joined together. The two cores are joined by the bar B, called a *yoke*.

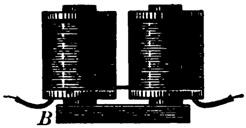

FIG. 280. — Common form of horseshoe electro-magnet.

A bar of iron on or near the poles is called an *armature*. Figure 281 shows the magnetic circuit of a horseshoe magnet.

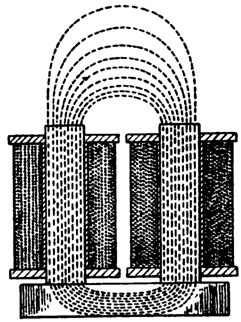

FIG. 281. — Diagram of the magnetic circuit of an electro-magnet.

482. The strength of an electro-magnet depends mainly upon two things, its magnetic circuit and the *ampere turns* encircling it.

For a strong magnet the air gap of the magnetic circuit should be as short as possible; even the joints in the iron lessen the lines of force and weaken the magnet; and the iron should be pure and soft, and short and thick, rather than long and thin.

With a good magnetic circuit, the strength of the magnet depends mainly upon the number of *ampere turns*, a term denoting the product of the current strength in amperes by the number of turns of wire around the core. Thus, a current of 1 ampere flowing 100 times around the core, or one of $\frac{1}{10}$ ampere flowing 1000 times around the core, constitutes 100 ampere turns, and each will produce a magnet of the same strength.

A magnet (Fig. 282) composed of two thick half rings of soft iron and a comparatively small helix may easily be made so strong that two boys cannot pull the ring apart. The magnet is strong because the magnetic circuit is

almost wholly iron. Even the thin layers of air between the halves of the ring weaken the magnetism somewhat; and if the ends of the half rings are rough, or do not fit closely together, the air gap will lessen the magnetism very much indeed. This apparatus illustrates especially the permeability of soft iron.

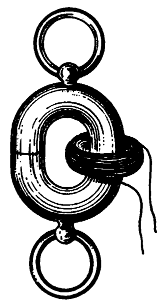

FIG. 282. — Ring electro-magnet with magnetic circuit in an iron ring.

The electro-magnet is put to many practical uses. It forms an essential part of the dynamo, the electric motor, telegraph instruments, the electric bell, and many other instruments. In some machine shops large masses of iron are picked up and moved about by means of powerful electro-magnets (Fig. 283).

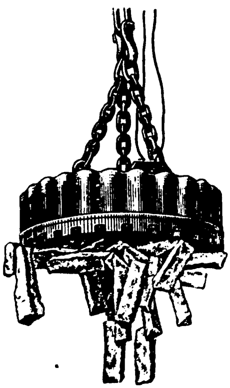

FIG. 283. — Electro-magnet lifting bars of iron.

483. The electric bell. — The arrangement of the parts of an electric bell is shown in Figure 284. It consists of a gong G, an electro-magnet E, with its armature ending in a hammer H, and swinging on a spring A, which presses against an adjusting screw C. When the push button P (shown in section at the right) is pressed, the current from the battery B flows through the binding post D, the electro-magnet E, the spring A, the screw C, and back to the battery. As the current passes, the electro-magnet attracts the armature, causing the hammer to strike the gong and also breaking the circuit between A and C. Then A springs back against C, completing the circuit again. This operation is rapidly repeated, causing a succession of blows on the gong.

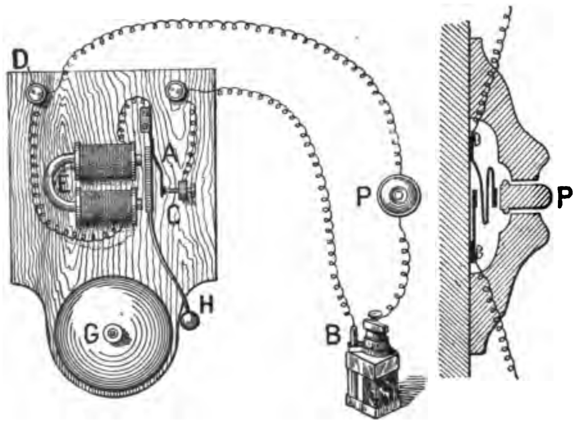

Fig. 284. — Electric bell.

484. The electric telegraph is essentially an electromagnet operated at a distance by means of a battery and a long wire. The wire extending from one station to another, joining the instruments, and carrying the current, is called the *line*. It is not necessary that the circuit be completed by a return wire, but each end of the line is connected to the earth by being attached to a water or gas pipe or a metallic plate buried in the ground. The earth then forms the return circuit.

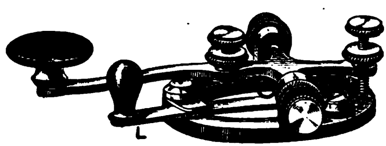

Fig. 285. — Telegraph key.

Messages are sent by a *key*, which is simply a lever (Fig. 285) used to make and break the circuit. When it is pressed down, the circuit is made by the contact of two platinum points. A spring raises the lever and breaks the circuit when the pressure is removed. When the key is not in use, the circuit is closed permanently by a switch *L*.

485. The sounder, or the instrument used for receiving messages, consists of an electro-magnet (Fig. 286) with a lever carrying an iron armature and held just above the poles of the magnet by a spring. The line is attached to this magnet. When the circuit is closed at the distant station by a key, the current energizes the magnet and the armature is attracted. This pulls the lever down until it strikes on an adjusting screw and makes a click or sound. When the circuit is broken at the key, the armature is released and the lever, actuated by the spring, strikes an adjusting screw above, making another sound. If the interval between the two sounds is very short, it is called a *dot;* if longer, it is called a *dash.* These terms, dot and dash, are used because at first a stylus on the end of the lever made dots and dashes on a ribbon of paper drawn along by clockwork. Nowadays operators read by sound, not by sight. A dot and a dash, for example, make the letter A, and a dash and three dots, the letter B. (See Appendix.)

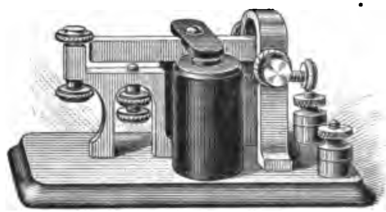

FIG. 286. — Telegraph sounder.

Several sounders and as many keys may all be on the same line, all of the sounders working at the same time. All the keys, however, but the one in use must be closed.

486. The relay. — On a long line between distant cities the current is not strong enough to operate the sounders at the way stations and at the terminal stations, and hence a *relay* is used at each station, which brings into action a local battery to operate the sounder at that station. The relay is in reality an electrically operated key. It

consists of an electro-magnet (Fig. 287) which is in the main line circuit. When the current flows through this magnet, it attracts the armature and thus brings in contact two platinum points. This contact makes the circuit of another battery and a sounder. The lever to which the armature is attached is so small and so nicely adjusted that it moves very easily. The relay is often used to repeat a message and to send it on to a more distant point. A diagram showing the general arrangement of the parts of the telegraph is shown in Figure 288.

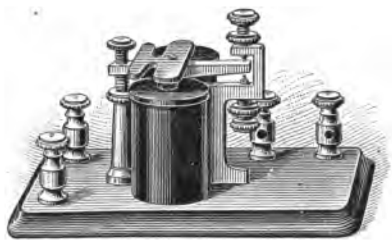

FIG. 287. — Telegraph relay.

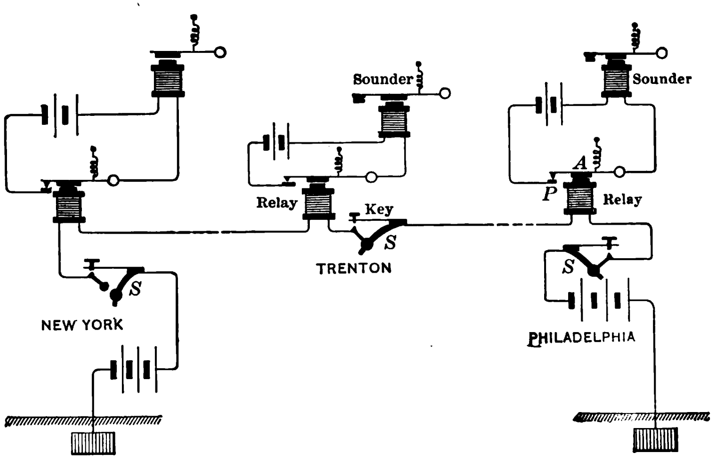

FIG. 288. — Diagram showing the arrangement of a telegraph system.

487. Influence of one electric current on another. — Since electric currents are surrounded by magnetic fields,

we should expect them to react upon each other, and they do.

(1) *Parallel currents flowing in the same direction attract each other.*

(2) *Parallel currents flowing in the opposite direction repel each other.*

(3) *Currents not parallel tend to become parallel and to flow in the same direction.*

The following experiments will illustrate these laws:

Experiment. — Having a wooden rectangular frame a little broader and longer than *R* (Fig. 275), with about 250 turns of wire about it, let it be connected to a battery. We will designate this frame as coil *A*. Let the coil also of Apparatus *M* be joined to a battery. (Two batteries are not necessary for this, as the same current may be passed through both coils.) Determine the direction of the current in each coil by means of a compass needle, and mark each coil in some way, as by tying a string to some part of it, so that the direction of the current can be remembered. Hold one side of coil *A* near and parallel to one side of coil *M*. If the currents are both up or both down in these adjacent sides, coil *M* will follow *A* as it is moved to and fro; but if the current is up one of them and down the other, they will repel each other.

Experiment. — Let coil *A* be placed around coil *M*, and let the latter be turned until the planes of the two coils are perpendicular to each other (Fig. 289). As soon as *M* is released, it will turn so as to be parallel to *A*, with the currents in the two coils in the same direction. Again,

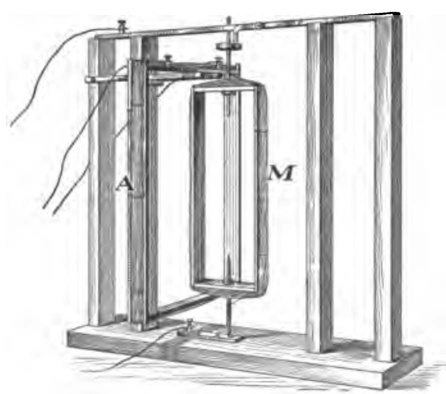

FIG. 289. — Apparatus for showing that currents not parallel tend to become parallel.

if one side of A is placed across the top of M and above it, the latter will turn so as to become parallel to A.

Maxwell summed up these laws in one law, viz.:

The two circuits tend to place themselves with reference to each other so that as many as possible of their lines of force shall be common to the two.

A careful study of the magnetic fields of the two coils in the above experiments will show that when they are in equilibrium the lines of force that thread one coil also pass through the other, the two sets of lines of force being in the same direction.

III. GALVANOMETERS

488. Galvanometers are instruments used to measure strength of currents by means of their magnetic effect; but often they are used to detect and not to measure currents, and in that case they are more properly called *galvanoscopes*. Many different forms of galvanometers are in use.

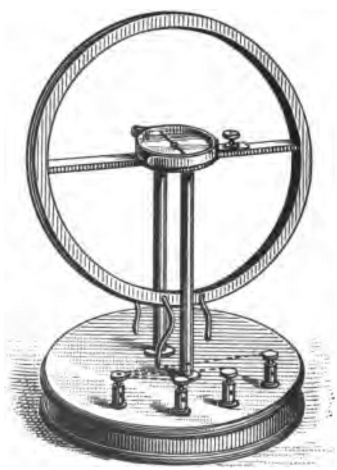

Fig. 290. — Tangent galvanometer.

489. The tangent galvanometer (Fig. 290) is very simple in construction. It consists of a ring of brass or wood, perhaps 30 cm. in diameter, around which a coil of wire is wound. This ring is attached to a base so as to stand in a vertical position, and at its center there is a compass box having a magnetic needle about 2 cm. long. Since this needle is so short, a long light pointer is

attached to it which passes over a circular scale graduated in degrees. The coil of the instrument stands north and south, and the current to be measured is sent through the coil. *The current*, however, *is not proportional to the angle of deflection*, as one might think, *but to the tangent of the angle of deflection*. For example, a current giving a deflection of 30° is not half as strong as one giving 60°. The tangent of 30° is .577 (see Appendix); and of 60°, 1.732. Hence a current giving a deflection of 60° is three times as strong as one giving a deflection of 30° (1.732 ÷ 3 = .577). In order to measure a current by this galvanometer, *the constant of the instrument*, *i.e.* the current that will cause a deflection of 45°, must be known.

490. The D'Arsonval galvanometer is the most useful and common type of galvanometer. In it the coil moves and the magnet is stationary. The principle of its construction is shown in Figure 291. It consists of a small coil of many turns of very fine wire placed between the poles of a strong permanent horseshoe magnet. This coil is suspended by a fine wire (*a*) which also serves to carry the current to the coil. The current leaves the coil by another fine wire (*b*), and the two wires, *a* and *b*, form the axis about which the coil turns. When a current flows through the coil, it becomes a magnet like coil *R* (§ 477) with lines of force threading it, and it tends to turn so as to face the poles of the large magnet. The torsion of the wires *a* and *b* opposes this turning, and when the current ceases, brings the coil back to its first or zero position. Often the magnet, or several magnets fastened together, are placed in a horizontal position.

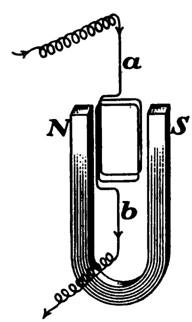

FIG. 291. — Diagram of a D'Arsonval galvanometer.

A pointer attached to the coil and extending over a circular scale may be used to indicate the amount of the deflection; but usually a tiny mirror is fastened to the coil, and the reflection of a straight centimeter scale is viewed in this mirror by a telescope. When the coil and mirror turn, the scale appears to move, and the distance it moves is read in centimeters by the telescope. With such an arrangement, the current is proportional to the deflection. Very minute fractions of an ampere are detected and measured by such instruments. The receiving instruments for ocean cables are of this type.

491. Ammeters and voltmeters. — The galvanometers so far described have scales graduated either in degrees or in

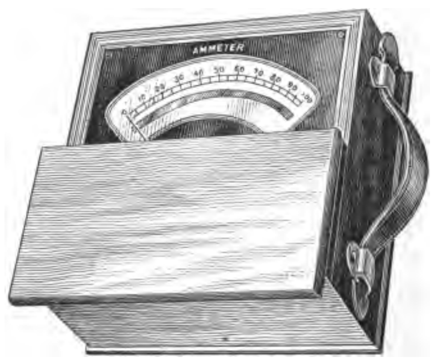

FIG. 292. — Portable ammeter.

centimeters; but a galvanometer, after being tested or calibrated, may have a scale graduated in amperes. It is then called an *ammeter*. If it is adapted to measuring very small currents and has a scale graduated for thousandths of an ampere, it is called a *mil-ammeter*. Ammeters are galvanometers of very small resistance. Galvanometers having a high resistance are often used

to measure E.M.F. or P.D., and when a scale is attached graduated in volts, they are called *voltmeters.*

The best portable voltmeters and ammeters are of the D'Arsonval type; but since a wire suspension is easily broken, the coil is supported on jeweled bearings like the balance wheel of a watch. Coiled springs, like the hairspring of a watch, react against the current and bring the coil back to its zero position; they also serve to carry the current to and from the coil. Such instruments have a pointer and not a mirror. Figure 292 represents the exterior of such an instrument. It has a horseshoe magnet (Fig. 293) within, with curved pole pieces. Within the circular space between these pole pieces the coil, shown in detail in Figure 249, is mounted.

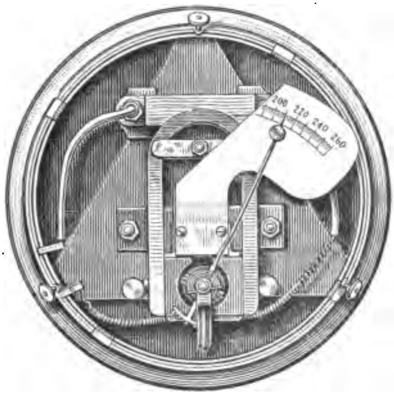

FIG. 293. — Interior of an ammeter.

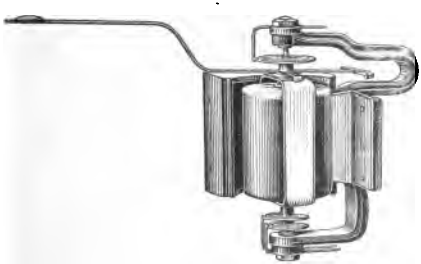

FIG. 294. — Coil of an ammeter; it is wound about a cylinder of soft iron.

Figure 295 represents the conventional symbol of a galvanometer used in diagrams of electrical apparatus.

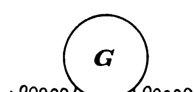

FIG. 295. — Symbol for a galvanometer.

IV. RESISTANCE

492. Conductivity and resistance. — When the terminals of a cell of a given E.M.F. are connected by different wires, it is found that some of the wires will carry or conduct more electricity than others. Conductors are therefore said to differ in their ability to conduct electricity, that is, in their *conductivity.* For example, if one conductor conveys twice as great a current as another with a given P.D. between its ends, then the conductivity of the first is twice that of the second. No conductor, however, has perfect conductivity, but it always obstructs the current more or less. This obstruction which a conductor offers to the flow of an electric current is called *resistance.* In the example just given, the second wire has half the conductivity of the first, or it obstructs the current twice as much as the first; its resistance is therefore twice as great as the first. This illustration shows that the *resistance of a conductor is the reciprocal of its conductivity.*

Different metals differ in their conductivity. Silver has the highest conductivity of all substances, and copper ranks next to it, the latter being more than six times as good a conductor as iron.

493. Unit of resistance. — The name of the unit in which resistance is expressed and measured is called the *ohm*, from Georg S. Ohm, a German who first established the laws of resistance. A column of mercury 106.3 cm. long and 1 sq. mm. in cross section has a resistance of one ohm at 0° C., and this has been taken as the legal definition of the ohm by the United States and other governments. Ten feet of No. 30 or 100 feet of No. 20 copper wire have a resistance of about 1 ohm.

The purity of a substance or anything affecting its molecular structure affects its resistance. The resistance

of metals increases with a rise of temperature, but the reverse is true with carbon, most electrolytes, and some other substances.

494. Laws of resistance. — The resistance of a conductor depends in general on its length and size according to the following laws:

I. *The resistance of a conductor is proportional to its length;* that is, $r : r' = l : l'$, r and l being the resistance and length of one conductor, and r' and l', the resistance and length of another conductor of the same kind.

II. *The resistance of a conductor is inversely proportional to the area of its cross section, or to the square of its diameter if it is round;* that is, $r : r' = d'^2 : d^2$, r being the resistance of a wire whose diameter is d, and r' that of another wire of the same kind and length whose diameter is d'. If both length and diameter vary at the same time, the two laws may be expressed by the proportion, $r : r' = l \times d'^2 : l' \times d^2$.

Sizes of wire are designated by number, just as thread is, the larger the number the smaller the wire. No. 40 is so fine that it is rarely used. No. 0 is about as large as an ordinary lead pencil, and No. 18, 0.0403 in. in diameter, is a very common size and is about as large as a hat pin. In measuring wire a thousandth of an inch is called a "mil," and the square of the diameter of a wire expressed in mils is called "circular mils." Thus, the diameter of No. 18 wire is 40.3 mils and its circular mils = 1624. (See table in the Appendix.)

Problems

1. If one wire 80 ft. long has a resistance of 3.2 ohms, what is the resistance of a wire of the same kind 175 ft. long?

2. How long must a wire be to have a resistance of 100 ohms, if 7.2 ft. of it have a resistance of 0.45 ohms?

3. Calculate from Table II in the Appendix the resistance of 150 ft. of No. 16 copper wire.

4. If a wire 32 mils in diameter has a resistance of 10 ohms, what is the resistance of a wire of the same kind and length having a diameter of 72 mils?

5. If a wire 500 ft. long and 18 mils in diameter has a resistance of 17 ohms, what is the resistance of a wire of the same kind 125 ft. long and 9 mils in diameter?

6. Calculate from data given in Table II in the Appendix the resistance of a wire 1 ft. long and 0.001 in. in diameter.

7. A wire 10 ft. long is 1 mil in diameter. What must be the diameter of a wire 40 ft. long to have the same resistance?

8. What length of wire 25 mils in diameter will have the same resistance as 153 ft. of wire 75 mils in diameter?

9. A wire 25 m. long and .75 mm. in diameter has a resistance of 80 ohms. What must be the diameter of the same kind of wire 10 m. long to have a resistance of 8 ohms?

10. What is the diameter of a wire which is 60.5 m. long and which has a resistance of 10 ohms, if a wire of the same kind 40 m. long and 5 mm. in diameter has a resistance of 8 ohms?

11. If 100 ft. of wire weighs 5 lb. and has a resistance of .07 ohms, what will be the resistance of one mile of the same kind of wire weighing 100 lb.?

12. Two wires of the same length and material have respectively resistances of 12 and 20 ohms. If the diameter of the first is 24 mils, what is the diameter of the second?

13. If the length of a wire is increased 4 times and the diameter also is increased 4 times, what is the effect on its resistance?

14. A copper wire 10 ft. long and 10 mils in diameter has a resistance of very nearly 1 ohm. Assuming it to be exactly 1 ohm, what is the resistance of a copper wire 1 m. long and 1 mm. in diameter?

V. OHM'S LAW

495. Ohm's law, which is perhaps the most important law of electrical science, was established by him in 1827, and expresses the relation between the three quantities, current strength, E.M.F. or P.D., and resistance. It may be expressed as follows:

The strength of a current equals the electro-motive force or potential difference divided by the resistance, or in symbols,

$$I = \frac{E}{R},$$

I being the current strength in amperes, E the E.M.F. or P.D. in volts, and R the resistance in ohms. These three units, the ampere, the volt, and the ohm, have been so chosen and defined that when the current strength is 1 ampere, and the E.M.F. or P.D. is 1 volt, the resistance is 1 ohm; that is,

$$1 \text{ ampere} = \frac{1 \text{ volt}}{1 \text{ ohm}}, \text{ or in general, amperes} = \frac{\text{volts}}{\text{ohms}}.$$

When the law is applied to the whole circuit,

$$I = \frac{\text{E.M.F.}}{R};$$

but when it is applied to part of a circuit,

$$I = \frac{\text{P.D.}}{r},$$

P.D. representing the fall of potential in the part of the circuit under consideration and r the resistance of that part.

$I = \frac{E}{R}$ may be reduced to two other useful forms,

$$E = RI, \text{ and } R = \frac{E}{I}.$$

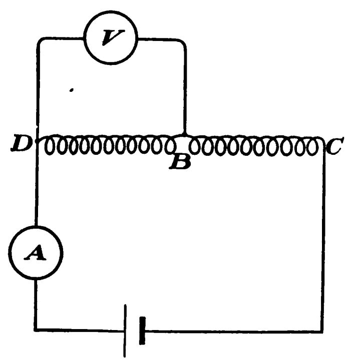

FIG. 296. — Diagram of the measurement of the resistance of a conductor by means of a voltmeter and ammeter and Ohm's law.

496. Measurement of resistance by Ohm's law. — It is evident that by Ohm's law any one of the three quantities, current strength E.M.F. or P.D., and resist-

ance, may be determined if two of them are known. For instance, suppose we wish to measure the resistance of a conductor from D to B (Fig. 296). It is joined to a battery with an ammeter A in the circuit, then a voltmeter V is connected to the ends of the conductor whose resistance is to be measured. The ammeter gives the current through DB; and the voltmeter, the P.D. between D and B, from which the resistance is easily calculated by Ohm's law. Thus, if $I = .2$ ampere, and P.D. $= .6$ volt, the resistance $\left(r = \frac{\text{P.D.}}{I}\right) = \frac{.6}{.2} = 3$ ohms. Conversely, if the resistance of a conductor is known, the current flowing through it may be determined by applying a voltmeter to its terminals and measuring the P.D.

497. Fall of potential along a conductor. — Figure 297 represents a voltaic cell, whose poles are joined by a con-

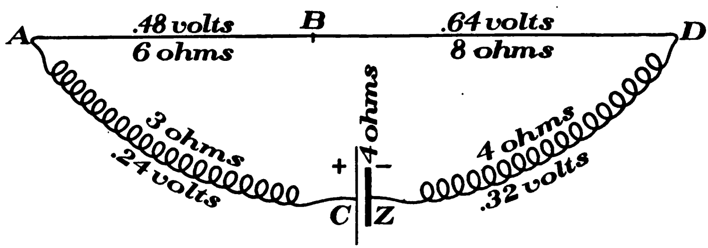

FIG. 297. — Diagram of the fall of potential in different parts of a circuit.

ductor $CADZ$. Let the E.M.F. of the cell, which is the P.D. between C and Z when the circuit is broken, be 2 volts, and the internal resistance of the cell be 4 ohms. Let the resistance from C to A be 3 ohms; A to B, 6 ohms; B to D, 8 ohms; and D to Z, 4 ohms. The resistance of the whole circuit is therefore 25 ohms, and by Ohm's law $\left(I = \frac{2}{25}\right)$ the current is .08 ampere. This is the strength of the current in every part of the circuit.

By applying Ohm's law to the different portions of the circuit, the fall of potential in each portion may be obtained; for example, the resistance from A to B is 6 ohms, and the current is .08 ampere. The P.D. between A and B is therefore (E or P.D. = RI, § 495) $6 \times .08$ or .48 volt. In like manner, the falls of potential between C and A, B and D, and D and Z can be found to be, respectively, .24, .64, and .32 volt.

The current flows around the circuit from points of higher to those of lower potential, for C is higher than A, A than B, B than D, and D than Z; but *in the cell* the current flows from lower to higher potential because the cell is expending energy upon it. The cell expends enough energy to maintain an E.M.F. of 2 volts; but, like other machines, it uses a part of this energy on itself, the loss caused by the resistance of the cell being .32 volt (P.D. $= 4 \times .08$). The P.D. between C and Z when the circuit is closed is therefore $(2 - .32)$ 1.68 volts. If we compare the P.D. and the resistance between A and B with the same quantities from B to D ($48 : 64 = 6 : 8$), we see that they are proportional. Hence the law:

With a given current the fall of potential along a conductor is proportional to the resistance passed over.

This in reality is only another way of stating Ohm's law.

498. The Wheatstone bridge is an instrument for measuring resistance. Its action and use are based upon Ohm's law as just stated, namely, the fall of potential along a conductor is proportional to its resistance. It consists of *four resistances*, called *arms*, and a galvanometer. The method of connecting the parts of the instrument is shown diagrammatically in Figure 298. CA, CB, AD, and BD represent the four resistances or arms. G is a galvanometer connected to the points A and B, and E is a battery joined to the bridge at C and D.

The current from E divides at C into two parts, not necessarily equal, one part going through CAD and the other by the path CBD, the two parts uniting at D. Some of the current may pass from A to B or from B to A; but the resistances can be adjusted so that no current will flow between the points A and B.

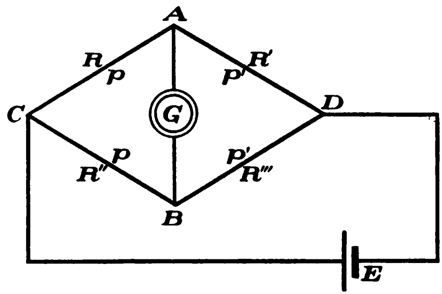

FIG. 298. — Diagram of the Wheatstone bridge.

When this adjustment is accomplished, the points A and B are at the same potential and the four resistances are proportional to one another; and if three of them are known, the fourth can be determined. The proof is as follows:

Let R, R', R'', and R''' be respectively the resistances of CA, AD, CB, and BD. The fall of potential from C to A and that from C to B are equal, because the fall is from a point of common potential to points of equal potential. (A and B are at the same potential, because no current flows through the galvanometer from one point to the other.) The falls of potential between A and D and B and D must also be equal, because the fall is from points of equal to a point of common potential. Let p represent the P.D. between C and A, also between C and B; and let p' represent the P.D. between A and D, and between B and D.

The current through CA is the same as that through AD; hence, applying Ohm's law as stated in § 497, $\frac{p}{p'} = \frac{R}{R'}$. For a like reason, $\frac{p}{p'} = \frac{R''}{R'''}$. Therefore $\frac{R}{R'} = \frac{R''}{R'''}$; or, *the resistances of the four arms of a Wheatstone bridge are proportional, if no current flows through the galvanometer of the bridge.*

499. There are two methods of joining instruments or conductors in an electric circuit, *series* and *parallel.* If

they are joined so that the current which passes through one passes through all in succession, they are said to be joined in series. Figure 299 represents three bells connected in series in the circuit of a cell. If a circuit branches off so that the current divides, a part going through one conductor and a part through another, then the instruments in the various branches are connected in parallel. Figure 300 shows three bells thus connected. Figures 302 and 303 represent cells joined in series and in parallel. All the street cars of a city line are in parallel, because the current branches off where each trolley touches the trolley wire, passing into the ground and back to the power house. The arc lamps along the street are usually joined in series, but the incandescent lamps in a house are in parallel.

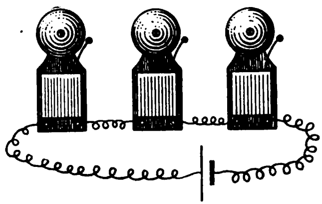

FIG. 299. — Diagram of three bells joined in series.

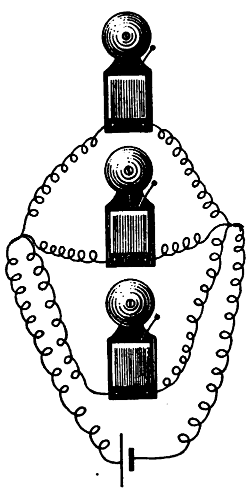

FIG. 300. — Diagram of three bells joined in parallel.

500. Shunts. — When a wire is connected in parallel with another wire, it is called a *shunt* to that wire, or each may be said to be a shunt to the other. The wire DB (Fig. 296) is a shunt to the voltmeter V. The joint resistance of two or more shunts equals the reciprocal of the sum of their reciprocals; thus, the joint resistance of two shunts of 6 and 12 ohms each is $(\frac{1}{6} + \frac{1}{12} = \frac{1}{4})$ 4 ohms.

The currents in the two shunts are inversely proportional to their resistances. If, for instance, we wish to reduce the sensitiveness of a galvanometer G (Fig. 301) we may arrange so that only $\frac{1}{100}$ part of the current shall pass through it by placing a shunt S between its terminals with a resistance $\frac{1}{99}$ of that of the galvanometer. Then $\frac{99}{100}$ of the current will pass through the shunt and $\frac{1}{100}$ through the galvanometer.

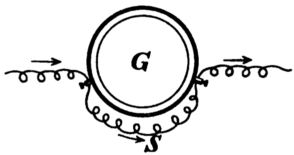

FIG. 301. — Diagram of a shunt to a galvanometer.

501. Joining cells. — When several cells are joined together, they form a *battery*. There are two methods of joining cells, series and parallel. Under some conditions the series method is the better because it gives the stronger current; under other conditions the parallel method is to be preferred. By means of Ohm's law we may determine which method should be used for any given conditions.

502. Joining cells in series consists in connecting the positive plate of the first cell to the negative of the second, the positive of the second to the negative of the third, and so on to the last cell, as shown in Figure 302. The

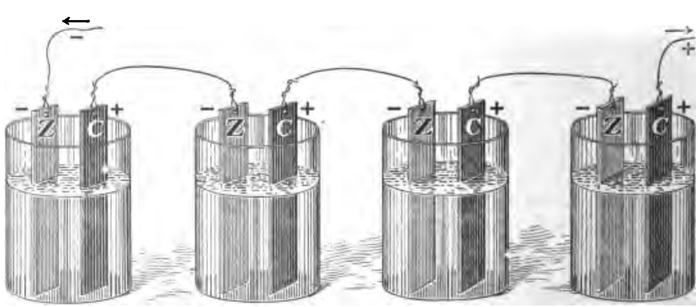

FIG. 302. — Diagram of cells joined in series.

negative pole of the first cell and the positive of the last form the two terminals of the battery. The current that passes through the first cell passes through all the others in succession. Experiment proves that *when cells are joined in series the E.M.F. of the battery is equal to the sum of the E.M.F.'s of the several cells; or if each cell has the same E.M.F., the E.M.F. of the battery equals the E.M.F. of one cell multiplied by the number of cells.* The reason for this is that the current flows from lower to higher potential in each cell, work being done upon it, and each cell adds its E.M.F. to that of the preceding.

The *internal resistance of a battery, when the cells are joined in series, is equal to the sum of the resistances of the several cells; or if they are all alike, to the resistance of one cell multiplied by the number of cells.* The reason for this is that the length of the liquid traversed by the current increases as the number of cells.

In applying Ohm's law to a battery joined in series, we find the current strength by the formula, $I = \frac{nE}{R + nr}$, in which E is the E.M.F. of each cell, r the internal resistance of each cell, R the external resistance of the circuit, and n is the number of cells. nE is the E.M.F. of the battery, and $R + nr$ is the resistance of the whole circuit.

503. Joining cells in parallel consists in joining all of the positive plates of the several cells together, and also all of the negative ones. This may be done as shown in Figure 303. P and D may be regarded as the terminals of the battery. The current divides at P, part of it passing through each cell and the several parts uniting again at D. The effect of this method of joining the cells is the same as enlarging the plates of a single cell. Since the E.M.F. of a cell is not affected by the size of its plates (§ 462), the E.M.F. of a battery with its cells joined parallel is

the same as that of a single cell. *The internal resistance, however, is less than that of one cell; it is equal to the internal resistance of one cell divided by the number of cells.*

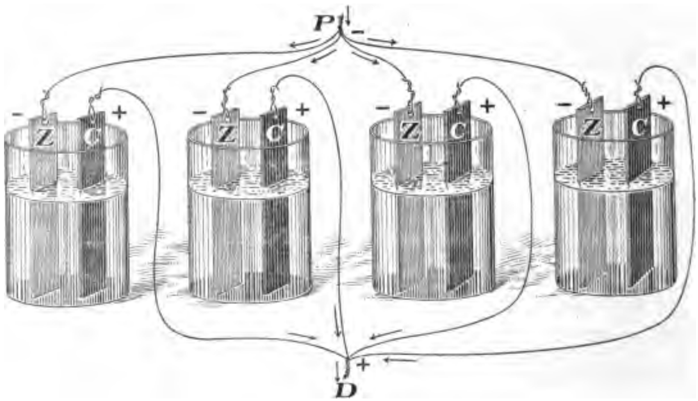

FIG. 303. — Diagram of cells joined in parallel.

The reason for this is that the cross-sectional area of the liquid conductor is increased.

Applying Ohm's law to a battery having its cells joined parallel, we have the formula for the current strength,

$$I = \frac{E}{R + \frac{r}{n}}.$$

504. How cells should be joined depends upon the relative values of the external and internal resistances. *When the external resistance is large compared with the internal resistance of one cell, the series method is the better; but when the external resistance is relatively small, the parallel method should be used.* The following problems illustrate this principle:

Let E, the E.M.F. of one cell, be 2 volts.

Let r, the internal resistance of one cell, be 1 ohm.

Let n, the number of cells, be 10.

Let R, the external resistance of the circuit, be 1000 ohms for the first three problems and 0.001 ohm for the last three.

When $R = 1000$ Ohms	No. of Cells	When $R = 0.001$ Ohm
(1) $I = \frac{2}{1000 + 1} = .001998 +$ $= .002$ amp. (nearly)	One cell	(4) $I = \frac{2}{.001 + 1} = 1.998 +$ $= 2$ amp. (nearly)
(2) $I = \frac{20}{1000 + 10} = .0198 +$ $= .02$ amp. (nearly)	Ten cells in series	(5) $I = \frac{20}{.001 + 10} = 1.999 +$ $= 2$ amp. (nearly)
(3) $I = \frac{2}{1000 + .1} = .001999 +$ $= .002$ amp. (nearly)	Ten cells in parallel	(6) $I = \frac{.2}{.001 + .1} = 19.81 +$ $= 20$ amp. (nearly)

Comparing the first three problems, we see that under the given conditions ten cells in series give practically ten times as strong a current as one cell; whereas ten cells in parallel give practically the same current as one cell.

Comparing the last three problems, we see that ten cells in parallel give under the stated conditions practically ten times as strong a current as one cell, but ten in series give the same current as one cell.

Can you see why in problem 3 even a million cells could not give a current of quite .002 ampere? Why in problem 5 with any number of cells could the current never quite equal 2 amperes?

The resistance of a Daniell cell is comparatively large, being from about 1 to 5 ohms. If its E.M.F. is 1.1 volts, what is the largest possible current to be obtained from a single cell?

Problems based on Ohm's Law

1. The E.M.F. of a Daniell cell is 1.1 volts. If it sends a current of .4 ampere through a wire having a resistance of 2 ohms, what is the internal resistance of the cell?

2. The P.D. between the terminals of an electric lamp is 60 volts and the current through the lamp is 8 amperes. What is the resistance of the lamp?

3. A current of 24 amperes is flowing through a wire. A certain portion of the wire has a resistance of 8 ohms. What is the fall of potential in that portion?

4. A trolley wire has a resistance of .001 ohm per foot. What current is the wire carrying when the fall of potential along the wire is .1 volt per foot?

5. A bichromate cell sent a current of 0.4 ampere through an electric bell having a resistance of 4.8 ohms, when it was connected directly to the bell; but when the bell was so far away from the cell that wires having a resistance of 3 ohms had to be used to join them, the current fell to 0.25 ampere. What were the E.M.F. of the cell and the resistance of the cell?

6. The E.M.F. of a battery is 6 volts and the total resistance of the circuit is 12 ohms. The external circuit is composed of two wires, A and B, joined end to end. If the fall of potential along A is 2.5 volts and the resistance of B is 2 ohms, what is the resistance of A and the fall of potential along B?

7. The E.M.F. of a battery is 10 volts. When producing a current of 5 amperes the P.D. between the poles of the battery is 8 volts. What is the resistance of the battery? *Ans.* 0.4 ohms.

8. A cell having an E.M.F. of 1 volt sends a current of 0.5 ampere through a galvanometer whose resistance is 1 ohm. What current would 3 such cells give if joined in series to the galvanometer?

9. The P.D. of a cell on open circuit is 1.32 volts. How much is it if the circuit is closed by a wire having a resistance of 2 ohms, the resistance of the cell being 0.2 ohm? *Ans.* 1.20 volts.

10. In problem 9, what is the loss of potential in the cell? *Ans.* 0.12 volt.

11. Solve problems 9 and 10 under the supposition that the wire has a resistance of 3.1 ohms instead of 2 ohms.

12. The E.M.F. of a cell is 1.8 volts and its resistance is 0.5 ohm. How much is the P.D. between the plates of the cell when connected by a conductor having a resistance of 179.5 ohms? What is the loss of potential in the cell itself?

13. In problem 12, substitute 89.5 ohms for 179.5 ohms and solve it.

14. What does joining the plates of a cell or battery by a conductor of small resistance do to their difference of potential? What is the effect of increasing this resistance on the P.D. between the plates?

15. Could the E.M.F. of a cell or battery be measured accurately by connecting it to a voltmeter of small resistance? (Small relatively to that of the cell itself.)

16. How much change is caused in the P.D. between the plates of a cell whose E.M.F. is 1.5 volts by connecting them to a galvanometer of 1500 ohms' resistance, the resistance of the cell being 0.5 ohm? *Ans.* A little less than .0005 volt.

17. The E.M.F. of a Daniell cell is 1.1 volts, but the P.D. between its poles when joined by a resistance of 10 ohms is 0.8 volt. What is the resistance of the cell?

18. A battery of 4 cells joined in series is joined to a sounder of 15 ohms' resistance. Each cell has a resistance of 0.8 ohm and an E.M.F. of 2 volts. What is the current strength?

19. In the last problem, how much would the strength of the current be changed by introducing into the circuit a galvanometer having a resistance of 18.2 ohms?

Could the current under the conditions like those given in problem 18 be accurately measured by introducing such a galvanometer?

20. Show that it could be measured quite accurately by a galvanometer or ammeter having a resistance of only 0.1 ohm.

This can be shown by adding the resistance of the galvanometer to external resistance given, working out the problem, and noting what difference is caused in the current by the added resistance of the galvanometer.

21. 10 cells joined in parallel, each having a resistance of 4 ohms, produce a current of 0.75 ampere, when the external resistance is 2 ohms. What is the E.M.F. of each cell?

22. 10 cells joined in series produce a current of 0.2 ampere when the external resistance is 60 ohms, and the internal resistance of each cell is 2 ohms. What is the E.M.F. of each cell?

23. How many cells in series, each of 1.8 volts E.M.F. and 1.1 ohms' resistance, will be required to send a 0.5 ampere current through an external resistance of 50 ohms?

24. Compare the current produced by 8 cells joined in series with that produced when they are joined in parallel, the external resistance being 2 ohms, the resistance of each cell being 2 ohms, and the E.M.F. of each cell being 1 volt.

25. Compare the current obtained from 200 cells joined in parallel with that obtained from one cell, if the external resistance is 12,000 ohms, and the resistance and E.M.F. of each cell are respectively 0.1 ohm and 1.6 volts.

VI. CURRENT INDUCTION

505. So far in our study of electric currents we have considered the voltaic cell as their source; but the cell plays an insignificant part in producing electric currents in the world of to-day. The E.M.F.'s for the production of currents for power and electric lighting, on a scale altogether impossible by the cell, are generated by means of the *dynamo* through the agency of *induction*, the study of which we are about to begin.

506. **Faraday's law of induction.** — The principles of current induction were discovered by Faraday in 1831–32, and he stated the law of induction somewhat as follows:

If a conducting circuit is placed in a magnetic field, and if by any means whatever the number of lines of force threading that circuit is changed, then an E.M.F. is caused in the circuit which is proportional to the rate at which the change occurs.

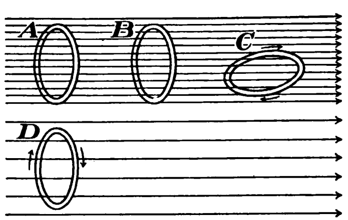

FIG. 304. — Diagram of a coil of wire in a magnetic field.

Let the horizontal lines (Fig. 304) represent the lines of force of a magnetic field and let the ring *A* represent either a single loop of wire or a coil of wire with its ends joined together. If *A* is moved to position *B*, no change is made in the

number of lines of force threading it and no E.M.F. is produced by the motion; but if it is turned over into position *C* or is moved into a weaker field, as at *D*, the number is changed, and an E.M.F. is produced which causes a current in the wire of the ring.

Current induction may be illustrated by a variety of interesting experiments, but all of them in one way or another involve the increase or decrease of the lines of force threading a circuit.

507. Current induction by the earth's magnetic field. — Let a coil of 300 or 400 turns of No. 25 wire about 30 cm. in diameter be joined by flexible wires to a sensitive galvanometer. (The coil *A* described in § 487 may be used, or the wire may be wound upon a barrel hoop.) Let this coil be held with its plane perpendicular to the lines of force of the earth (Fig. 305), as shown by the dipping needle. The lines threading it will then be at a maximum. If the coil is now turned over quickly, a half revolution being made about an axis lying in its plane, the galvanometer will show that a current is produced in it by the motion. If the revolution is completed, a current flows in an opposite direction in the coil during the motion. While the coil is turning through the first 90°, the lines of force threading it are decreasing, and during the next 90° they are increasing; but the current in the coil is in the same direction because it presents its opposite side toward the lines of force. Figure 306 illustrates the principle of this experiment, except

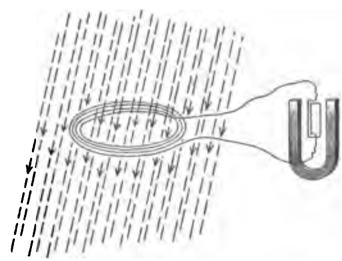

FIG. 305. — Diagram of a coil of wire in the earth's magnetic field.

that the field there shown is not due to the earth's magnetism.

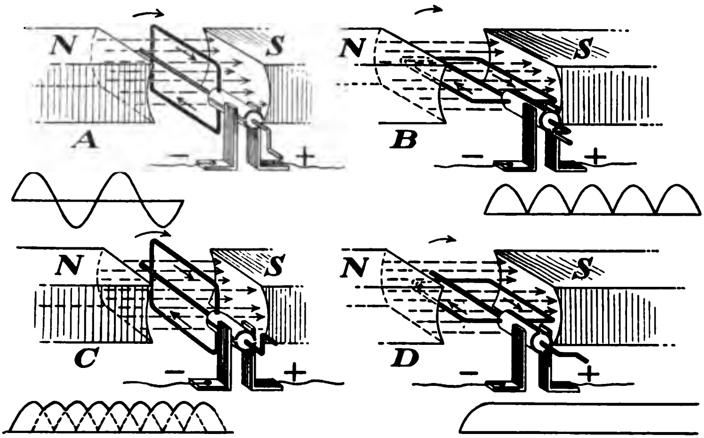

FIG. 306. — Diagram of currents produced by the revolution of a coil of wire in a magnetic field.

Experiment. — Connect a large spool of wire (Fig. 307) by flexible wire to the galvanometer, and repeat the last experiment. Very small deflections will be produced. Let a thick rod of soft Swedish iron be introduced into the spool, and the experiment be repeated. A large deflection of the galvanometer will now be produced. This is because the iron core concentrates and greatly increases the lines of force threading the spool.

FIG. 307.— Spool of insulated wire with soft iron core.

508. Current induction by the field of permanent magnets. — Let Apparatus *M* be connected to the galvanometer, and let some bar magnets be placed close to the coil, the north poles of some of them being on one side of the coil and

the south poles of the remainder being on the opposite side. With this arrangement the lines of force will pass from the north poles of one set of magnets to the south poles of the others, and the coil can revolve in this field.

Let the coil be placed with its plane perpendicular to the lines of force, and let it be turned quickly through half of a revolution. A current will be produced in the coil and galvanometer. Allow the galvanometer to come to rest and then turn the coil in the same direction as before through another half revolution; the galvanometer will be deflected in the opposite direction. This shows that during one complete revolution of the coil two currents are produced, a current in one direction in the coil during one half revolution, and a current in the opposite direction in the coil during the remaining half revolution. This experiment is like the preceding ones, but the induced currents are stronger because the field is stronger.

Again, let the coil be set in rapid rotation by the thumb and finger. Watch the action of the galvanometer while the motion of the coil gradually dies away. The galvanometer coil or needle will be given an oscillatory motion because the currents induced by the rotating coil change direction at each half revolution.

509. Alternating currents. — Electric currents such as we have studied hitherto, being continuously in one direction in the conductor, are called *direct currents;* but a current which, like the last one shown, flows for an instant in one direction, then stops and flows in the opposite direction, thus continually reversing its direction, is called an *alternating current.* The reversals usually occur many times a second. Two reversals are called a *cycle.*

510. Current induction by motion of the magnet. — In the preceding experiments the coils have been moved

while the magnets or the field have been stationary; but the coil may remain at rest while the field is moved.

Connect a coil of quite a large number of turns of wire to the galvanometer (Fig. 308), and then thrust one of the poles of a magnet into it. A current is induced in the coil. Allow the magnet to rest in the coil, and no current is produced. Withdraw the magnet, and a current opposite to the first results. If the magnet is introduced into the coil or removed very slowly, the induced currents are much weaker because the E.M.F.'s developed are less.

FIG. 308. — Apparatus for showing that currents may be induced in a coil by means of a magnet.

511. Direction of the induced current. — If the direction of the induced currents in the preceding experiments are traced out through the coil and galvanometer, it will be found in each case to agree with the following rule: *When the lines of force are increasing, grasp the coil with the left hand with the fingers through it in the direction of the lines of force, and the extended thumb will indicate the direction of the induced current; when the lines of force are decreasing, use the right hand.*

Test this rule by a study of Figures 305 and 306, and also determine at what point in the revolution of the coil (§ 508) the direction of the current changes.

512. The law of Lenz. — *The direction of the induced current is always such as to oppose the motion which causes the induced current.* This opposition to the motion is due to the reaction between the magnetic field causing the induction and the field developed by the induced current.

For example, in the experiment with Apparatus M (§ 508), when one face of the coil is approaching a north pole of the magnet, the induced current in the coil makes that face a north pole, and hence there is repulsion between it and the pole it is approaching. The law of Lenz is a necessary consequence of the principle of conservation of energy; for if it were otherwise, electrical energy could be developed at the expense of no other energy.

Experiment. — Let a solid brass cylinder about 2 cm. in diameter and 3 cm. long be suspended by a stout thread between the poles of a strong electro-magnet (Fig. 309). Twist the thread considerably so that it will cause the cylinder to revolve rapidly between the poles of the magnet without touching them. When there is no current in the magnet coils, it will revolve easily, but when the circuit is closed and the magnet energized, it is instantly brought to rest; when the circuit is broken, it begins to revolve again. The motion of the cylinder is more easily seen at a distance if it has one or two stripes of black paint upon it.

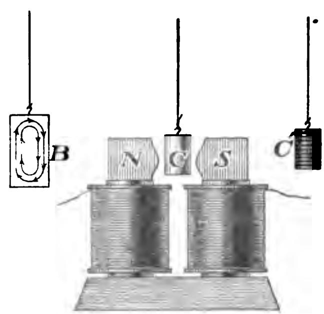

Fig. 309. — Brass cylinder revolving between the poles of an electro-magnet.

The cylinder stops revolving, not because it touches anything, nor because it is magnetic and is attracted by the magnet, for it is not; but the parts of it form circuits in which currents are induced. These induced currents (B, Fig. 309) within the cylinder oppose the motion. Currents caused in this way within a mass of metal are called "eddy" or Foucault currents. If a cylinder, C', built up of copper washers with disks of paper between them, is used, these currents are largely prevented, and the motion of the cylinder is little affected by the magnet.

It is on the principle illustrated here that the vibrating coil of a D'Arsonval galvanometer is quickly brought to rest when it is short circuited. As the coil turns in the magnetic field, currents are induced in it which oppose the motion and stop it.

VII. THE DYNAMO AND ELECTRIC MOTOR

513. The dynamo is a machine for transforming the energy of mechanical motion into the energy of the electric current by means of current induction. The mechanical energy is usually supplied through the agency of a steam engine or a water wheel.

The main parts of a dynamo are (1) the field magnet, (2) the armature, (3) the brushes, and (4) the collecting rings. The experiment with Apparatus *M* (§ 508) illustrates these parts as well as the action of the dynamo. The magnets which furnish the magnetic field are the field magnets, the revolving coil in which the current is induced is the armature, and the mercury performs the work of the collecting rings by which the current is transferred from the armature to the rest of the circuit. However, when the field is furnished by permanent magnets as in this case, the machine is called a *magneto* rather than a dynamo.

514. The field magnet of a dynamo is a powerful electromagnet furnishing the field in which the current induction may take place. It consists of four parts: (1) the *pole pieces* *N* and *S* (Fig. 310); (2) the *field cores* *C* and *C*; (3) the *field coils* which are wound around the field cores; and (4) the *yoke* *Y*. The pole pieces, the cores, the yoke, and the space *A* form the magnetic circuit. The lines of force in passing from *N* to *S* make a strong field in the space *A*, and it is in this circular space that the armature revolves.

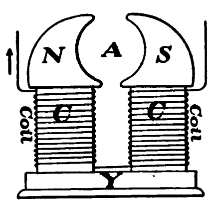

FIG. 310.—Diagram of the field magnet of a dynamo.

A dynamo with such a field magnet is called a *bipolar* machine because it has two poles. Most modern machines

have more than two poles, and in that case they are called *multipolar* machines. In such machines the yoke is a massive hollow cylinder of iron (Figs. 311 and 312) and the cores extend from it toward the center; in the central space the field is formed in which the armature revolves. Dynamos of moderate size have four or six poles, but larger ones usually have more.

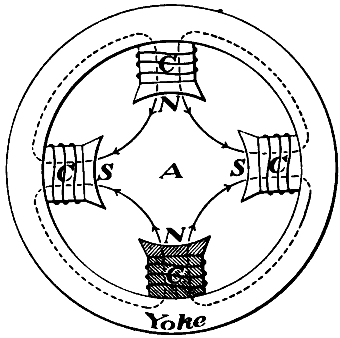

FIG. 311. — Diagram of a field magnet with four poles.

515. Brushes and collecting rings. — Figure 313 represents a coil of wire between the poles of a magnet. This coil is supposed to revolve on an axis or shaft in the line ST. The ends of the coil are attached to two metal rings R and R' which are on the same shaft and revolve with it. These rings are insulated from the shaft and from each other. B and B' are bars of copper or carbon fixed in position which rest on the rings and rub against them as they revolve. These bars are called *brushes*, and the rings, *collecting rings*. The alternating currents induced in the revolving coil pass to these rings,

FIG. 312. — Multipolar dynamo.

from them to the brushes, and by the wires to the external circuit. The current leaves the machine by B and B' alternately. A dynamo which furnishes in this way an alternating current is called an *alternator*.

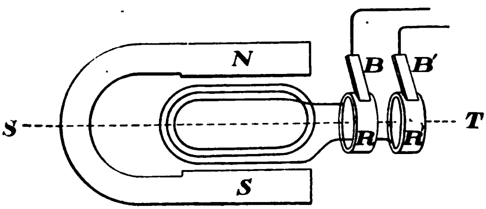

Fig. 313. — Diagram of collecting rings and brushes.

516. The commutator is a device used instead of collecting rings when a direct current is wanted from the dynamo. It may be considered as a collecting ring cut into parts or segments as shown in Figures 314 and 315. Each segment is carefully insulated from the others and connected to the end of a coil of wire in which the currents are generated. As the coil and commutator revolve (Fig. 314), the two brushes rest alternately on c and c', and they are so placed that the spaces between c and c' pass under the brushes at the instant the current changes direction in the coil. In this way B remains positive and B' negative, and the current is direct in the external circuit while it is alternating in the coil or armature. The commutator therefore is a device for changing alternating currents into direct currents.

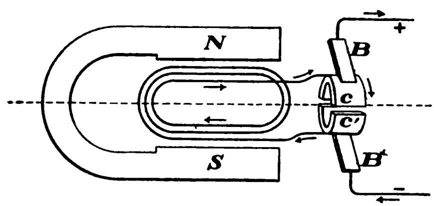

Fig. 314.—Diagram of a commutator and brushes.

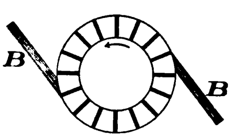

Fig. 315. — Diagram of the segments of a commutator.

When there is only one coil (Fig. 314), the commutator has two segments; but when there are more coils, a greater number of segments are necessary, as shown in Figure 315.

517. The armature consists of the *coil* or *coils* in which the current is induced, together with the *core* of iron upon which the coils are wound. It is supported on an axle, or shaft, upon which it is caused to revolve rapidly in the field of the field magnet. The loop of wire shown in Figure 306 and the coils shown in Figures 313 and 314 represent the armature. In practical machines, however, the armature has several coils instead of one, and these coils are wound upon an iron core, called the *armature core.* The simplest form of armature consists of one coil of wire wound in two grooves cut lengthwise on opposite sides of an iron cylinder (Fig. 316). Such an armature is called a *shuttle armature.*

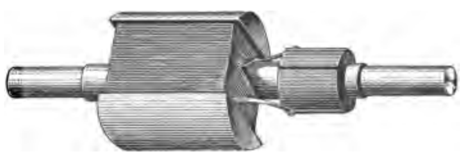

FIG. 316. — Shuttle armature.

518. The drum armature, which was developed from the shuttle armature, is one very much used in direct current dynamos. Instead of one coil in one pair of grooves, there are several coils in as many pairs of grooves, spaced equally round the cylindrical core. Figure 317 illustrates such an armature with a commutator attached. An armature with only one coil gives a current which rises and falls in strength, there being

FIG. 317. — Drum armature.

two points in every revolution of the armature when it is zero; but with several coils the current becomes steady, because some one coil is always in the position giving the maximum effect.

519. The Gramme ring is another form of armature that is much used. The core of this armature is an iron ring *CDEA* (Fig. 318), which is wound with a continuous length of wire uniformly over its whole surface, or it is wound with a number of coils which are joined together in series so as to form a continuous wire. At equal intervals the windings are joined to the different segments of the commutator. The lines of force pass through the core as shown by the dotted lines; when the armature is revolving in the direction shown by the arrow, the lines of force threading the coils between *C* to *D* and *A* to *E* are increasing, while in the coils from *C* to *A* and *E* to *D* they are decreasing.

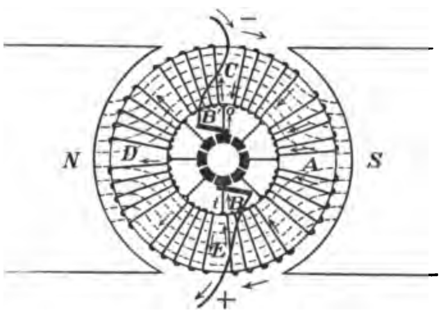

FIG. 318. — Diagram of a Gramme ring armature.

By applying the rule for finding the direction of induced currents (§ 511) to the different parts of the ring, one will see that the directions of the currents in the various coils are as shown by the arrows in the figure. If the current in the coils on the right half of the ring, *CAE*, are traced out, they will be seen to be going from *o*, or brush *B'*, to *t*, or brush *B*; also on the left side of the ring, *CDE*, they will be found to be going from *o* to *t*. Thus the currents induced in the two halves of the

ring meet at *t*, passing out by the brush *B*, and return to the armature by *B'*, dividing at *o*.

520. Laminated cores. — The core of an armature is not made of one solid piece of iron, for if it were, eddy currents (§ 512) would be induced in it as well as in the wire wound upon it. The energy of such currents, being transformed into heat, would not only be wasted, but would make the machine too hot to be operated. Armature cores are therefore made of thin disks of iron packed together but insulated from one another by varnished paper.

521. The excitation of the field magnets. — The current necessary to energize the field magnet of a dynamo is taken from the dynamo itself, if it is a direct current machine; if it is an alternator, a small auxiliary dynamo called an *exciter* is used to supply the current for the field coils.

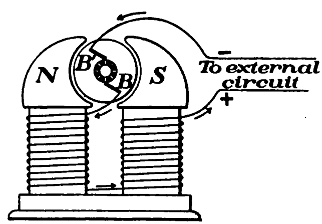

FIG. 319. — Diagram of a series-wound dynamo.

When the current leaves the armature by the brush *B* (Fig. 319), it may pass through the field coils and then to the external circuit and back to the other brush. Such a machine is called a *series-wound* dynamo.

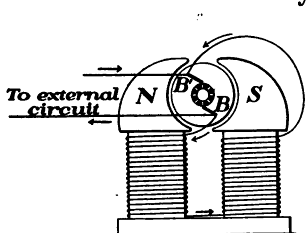

FIG. 320. — Diagram of a shunt-wound dynamo.

When the current divides, as at brush *B* (Fig. 320), a part going through the field coils and part through the external circuit, the two parts uniting again at *B'*, the dynamo is said to be *shunt-wound*. In series-wound dynamos the field coils are composed of a

few turns of coarse wire, but in shunt-wound machines the field coils have many turns of small wire.

When a dynamo is started up again after being at rest, there is of course little or no current to actuate the field magnets, but there is always sufficient residual magnetism left in the magnet to start the currents, and after a short time the machine generates its full E.M.F.

522. The electric motor is a machine for transforming the energy of the electric current into mechanical energy, being in this respect the converse of the dynamo. The direct current motor has the same parts as the direct current dynamo, namely, field magnet, commutator and brushes, and armature. In fact every direct current dynamo may be run as a motor, if a current is passed through it. The armature may be either a Gramme ring or a drum armature.

For example, let us study how the Gramme ring (Fig. 318) acts when used as a motor. A current entering the coils of the ring by the brush B' divides at o, half of it going through the coils on the side D and half through the coils on the side A. The two parts of the current unite at t and pass out by the brush B. These currents make an electro-magnet of the ring with its north pole at E and its south pole at C. (See rule, § 480.) Although the ring revolves, the poles remain stationary near the points C and E because the brushes by which the current enters and leaves the armature are stationary. The rotation of the armature occurs because the south pole at C is repelled by S and attracted by N, and the north pole at E is repelled by N and attracted by S. Observe that the direction of rotation is opposite to that which it had as a dynamo. The motor may be reversed by changing the direction of the current either in the field coils or in the armature, but not by reversing the current in both at once.

Apparatus M (Fig. 275) may be made into a motor to illustrate the action of a drum armature in a motor. The action is very similar to that of the Gramme ring. Let a two-part commutator be placed on the rod d and a brush holder be attached to the rod A. Then let one part of the commutator be connected to d and the other part to c by wires, and let magnets be placed on each side of the coil to furnish the field. If a current is then passed through the coil, it will revolve rapidly.

Motors, like dynamos, may be either series-wound or shunt-wound. In the former, the fields are wound with a few turns of heavy wire which are joined in series with the armature ; in the latter, the field coils have finer wire and are connected in parallel with the armature. Street car motors are usually series-wound, while other motors are more often shunt-wound.

523. The counter E.M.F. of a motor. — When the armature of a motor is revolving, it tends to act like a dynamo and to set up an E.M.F. in opposition to the current which is passing through it. Indeed, in a good motor running at full speed, this opposing or counter E.M.F. is so great as to diminish the current through the armature very greatly. In a motor just starting, this counter E.M.F. is at first zero and the current through the armature might be so great as to burn the wire and insulation. To prevent this the full pressure is not applied to the armature all at once, but gradually through a rheostat.

It is by overcoming this counter E.M.F. of the motor that the current does work. *The counter E.M.F. × the amperes = the rate at which the electrical energy is changed into mechanical energy.*

VIII. CURRENTS INDUCED BY CURRENTS

524. Current induction by the magnetic field of another current. — It has been shown that an electric current has a magnetic field about it, and that a coil carrying a current acts like a magnet. We should therefore expect that a current in one coil could produce a current in another coil

by induction. Experiment proves this to be true. The coil in which the inducing current flows is known as the *primary* coil and the one in which the currents are induced, as the *secondary* coil.

Almost any two coils may be used to illustrate this principle experimentally; but coils made by winding wire about short lengths of pasteboard mailing tubes answer admirably for the purpose. Let one coil be made by winding 500 turns of No. 28 wire on a tube about 12 cm. long and 5 cm. in diameter; and another coil, by winding about 300 turns of No. 25 wire on a tube about 3 cm. in diameter.

Experiment. — Let the 500-turn coil, *S* (Fig. 321), be joined to the galvanometer *G* and constitute the secondary, and the other coil, *P*, be joined to two bichromate or dry cells to act as a primary coil, the two coils being supported on a wooden rod. The experiment may be divided into six distinct operations:

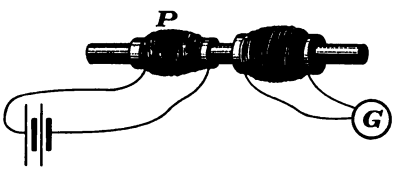

FIG. 321. — Apparatus for illustrating current induction by means of another current.

(1) Let one end of the primary coil be brought near to one end of the secondary coil, or let the primary coil be introduced into the secondary. The galvanometer will show the presence of an induced current in the secondary during this movement.

(2) Remove the primary coil away from the secondary, and again a current will be induced in the secondary.

(3) While the primary coil rests within the secondary, let the circuit of the primary be made. A momentary current will be induced in the secondary.

(4) Again with the coils in the same position as in the last case, let the primary circuit be broken. A momentary induced current will occur in the secondary at the instant the primary circuit is broken.

(5) Let some kind of rheostat be placed in the primary circuit, so that the current may be increased or decreased at pleasure. With the primary coil within the secondary or close to it, let the primary

current be suddenly increased. This will cause a current in the secondary.

(6) Let the current in the primary be decreased and a current will be induced in the secondary by the operation.

Observe that the 1st, 3d, and 5th operations are alike in that they increase the lines of force threading the secondary. If the directions of the primary and secondary currents are traced out in these cases, the direction of the secondary current will be found in each case to be opposite to that of the primary.

The 2d, 4th, and 6th cases are alike in that they decrease the lines of force threading the secondary, and the direction of the induced current is the same as that of the inducing current.

The 1st and 2d of these methods are used in the induction coil (§ 528), and the 5th and 6th are used in connection with the telephone (§ 530).

Let the two coils be supported upon a rod of soft iron instead of wood and the six operations be repeated. The induced currents will be very much stronger than before because the number of lines of force is greatly increased by the presence of the iron.

525. The transformer. — If an iron ring *rr* (Fig. 322) has two independent coils of insulated wire *A* and *B* wound upon it, an alternating current in one of them will induce an alternating current in the other. Let us suppose coil *A* to be connected to an alternating generator. The current starting at zero in the direction of arrow 1 rises to a maximum, and then, falling to zero, starts in the direction of arrow 2, again rising to a maximum in that direction and falling to zero. These changes go on continuously and

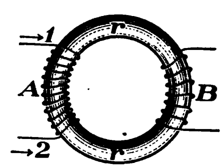

FIG. 322. — Diagram of a ring transformer.

rapidly. The current in the first direction magnetizes the ring with the lines of force extending around in the ring in a clockwise direction; but the ring loses its magnetism and is immediately remagnetized, when the current changes direction, with the lines of force counter clockwise in the ring. In this way an alternating current is induced in coil *B* because lines of force are made to thread it alternately, first in one direction and then in the other.

The E.M.F.'s in the two coils are directly proportional to the number of turns of wire in them. If coil *B*, which is the secondary, has 100 times as many turns as *A*, then the E.M.F. in *B* is 100 times as great as in *A*. Such an arrangement, which increases the E.M.F., is called a "step-up" transformer. On the other hand, if *A*, the primary, has the greater number of turns, the E.M.F. in *B* will be less than in *A*. It is then called a "step-down" transformer. The E.M.F. of an alternating current may in this way be greatly increased or reduced with little loss of energy, and it is for this reason that the alternating current is so largely used for many practical purposes, especially for transmitting electrical energy over long distances.

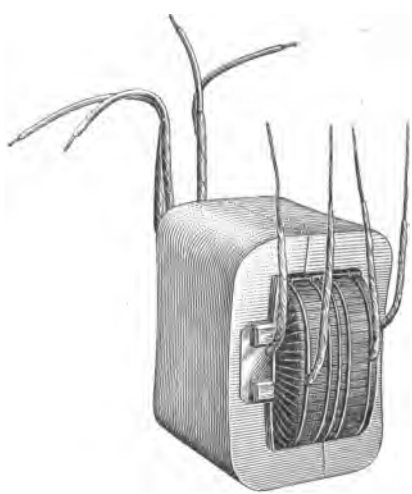

FIG. 323. — A commercial transformer, the outside shaped like a figure 8, is the laminated core; the coils are wound upon the crosspiece of the 8.

If the iron ring is one piece of metal, eddy or Foucault currents are induced in it as well as in the wire upon it, and the energy of the current is wasted in heat. In commercial transformers (Fig. 323) the magnetic circuit is built up of thin sheets of iron to prevent this loss.

526. Self-induction. — We have seen how a current may be induced in one coil by means of a current in another coil not connected with it; but two separate coils are not necessary for induction. For example, when a current is started through A (Fig. 324), each separate turn of it acts inductively on all of the others; the turn a develops lines of force which thread b, c, etc., and b and c and all of the others do likewise. When the current starts in A, the induced E.M.F. is in opposition to the P.D. sending the current through A. This retards the current so that it does not reach its full strength at once. When the circuit of A is broken, the induced E.M.F. is in the same direction as the P.D. sending the current through the coil. This tends to prolong the current so that it cannot stop instantly. This action gives to a current a property like inertia. This inductive action occurring in the same conductor as that in which the inducing current flows is called *self-induction.*

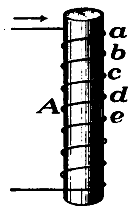

FIG. 324. — Diagram illustrating self-induction in a coil.

Because self-induction retards the starting of a current in a coil, the E.M.F. induced by making a circuit is always very much less than that induced by breaking the circuit. The current caused by self-induction is often called the *extra current.*

Experiment. — Let a large electro-magnet be placed in the circuit of one or more dry cells. Arrange the wires so that the circuit may be made and broken by touching two free ends and separating them

again. Grasp the bare ends of the wires with the moistened thumb and finger of each hand and make and break the circuit with them. At the break of the circuit, the E.M.F. due to the self-induction may be great enough to give quite a shock to the arms, although the E.M.F. of the battery is imperceptible.

Again complete the circuit by dipping the ends of the wire into some mercury, then break it by lifting one wire quickly from the mercury. A bright spark will be produced at the break by the self-induction.

527. Induction by alternating currents. — Many towns have the alternating current for electric lighting. If such a current is available, some very interesting experiments in induction may be made with an apparatus constructed as follows: Sufficient soft iron wire, such as stovepipe wire, is cut into 15-inch lengths to make a bundle about 4 in. in diameter. This core is covered with insulating tape and supported in an upright position by being set in a round hole in a suitable base. It is then wound for its whole length with two layers of No. 12 magnet wire, the ends of the wire being attached to two binding posts on the base. The terminals of a 110-volt alternating circuit may be safely joined to this coil without other resistance. We will designate this apparatus as coil *P*.

Experiment. — Make a coil 6 in. in diameter of four turns of No. 12 wire, joining the ends of the wire by a short piece of fine platinum wire. When this coil is placed over coil *P*, the currents induced in it will heat the platinum wire red-hot.

Experiment. — Make a compact coil of 500 turns of No. 22 wire. The opening in the coil should be about 6 in., or large enough to slip over coil *P* without touching it. Join the ends of this coil to a lamp socket in which a 110-volt incandescent lamp is placed. Lower this coil over coil *P*. The lamp will be lighted very brilliantly by the induced currents.

Experiment. — Let a heavy copper ring about 6 in. in diameter be supported about the upper part of coil *P*. At the closing of the circuit of coil P, this ring will be thrown upward and away from *P*. This repulsion occurs because the alternating currents induced in the ring are for the most part opposite to those in coil *P*.

If the ring is held in position about coil *P*, it soon becomes very hot. The induced currents in the ring are very large and they cause large currents in coil *P* also. The operator should make sure that the wires and fuses leading to his coil are sufficient for this purpose.

Experiment. — Secure from the tinner a ringlike dish of copper having an opening large enough so that it may be placed over or around coil *P*. Fill this dish with water and place it so that it encircles coil *P* near its center. The water will soon be made to boil by the heat caused by the induced currents in the dish.

528. The induction coil (Fig. 325) consists of a large coil of very many turns of fine insulated wire, within which there is another coil consisting of one or a very few layers of coarse wire. A core consisting of a bundle of iron wires extends through the inner coil. The coarse wire, which is connected to a battery, forms the primary coil. The coil of fine wire, often many miles in length, is the secondary in which currents are induced by the making and breaking of the primary circuit (§ 524). A current interrupter, a device for making and breaking the primary circuit rapidly, is a necessary adjunct to the induction coil. Usually it is a part of the instrument itself, as described here; but often it is a separate affair.

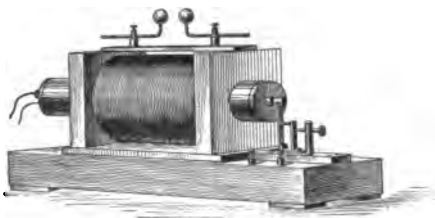

Fig. 325. — Induction coil.

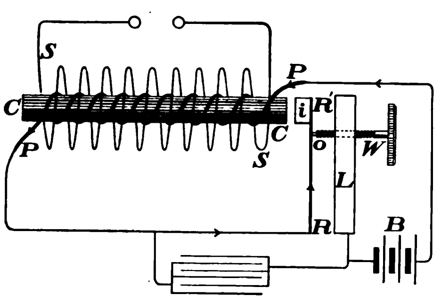

Fig. 326. — Diagram of an induction coil.

Figure 326 shows the arrangement of the parts of an induction coil. *CC* represents the iron core, the heavy wire *PP* the primary coil, and the fine wire *SS* the second-

ary coil. RR' is a flat strip of spring brass fixed at R and carrying a block of soft iron i; this is the current interrupter. The current passes from the battery B through the primary coil PP to R, up the spring RR' to o, then to the screw W, down the post L, and back to the battery. The passage of the current through the primary coil magnetizes the core CC, which attracts the iron block i, and bends the spring RR' away from the screw W. This breaks the circuit at o.

As a result of this, the core loses its magnetism and the spring flies back against the screw W, again making the circuit at o. In this way the spring is caused to vibrate and to make and break the circuit rapidly at the point o. At each make, a current is induced in the secondary opposite in direction to that in the primary; at each break, a current is induced in the secondary having a direction the same as that in the primary.

The E.M.F.'s induced in the secondary are extremely high because of the very great number of turns of wire in that coil (§ 525). If the distance between the terminals is not too great, the current induced at the break in the primary leaps across the gap, forming a spark. Coils giving sparks more than three feet in length have been made, and those giving 12-inch or even longer sparks are not uncommon. The P.D. between the terminals of these large coils is very great, sometimes more than 2,000,000 volts.

Currents from small coils are passed through the human body for therapeutic purposes, but the shock from a coil of even moderate size may be dangerous. The induction coil has many other practical uses. The spark of the coil is used in chemistry for igniting gases; it is also used for firing mines and blasts, and in gasoline engines to ignite the mixture of air and gasoline vapor. Wireless telegraphy,

the X-ray, and the telephone, all make use of the induction coil.

529. The condenser. — On account of self-induction in the primary, a condenser (§ 428) is usually attached to an induction coil (Fig. 326). It consists of many layers of tin foil and is contained in the base of the instrument. When the circuit is broken at *o*, the extra current jumps across the break, forming a spark at that place. The heat of this spark burns the end of the screw as well as the point of contact on the spring. The spark also prolongs the time of breaking the circuit and thus lessens the induced E. M. F. in the secondary. The condenser, which is joined to points on each side of *o*, prevents these sparks at *o* to a great extent and thus makes the break more abrupt. Instead of jumping across the gap at *o*, the extra current rushes into the condenser and charges it. The condenser, however, is immediately discharged around through the primary and the battery. As this discharge is opposite to the battery current, it helps to demagnetize the core at the break. Since the starting of the current in the primary at the make is considerably prolonged by self-induction, the E.M.F. in the secondary at the make is too small to cause a spark at the terminals. The discharge, therefore, occurs only at the break and only in one direction.

IX. TELEPHONE

530. The telephone, invented by Alexander Graham Bell and first exhibited by him in Philadelphia in 1876, is wonderfully simple in construction, although the telephone as we know it to-day is a somewhat complicated combination of several instruments. The Bell telephone proper is the instrument which we place at the ear and now call the *receiver*. The instrument into which we speak is known as the *transmitter*. The telephone receiver may be used as a transmitter and, indeed, was so used at first.

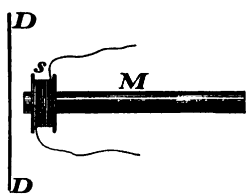

FIG. 327. — Diagram of a telephone receiver.

The receiver consists of a permanent bar magnet *M* (Fig. 327) having a small spool *s* of fine insulated wire

around one end of it. Near the same end of the magnet is a disk or diaphragm of thin soft iron *DD*. The parts are inclosed in a case of hard rubber, the diaphragm being clamped firmly all around its edge so that it may vibrate at its center like a drumhead. It must not, however, touch the end of the magnet as it vibrates. The receiver may be actuated by either an alternating or a fluctuating current which passes through the wire wound upon the spool. To one understanding the electro-magnet it is obvious that the attraction of the magnet *M* for the center of the disk is changed by each alternation or fluctuation of this current. This variation in the strength of the magnet causes the disk to vibrate in harmony with the variations of the current. The vibrations of the disk produce sound waves in the air, and hence the receiver transforms the energy of the electric current into the energy of sound, a form of mechanical energy.

FIG. 328.—Section of a telephone receiver.

In recent years the receiver has been slightly modified by the substitution of a U-shaped magnet for the bar magnet (Fig. 328).

When two receivers are connected by wires as shown in Figure 329, either one may be used as a transmitter while

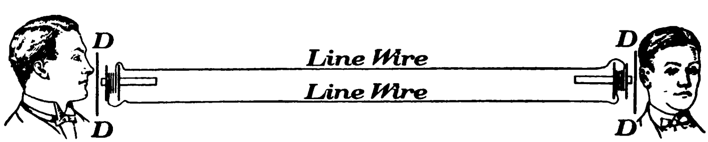

FIG. 329.—Diagram of two receivers connected.

the other is used as a receiver. The sound waves striking the diaphragm of the one used as a transmitter cause it

to vibrate so that its center alternately approaches and recedes from the end of the magnet. When it approaches the magnet, it increases the lines of force that thread the spool of wire; when it recedes, the lines of force are decreased. This action induces alternating currents in the wire, which, passing to the other receiver and through its coil, cause its diaphragm to vibrate and reproduce the words spoken at the first one. Conversation may in this way be carried on by two receivers a few miles apart. The earth may be substituted for one of the wires connecting the two instruments.

531. The microphone. — A simple form of this instrument is shown in Figure 330. *B* and *B* are two short rods of carbon fastened to an upright support, and *C* is a third rod of carbon with pointed ends which rest loosely in two little cavities in the rods *B* and *B*. These three rods form part of a battery circuit in which a telephone receiver is placed. The slightest disturbance of the contacts between the rod *C* and the rods *B* and *B* changes the resistance to the current at those points and hence causes a change in the strength of the current throughout the circuit. It is said that even a fly walking on the base of the instrument may cause sufficient disturbance of the contacts to produce an audible sound in the receiver. This action of loose carbon contacts, which has proved of great practical importance, was discovered in 1878 by Professor Hughes in England, and independently by Edison in this country in the same year.

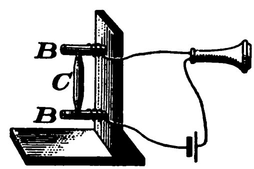

FIG. 330. — Microphone.

532. Transmitters. — The Bell receiver is exceedingly sensitive as a receiver, but it is not a very satisfactory

transmitter. For that reason some form of a microphone or instrument depending on loose carbon contacts is used as a transmitter in connection with the receiver. In one form, now much used, there is a small space E (Fig. 331), between two carbon disks, A and B, which is filled with grains of hard carbon. The back disk A is fixed, the front disk B is attached to the diaphragm C. The two disks and the grains of carbon between them form part of a battery circuit, the current passing by a wire to one disk, through the granules to the other disk, and then by a wire back to the battery. When sound waves strike the diaphragm C, they cause vibrations in it which are communicated to the disk B and to the carbon granules. Every variation of contact among these granules causes a change of resistance to the current passing through them, and consequently a change in the current strength. The current therefore fluctuates with every movement of the diaphragm. This fluctuating current may itself be sent to a distant receiver, but usually it is sent through the primary of a small induction coil, and the alternating currents induced in the secondary pass to the distant telephone.

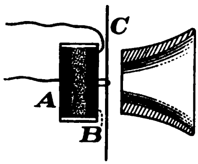

FIG. 331.—Diagram of a transmitter.

X. CHEMICAL EFFECTS OF THE ELECTRIC CURRENT

533. The electrolytic cell. — In order to conduct a current through a liquid, wires from a battery or a dynamo are joined to two metal plates A and B (Fig. 332), which are immersed in the liquid. These plates are the *electrodes*, the one A by which the current enters the liquid being the *anode*, and the other, by which it leaves, the *cathode*. The anode is positive and the

cathode is negative. If the liquid contains molecules capable of dissociation into two parts or ions, it is called

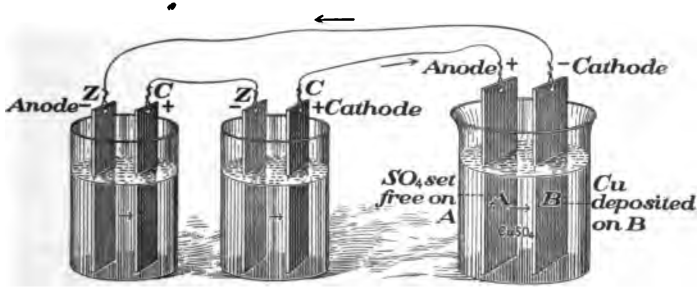

Fig. 332. — Cells arranged for electrolysis of $CuSO_4$.

an *electrolyte*, and the whole arrangement constitutes an *electrolytic cell.*

Experiment. — Let two strips of platinum foil be attached to the ends of wires joined to the poles of a battery of two or more cells. The strip joined to the positive pole or cathode of the battery will be the positive electrode, or anode, of the electrolytic cell. The other one will be the negative electrode, or cathode. When these strips of foil are dipped into a tumbler containing some copper sulphate ($CuSO_4$) dissolved in water, bubbles of oxygen gas will be seen escaping from the positive electrode, and after a short time the negative electrode will be covered with a bright coating of the metal copper. It is, indeed, electroplated with copper. The tumbler with the solution and plates is an electrolytic cell.

When copper sulphate ($CuSO_4$) is dissolved in water, some of its molecules break up into two ions, Cu and SO_4. The Cu ion is positively charged, and the sulphion (SO_4) negatively charged. The negative electrode attracts the + Cu ions and the positive electrode repels them, so that they travel toward the cathode and are there deposited, delivering up their charges to the plate. On the other hand the negative SO_4 ions are urged toward the anode, to which they deliver their negative charges. It

is in this way that all liquids, except molten metals, are supposed to conduct electricity. Pure liquids like water and alcohol are not electrolytes and cannot conduct electricity.

The process of separating a liquid into two parts by passing a current through it is called *electrolysis*.

534. The products of electrolysis appear only at the electrodes, no apparent action occurring in the liquid between them. The final products of the process are often the results of further chemical actions between the deposited ions and the electrodes or the solvent. For example, in the above experiment with platinum electrodes sulphions (SO_4) go to the anode, but they immediately attack the water present, taking its hydrogen to form sulphuric acid (H_2SO_4) and setting its oxygen free. Hence the final products are copper at one electrode and sulphuric acid and oxygen at the other.

The nature of the electrode itself often affects the final results. For instance, in the experiment just described, when the positive electrode is a copper plate instead of a platinum one, the final product at that electrode is copper sulphate ($CuSO_4$), instead of sulphuric acid and oxygen. This result is produced because the sulphions take atoms of copper from the plate itself and with them form copper sulphate. In this way a new molecule of copper sulphate is formed as often as one is destroyed by the electrolysis. The solution remains constant in strength, but the anode loses in weight while the cathode gains.

535. Electroplating. — The principle just described is utilized in electroplating. The object to be plated is made the negative electrode of the electrolytic cell (Fig. 333). The electrolyte is a solution of some salt of the metal with which the object is to be plated, and the

cathode is a plate of the same metal. Thus, in making silver-plated ware, the liquid is a solution of cyanide of silver, and the anode is a piece of solid silver, from which silver passes into solution as fast as it is deposited on the objects being plated. For this reason the solution may be used indefinitely.

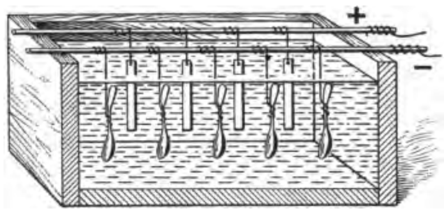

FIG. 333. — Diagram of an electroplating tank.

536. Electrotyping. — Books as a general thing are not printed from type, but from electrotype plates. The page is first set in common type, and an exact impression of it is made in a wax plate. This plate or mold is to be made the negative electrode in an electroplating bath of copper sulphate; but as it is composed of nonconducting material, its surface must first be rendered conducting before the electrolysis can proceed. This is done by sifting finely powdered graphite upon it, and sometimes its surface is sifted over with fine iron filings and then washed with a solution of copper sulphate. This last process deposits a thin film of copper by a chemical method on the plate. The mold is now ready for the plating bath, in which it remains until a sheet of copper has been deposited upon it as thick as a visiting card. This sheet of copper, which is an exact complement of the impression in the wax and hence a copy of the type which made the impression, is now strengthened by having molten type metal poured over its back to the depth of an eighth of an inch.

537. Electrolysis of water. — Since pure water is not an electrolyte, a small quantity of sulphuric acid is added to it to render it conducting. The most convenient means for the electrolysis of water is Hofmann's apparatus

(Fig. 334). If, however, this is not at hand, two test tubes filled with the electrolyte and inverted in a dish of it may be used. The platinum electrodes, sealed in bent glass tubes and connected to the wires of the battery by mercury placed in these tubes, are inserted under the mouths of the test tubes. When the circuit is closed, bubbles of gas form on the electrodes and rise to the top of the tubes, collecting twice as fast over the negative as over the positive electrode. When the tube over the negative is full, the gas in it may be shown to be hydrogen by the application of a lighted match to it. It will burn with almost a colorless flame. The gas in the other tube may be shown to be oxygen by its causing a glowing splinter to burst into flame.

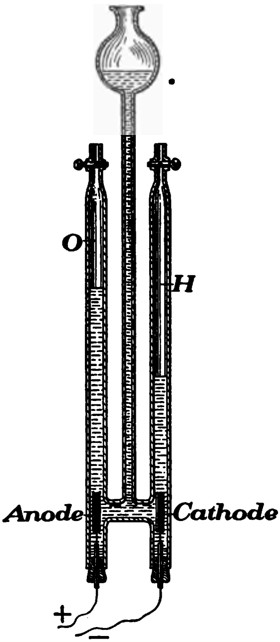

FIG. 334.—Diagram of Hofmann's apparatus for the electrolysis of water.

It is supposed that in this decomposition of water by electrolysis the hydrogen ions of the sulphuric acid (H_2SO_4) pass to the negative electrode, and, giving up their positive charges, appear as hydrogen gas; also that the SO_4 ions pass to the positive electrode, where, giving up their negative charges, they unite with the hydrogen of the water (H_2O) to form a new molecule of sulphuric acid. The oxygen of the water left after the hydrogen has been taken from it rises in bubbles from the electrode. Thus none of the acid, which is the real electrolyte, is destroyed, while the water is in the end

decomposed. Note that the action here described at the positive electrode is the same as that given in § 534.

538. The storage battery. — Experiment. — Let two lead plates, about 4 × 6 in., be fastened to a strip of wood a half inch thick (Fig. 335), and suspended in a jar containing about ten parts of water to one of sulphuric acid. Join two wires to these plates and connect them to a battery of two bichromate cells, placing an ammeter or a galvanometer of low resistance in the circuit. This forms an electrolytic cell, and when the circuit is closed, the water is decomposed as in the last experiment. The oxygen, however, instead of escaping in bubbles, unites with the lead of the positive plate, forming lead peroxide (PbO_2), which covers the plate as a dark brown coating. The hydrogen escapes at the negative plate. While this is going on, the ammeter shows that the current from the battery is diminishing. After a time disconnect the wires from the battery and join them to an electric bell. The bell will now ring, and the ammeter will show that the electrolytic cell is sending a current back through it opposite in direction to the original current.

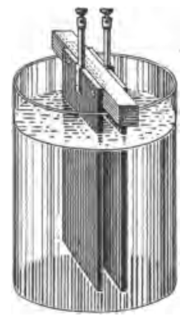

FIG. 335. — Electrolytic cell with lead electrodes, used as a storage cell.

This experiment illustrates the principle of the storage battery. While the current is being sent through it from the battery it is being "charged," and during the electrolytic process chemical potential energy is being stored, *not electricity*. Just as much electricity comes out of the cell at the cathode as passes into it at the anode, but some of the energy of the current remains in the cell and is transformed from a kinetic to a potential form. When the cell is used to ring the bell, it is being "discharged" and is then virtually a voltaic cell. The dark brown plate is the positive plate both during the charging and the discharging of the cell. During the charging an opposing E.M.F. develops in the cell, which opposes and weakens

the charging current and which causes the current of the discharge. The transformation of energy is reversed during the discharge, for the stored potential energy is changed back again into the kinetic energy of the electric current. During this double transformation of energy some energy is wasted, so that the efficiency of the storage battery is only about 75%.

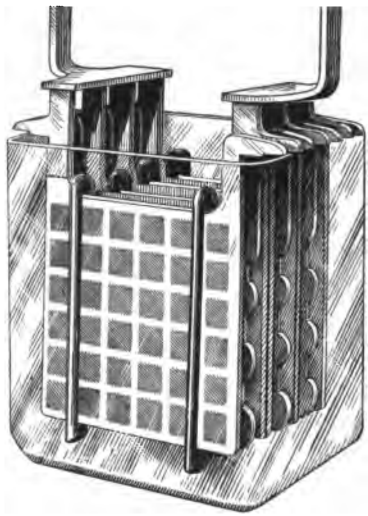

FIG. 336. — Storage cell.

In commercial cells (Fig. 336) the plates are prepared by pressing a paste made of lead oxide and sulphuric acid into interstices in them. In this way a large amount of "active material" is provided. In practice the two electrodes consist of many plates placed close together, positive and negative plates alternately. In this way the internal resistance of the cells is made very small, and large currents can be obtained from them. The E.M.F. of a lead storage cell is usually 2 volts.

539. Faraday's laws of electrolysis. — Faraday determined experimentally two laws of electrolysis (1834).

I. *The quantity of an electrolyte decomposed by electrolysis is directly proportional to the quantity of electricity passing through it.*

Because of this law it is possible to measure the quantity of electricity passing through a circuit by electrolysis ; also if the current is uniform, its strength

may be determined. An electrolytic cell used for the purpose of measuring current strength is called a *voltameter*. A few years ago electric lighting companies measured the electricity used in our homes by means of a voltameter consisting of a little bottle containing a solution of zinc sulphate and two zinc plates. Each month the quantity of electricity used was determined by weighing the negative plate and finding its gain in weight. In fact the ampere itself is defined as that strength of current which under suitable conditions deposits by electrolysis 0.001118 g. of silver per second.

The amount of an element deposited by one ampere in one second is known as the *electro-chemical equivalent* of that element.

II. *The electro-chemical equivalent of an element is proportional to its chemical equivalent.*

For example, 31.7 g. of copper, or 32.5 g. of zinc, or 108 g. of silver are required to take the place of 1 g. of hydrogen in chemical compounds, and hence these numbers are respectively the chemical equivalents of copper, zinc, and silver. A current that sets free 1 g. of hydrogen would in the same time deposit 31.7 g. of copper, 32.5 g. of zinc, or 108 g. of silver.

540. Uses of electrolysis. — Besides the several uses already mentioned, electrolytic processes are now conducted on a great scale for commercial purposes. Copper and other metals are refined by it. The metals aluminum and sodium, and other chemical products, are manufactured by electrolytic processes. The metals sodium and potassium were first discovered by electrolysis by Sir Humphry Davy in 1807.

XI. ENERGY OF THE CURRENT — HEAT EFFECTS

541. The energy of an electric current. — To develop an E.M.F. and to maintain an electric current requires an expenditure of energy. In a voltaic cell chemical potential energy is used for the purpose; in a dynamo, mechanical energy obtained by means of steam engines or water wheels is used. The chemical energy or the mechanical energy is transformed into electrical energy which somewhere in the circuit is given back again, in the form of heat, mechanical energy, or chemical energy.

The energy of the current depends upon two factors: (1) the *quantity* of electricity transferred, and (2) the P.D. of the points between which it is transferred. If $Q =$ the quantity of electricity, and $E =$ the P.D., then

$$\textit{energy of the current} = E \times Q.$$

If the quantity is expressed in coulombs and the P.D. in volts, then the energy is given in joules.

$$\textit{Volts} \times \textit{coulombs} = \textit{joules of work or energy.}$$

542. The power of the electric current means its *rate* of expending or absorbing energy. Since an ampere means one coulomb of electricity per second,

$$\textit{volts} \times \textit{amperes} = \textit{joules of work per second.} \tag{1}$$

By definition (§ 92) one joule of work per second = one watt. Hence,

$$\textit{volts} \times \textit{amperes} = \textit{watts, or } EI = \textit{watts.} \tag{2}$$

Since volts × amperes = joules per second, volts × amperes × seconds = the total amount of work done in any number of seconds, or in symbols,

$$EIt = J, \tag{3}$$

J representing work or energy in joules.

If the energy of a current is used simply in overcoming resistance, we may substitute RI for E (§ 495) in the above equations. We thus obtain from equation (3)

$$I^2Rt = J.$$

If we wish to express electrical power in horse power, we may do so by dividing the number of watts obtained by 746, since 746 watts equal one horse power. Electrical machinery is usually rated in kilowatts. A kilowatt equals 1000 watts and is almost exactly $\frac{4}{3}$ of a horse power.

Problems

1. How many amperes represent a horse power when the P.D. is 2 volts?

2. How many volts P.D. are necessary for a horse power when the current is 2 amperes?

3. An ordinary 16 candle power incandescent lamp requires 0.5 ampere of current when the P.D. between its terminals is 110 volts. How many watts are required to operate the lamp and how many per candle power?

4. A certain building is lighted by 1017 such lamps. A steam engine of what horse power would be required to furnish the current when all of the lamps are in use, provided no energy were wasted?

5. In an Edison station the dynamos are rated at 6000 kilowatts, and the P.D. between their terminals is 4600 volts. How many amperes does a dynamo furnish when running at its full capacity?

6. What horse power is required to maintain a current of 4 amperes through a resistance of 373 ohms?

7. An arc lamp requires a current of 9.75 amperes and the P.D. between its terminals is 50 volts. What power does it absorb?

8. A 10 horse power engine is required to run a dynamo which maintains a current of 16 amperes through an external resistance of 25 ohms. Show that the efficiency of the dynamo is about 86 %.

9. How many joules of energy are absorbed by a current of 1 ampere flowing for 10 sec. through a wire having 50 ohms of resistance? By a current of 2 amperes? 4 amperes? 10 amperes?

10. How large a steam engine is necessary to maintain a P.D of 220 volts between two points connected by a conductor having a resistance of 4 ohms?

543. Heat produced by the electric current. — All that part of the energy of the electric current which is used simply in overcoming the resistance of the conductor is converted into heat by the resistance. This conversion of kinetic electrical energy into heat by resistance is analogous to the transformation of mechanical energy into heat by friction. We have already studied how some of the energy of the current is transformed into mechanical energy by the electric motor, also how it may be converted into chemical energy by electrolysis.

The heating effect of a current may be illustrated by joining the terminals of a bichromate cell or of a dry cell by a very short, thin platinum wire. This will be heated to incandescence by the current.

Experiment. — Let a thin iron wire about 1 m. long be stretched between two supports and a current passed through it. At first if the current is not very strong, the only observable effect will be a sagging of the wire because it expands by the heat. If, however, the wire is in the lighting circuit of the building or is connected to a storage battery and the current can be gradually increased, it may be made white-hot or even easily melted.

Instead of using an iron wire a meter long, join a thin copper wire of the same size to the end of the iron wire by twisting their ends together and stretch the two between the supports so that about a third of the length is iron and the remainder is copper. If the current is sent through this, the iron part may be made red-hot or even melted, while the copper wire is warmed very little. The iron offers more resistance than the copper and for that reason is heated more.

544. Joule's law. — We have learned (§ 370) that 4.187 joules of mechanical energy are equivalent to 1 gram-calorie of heat; 1 joule must therefore be equal to $\frac{1}{4.187}$ of a calorie, or in the decimal form to 0.2388 calorie. It is

customary to use the number 0.24 instead of 0.2388 as the equivalent of a joule in heat.

Since the energy expended by a current in t seconds in overcoming resistance is expressed in joules by I^2Rt, then $0.24 \times I^2Rt$ gives the amount of heat produced by the current in gram-calories. Let H represent the heat thus produced; then,

$$H = 0.24\ I^2Rt.$$

This equation teaches that

The heat developed in a conductor by an electric current is proportional (1) *to the square of the current strength,* (2) *to the resistance of the conductor, and* (3) *to the time the current flows.*

These facts were worked out experimentally by Joule in 1841, and the above statement is known as Joule's law.

545. Practical applications of the heating effect of the electric current are very numerous. Electric flatirons, soldering irons, griddle plates, and many other devices are in common use. These are heated by currents passing through coils of wire of considerable resistance which are placed in them. Houses are sometimes heated and cooking is done by the heat of the electric current. Electric welding is also one of the practical applications of this principle. To protect electric wires from the heat of too strong currents, fuses, consisting often of short pieces of easily melted wire, are placed in the circuit. Should the current rise above a given limit, these fuses melt or "blow" and the circuit is broken.

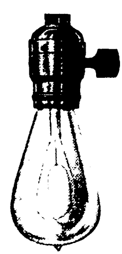

FIG. 337.—Incandescent electric lamp.

546. The incandescent electric lamp (Fig. 337) is the most common example of the heating effect of the electric current. It consists of a fine thread or filament of carbon, resembling a

horsehair, which is placed in a glass bulb. The ends of this filament are joined to two platinum wires which are fused into the end of a glass tube entering the bulb. These platinum wires are joined to two brass parts at the base of the lamp which are insulated from each other. When the lamp is screwed into its socket, these two brass pieces are brought into electrical connection with the two wires leading to the dynamo. Recently lamps have been produced in which the filament consists of thin wires made of the metal tungsten and other rare metals.

The filament has a high resistance, so that a small current passing through it heats it to incandescence. The air is exhausted from the bulb, otherwise the oxygen would unite with the filament and burn it up.

An ordinary 16 candle power lamp used on a 110-volt circuit requires about 0.5 of an ampere. This by Ohm's law gives the resistance of the lamp as 220 ohms when the lamp is hot, and the rate of consumption of energy as 55 watts, or about 3.5 watts per candle power. The lamps with metallic filaments require from 1 to 2 watts per candle power.

547. The Nernst lamp has, instead of a carbon filament, a tiny rod or glower, composed of oxides of rare earths, which is heated by the current passing through it. This glower is a nonconductor when it is cold and must be heated in some way before it will operate. This is usually done by some coils of fine platinum wire placed near the glower and heated by the current passing through them; after the lamp begins to glow, the current is cut out of the platinum coils and passes only through the glower. The glower of the Nernst lamp is incombustible and does not need to be inclosed in a bulb. It requires about 1.7 watts per candle power.

548. The electric arc. — When two conductors which are connected to the poles of a large battery or of a dynamo are brought together for an instant and then

separated a short distance, a kind of electric flame, called the electric arc, is produced between them. This was discovered by Sir Humphry Davy and was exhibited by him on a grand scale at the Royal Institution in 1808. He used two pieces of charcoal for his conductors, which were joined to a battery of about 2000 cells. The heat and the light of the electric arc are very intense. So great is the heat that the most refractory substances are melted and vaporized in it. It is the vapor thus formed at the end of the negative conductor that forms the arc and transmits the current across the space. The temperature of the arc is between 3500° and 4000° C.

549. The arc lamp (Fig. 338) consists of an electric arc between two rods of dense carbon. The greater part of the light comes from the ends of the carbons, which are heated to a very high temperature, the positive being much hotter than the negative. In the tip of the positive carbon there is a small indentation, called the crater, which is the brightest part of the arc. In the open air the carbons burn away quite rapidly, the positive about twice as fast as the negative. "Inclosed arc" lamps have a small globe, nearly air-tight, surrounding the arc, which prevents the free access of oxygen, and the carbons last about ten times as long as in the open arc. As the carbons waste away the arc lengthens until it soon breaks, unless the distance between them is readjusted. If the arc is broken, the carbons must be brought together again to reëstablish it. The movement of the carbons is usually controlled by the current itself through an electro-

FIG. 338.—Poles of an arc lamp.

magnetic mechanism too complicated to be described here. Such lamps are called "automatic" lamps. So called "hand-feed" lamps, in which the carbons are moved by the hand, are often used in electric stereopticons or lanterns.

550. Loss of energy by transmission.—Suppose a house to be lighted by 100 incandescent lamps, each requiring a current of 1 ampere and a P.D. of 60 volts. By Ohm's law the resistance of each lamp will be 60 ohms. Suppose the line wires or feeders connecting the lamps to the dynamo to have a resistance of 1 ohm, what is the loss of potential or "drop" in the line when one lamp is in use? What is it when all of the lamps are in use? When one lamp is in use, the current is 1 ampere and the resistance of the line is 1 ohm. By Ohm's law, $E = RI$, or $1 \times 1 = 1$; that is, the loss of potential in the line is 1 volt. The dynamo must therefore produce a P.D. of 61 volts to maintain a P.D. of 60 volts at the lamp. The energy is consumed in the lamp at the rate of 60 watts ($I \times E = 1 \times 60 = 60$ watts); in the line, at the rate of 1 watt ($1 \times 1 = 1$ watt). This means that 60 times as much energy is consumed in the lamp as in the line.

When all the lamps are in use, the current required is 100 amperes, for the lamps are in parallel, and each lamp requires 1 ampere. The potential drop in the line is $RI = 1 \times 100$ volts. The energy is consumed in the line at the rate of $IE = 100 \times 100 = 10{,}000$ watts; the energy is consumed in the lamps at the rate of $IE = 100 \times 60 = 6000$ watts. The dynamo must therefore deliver to the circuit energy at the rate of 16,000 watts, more than half, or $62\frac{1}{2}$%, of which is wasted in heat in the wires leading to the lamps. This loss may be reduced by making the line wires larger so as to reduce the resistance; but if the distance becomes greater and the number of lamps increase, the weight and cost of the copper wire necessary to transmit the energy without undue waste become so great as to make this method of distribution practically impossible.

551. A more economical method.—It is to be noted that the energy of a circuit does not depend on one thing alone, but upon two factors; not upon the amperes solely nor on volts alone, but upon the product of the volts by the amperes. $EI =$ watts. In the problem just considered, the greater demand for energy was met by an increase of the current factor or the amperes. This we found to be a wasteful method. However, the watts can be increased by increasing the P.D. or volts factor. Let us consider this method.

Suppose the P.D. at the terminal of each lamp is 240 volts, and that a lamp requires 0.25 ampere; each lamp would then consume energy at the same rate as before; 240 volts × 0.25 ampere = 60 watts. When one lamp is in use, the loss of potential in the line is $RI = 1 \times 0.25 = 0.25$ volt; and the energy is consumed in the line at the rate of $IE = 0.25 \times 0.25 = 0.0625$ watts. This is $\frac{1}{16}$ of what it was before.

When all the lamps are in use, the current required is $100 \times .25 = 25$ amperes. The drop in the line is $RI = 1 \times 25 = 25$ volts, and the energy is consumed in the line at the rate of 25 volts × 25 amperes = 625 watts. This again is $\frac{1}{16}$ of what it was before. The lamps in both cases require the same amount of energy. These examples show that by increasing the P.D. 4 times with a corresponding decrease in the current strength, the same amount of energy may be transmitted over the same wire with a loss $\frac{1}{16}$ as great.

Joule's law, $H = I^2Rt \times .24$, also shows that the energy lost in transmission increases as the *square of the current* and as the resistance. Hence the transmission of large amounts of energy over long distances cannot be done efficiently by large currents and small differences of potential.

552. Long-distance transmission of electrical energy. — A dynamo which furnishes a current of 1000 amperes under a pressure of 10 volts delivers energy at the same rate as one furnishing 10 amperes under a pressure of 1000 volts; but we have seen that in the latter case the energy may be carried to a distance with much greater efficiency. Successful transmission of great power over long distances depends on the possibility of using very high potentials. Difficulties of insulation limit the potentials to be used, but pressures of 70,000 volts are now successfully employed.

By alternating currents and "step-up" transformers, very great E.M.F.'s are generated; and again at a distant station, by "step-down" transformers, the P.D. may be reduced to a low voltage suitable for operating motors and lights.

In 1891, 140 horse power were transmitted from Lauffen

to Frankfurt, Germany, 117 miles, by three small wires, with a net loss of only 26%. This was the first attempt at electrical transmission over a long distance on a large scale. Since that time there has been a wonderful development in the use of the alternating current for transmission of electrical energy. For example, at Niagara Falls generators develop a pressure of 2300 volts, which is transformed to 22,000 volts for transmission to Buffalo (22 mi.), where it is again transformed to low voltage for operating street cars, electric lights, and factories. Power is also transmitted over a line about 200 miles long from Niagara to Syracuse, N.Y., the pressure being 60,000 volts.

Problems

1. The main wires of a house lighted by electricity are often maintained at a constant P.D. of 120 volts, and each lamp which connects these two wires has a resistance of about 240 ohms. By applying Ohm's law and Joule's law, explain why, when the lamp is "short circuited," some portion of the circuit is melted or burned out. Short circuiting means connecting the terminals or the mains by something which has little resistance.

2. If the P.D. between the carbons of an arc lamp is 45 volts, and the current is 9.75 amperes, at what rate is the lamp consuming energy? How many such lamps could be operated by a 100 H.P. engine, if 20% of the energy is wasted?

3. How much heat is produced by a current of 10 amperes flowing for 30 min. through a resistance of 180 ohms?

4. How much heat would be produced if, in the last example, the current is 20 amperes instead of 10?

5. If the E.M.F. remains the same, show that the amount of heat generated in the circuit is doubled when the resistance is reduced one half.

6. A 110-volt lamp requires 0.5 ampere, and has a resistance of 220 ohms. How much heat will be generated by such a lamp in an hour?

7. Suppose a coil of wire of 3.5 ohms' resistance is placed in 3 l. of water at 20° C. What will be the temperature of the water after 4 amperes has been flowing through the coil for 10 minutes?

8. A current passed through a coil of wire having a resistance of 40 ohms which was placed in 200 g. of water. After 10 min. the temperature of the water had risen 15° C. What was the current strength?

9. In Problems 3, 7, and 8, find the power absorbed in watts.

10. How many 16 c.p. lamps can be run by a 4-kilowatt dynamo, each lamp requiring 3.5 watts per c.p.? If the lamps are 50-volt lamps, what current does it take?

11. A building is lighted by 100 incandescent lamps, each requiring 1 ampere and a P.D. of 60 volts. Suppose the line wires or feeders connecting the wires with the dynamo have a resistance of 0.6 ohm. What is the loss of potential or "drop" in the line when one lamp is in use? What portion of the energy of the current is consumed in the lamp and in the line?

12. In the last problem, what is the loss of potential when all of the lamps are in use? (100 lamps require 100 times the current that 1 lamp does.) How does the energy consumed in the line compare with that consumed in the lamps?

XII. RÖNTGEN RAYS — WIRELESS TELEGRAPHY

553. The electric discharge through rarefied gases has in recent years become of great importance. The gas through which the discharge takes place is contained in a glass tube (Fig. 339) in the ends of which two platinum wires are sealed. These wires are connected to the terminals of the secondary of a large induction coil or to those of a static machine, and form the electrodes by which the electricity enters and leaves the tube. The one by which it enters is

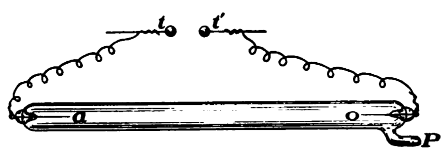

FIG. 339. — Tube for illustrating an electric discharge through a rarefied gas.

the positive, or anode, and the other is the negative, or cathode. These wires or electrodes often terminate at their inner ends in a metal disk or ball.

The discharge occurs much more easily through a partial vacuum than through air at ordinary pressure. This may be shown by connecting such a tube to an air pump at P. At first sparks will pass between t and t', but when the air in the tube becomes rarer the discharge takes the longer path from a to o. The passage of the electricity produces some very beautiful and peculiar luminous effects, which vary considerably with the degree of exhaustion of the tube and the nature of the residual gas. Tubes made in many forms similar to that shown in Figure 340, in which the exhaustion is carried to about $\frac{1}{500}$ of an atmosphere, are called Geissler tubes. The beauty of the color effects is enhanced by making parts of the tube of glass different colors and shapes.

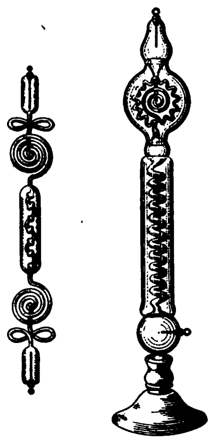

FIG. 340. — Geissler tubes.

554. Crookes tubes and cathode rays. — Sir William Crookes first investigated tubes in which the exhaustion was carried to an extreme degree, perhaps to the millionth part of an atmosphere. Such tubes are called Crookes tubes, and in them the discharge assumes an entirely different character. From the cathode there is projected a stream of negatively electrified corpuscles, called electrons, which are believed to be very minute parts of the atoms of matter. These streams of particles constitute the so-called *cathode rays*. They proceed in straight lines at right angles to the surface of the cathode. This is shown by the fact that a concave cathode focuses them at a point

as a mirror does light and also by the sharp shadows which pieces of mica produce when placed in their path (Fig. 341). They may, however, be turned aside by a magnet (Fig. 342).

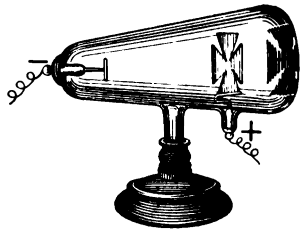

FIG. 341. — Crookes tube, showing shadow cast by a unica cross in cathode rays.

Cathode rays produce a faint glow in the traces of gas left in the tube, and brilliant fluorescent effects in objects upon which they strike. If they fall upon the walls of the tube, the glass glows usually with a greenish yellow light, although the color depends upon the nature of the glass. The impact of the rays upon certain gems and on various sulphides produces beautiful fluorescent colors.

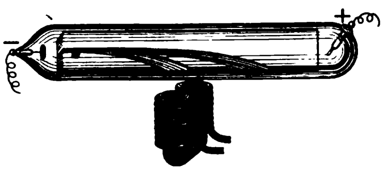

FIG. 342. — Crookes tube, showing cathode rays deflected by a magnet.

Another important characteristic of cathode rays is the production of heat in the objects upon which they strike. The walls of the tube may be melted by them; and if they are concentrated on a piece of platinum, it may be raised to a white heat or even melted.

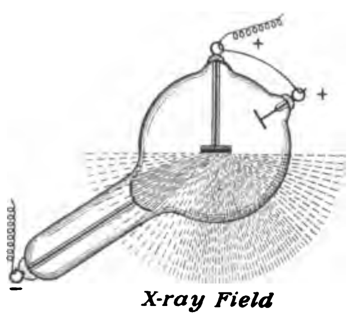

FIG. 343. — Röntgen tube.

555. Röntgen rays, or X-rays. — When cathode rays strike a solid, they give rise to another kind of rays, called *Röntgen rays* or *X-rays* which start from

the surface upon which the cathode rays fall. Figure 343 represents an X-ray tube, showing the curved cathode and the plate from which the X-rays emanate.

X-rays are like cathode rays in that they produce fluorescence in the glass walls of the tube and also in other substances upon which they fall; like them also they proceed in straight lines. They further resemble cathode rays in not being reflected, refracted, or polarized as light is. X-rays are unlike cathode rays because they are not deflected by a magnet or by an electrostatic charge, and the latter have less penetrating power than the former.

From these and other considerations it seems certain that they do not consist of electrically charged particles, nor on the other hand are they composed of trains of ether waves like light. It is thought, however, that they may consist of irregular wave-like pulses in the ether.

556. X-ray photographs. — X-rays affect the photographic plate in the same manner as light, but they penetrate many substances which are opaque to light. Dense substances like iron and zinc cut off the rays, while light substances like wood and flesh are transparent to them. When the hand is placed between a photographic plate and the X-ray tube, a shadow picture (Fig. 344) of the more opaque parts, such as the bones, is produced on the plate.

In the *fluoroscope*, which consists of a darkened box having an opening fitting closely about the eyes, there is a card covered with a fluorescent salt. When the hand is placed between the tube and this card, the X-rays cause the card to glow except where they are cut off by the bones of the hand or by other dense substances. Thus the shadow of the bones of the hand is at once visible on the card.

557. Electric waves. — In 1864 James Clerk-Maxwell of Cambridge, England, developed mathematically a theory of electro-magnetic waves propagated through the ether with the velocity of light. In 1888 Heinrich Hertz of Germany proved the existence of such waves experimentally. He produced reflection, refraction, interference, and other phenomena with them which are exactly similar to those we studied under the subject of light, and moreover he showed the velocity of propagation of these waves to be the same as that of light.

FIG. 344. — X-ray picture of a child's hand.

The only difference between light waves and electric waves appears to be that of length. The longest light waves that are capable of affecting the human eye have a length of about 0.00008 cm.; the shortest electric waves so far measured are about 0.3 cm. long. They vary in length from that to many meters. Intermediate between the longest light and the shortest electric waves are certain so-called heat waves, the longest of which yet detected are about 0.006 cm. All of these waves, however, long or short, are of the same kind, and are regarded as

electro-magnetic waves. The honor of establishing the electro-magnetic theory of light belongs to Maxwell and Hertz. To them also must be given the credit for opening the way for wireless telegraphy.

Joseph Henry, an American, discovered in 1842 that the spark discharge of a Leyden jar is oscillatory; that is, the electricity does not pass once merely from one coating to the other, but surges back and forth at a rate perhaps of several million times per second. Spark discharges of an induction coil are also oscillatory. Such oscillatory discharges set up electric waves in the ether. For his experiments Hertz used spark discharges produced by an induction coil between polished metal balls; and electric waves for wireless telegraphy are produced in this manner.

558. Electrical resonance. — We have seen (§ 214) how sound waves in air produced by one tuning fork will cause another fork to vibrate, provided the two forks have the same period of vibration. We called this resonance. In a strictly analogous way, electrical waves caused by the oscillatory discharge of one Leyden jar will cause an oscillatory discharge in another jar, provided the rates of the two jars and their discharging circuits are the same. This may be shown by the following experiment:

Experiment. — Two exactly similar Leyden jars, *A* and *B* (Fig. 345), are fitted with rectangular loops of wire which join the inner and outer coatings. The rectangle of *A* is adjustable in size by a slider *S*, and that of *B* is broken by a spark gap at *bb*. A strip of tin foil extends from the inner coating of *A* up over the edge of the jar and down the outside almost to the

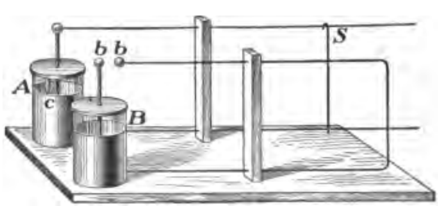

FIG. 345. — Apparatus for illustrating electrical resonance.

outer coating, a gap of about a millimeter being left between them at *c*. The jars are placed side by side with the loops of wire parallel to each other, and the two coatings of *B* are joined to the terminals of an induction machine or an induction coil. Every discharge of the jar *B* across the spark gap *bb* produces surgings in the rectangle of *B* and sends out electrical waves toward *A*. When the jars and rectangles are tuned to unison by moving the slider *S*, these waves produce similar oscillations in jar *A*, which become strong enough to make *A* overflow and produce a spark at *c*.

559. Wireless telegraphy. — Electric waves in ether are utilized for sending messages without wires. To send a message some instrument must be used to set up such waves; and to receive the message, one capable of detecting electric waves is necessary. The sending instrument, often called an oscillator, consists of a powerful induction coil whose terminals are connected to two polished metal balls separated a short distance from each other. The oscillatory discharge between these terminals sets up the waves in the ether. To intensify these waves Hertz joined a square metal plate to each of these balls (Fig. 346); but later Marconi invented the method of joining one of them to a high mast or tower of wires, the other being connected with the earth.

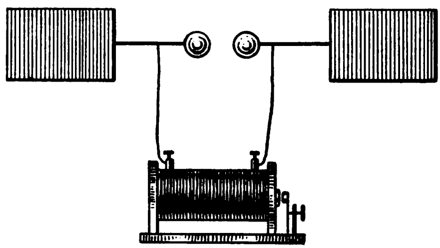

FIG. 346. — Hertz oscillator for sending out electric waves.

560. The coherer. — Although there are several methods of detecting electric waves, the coherer is the one that has been most used thus far. It is very simple in construction and was invented by Branley in 1890. It consists of a glass tube *C* (Fig. 347), perhaps as large

in diameter as an ordinary lead pencil and half as long. Two metal plugs are pushed into this tube, a small space being left between them. This place is nearly filled with filings of nickel and silver or other metals. When these plugs and the filings between them are made a part of the circuit of a battery, very little current flows in the circuit, because the filings offer great resistance. This may be shown by placing a galvanometer in the circuit. But when electric waves fall upon the circuit, the resistance of the filings is greatly reduced and the current is much stronger. In this way the waves are detected. The filings are restored to their original or non-conducting condition by tapping the tube after the passage of a current.

FIG. 347. — Diagram of a coherer.

Figure 348 shows the arrangement of the apparatus for receiving wireless messages. A battery, *B*, and a relay, *R*, are included in the coherer circuit. The relay brings into action another battery, *B′*, having in its circuit a telegraph sounder, *S*, and an electric bell, *L*. The hammer of the latter acts as a tapper or *decoherer* to shake up the filings after each signal has been received. The bell and sounder may be

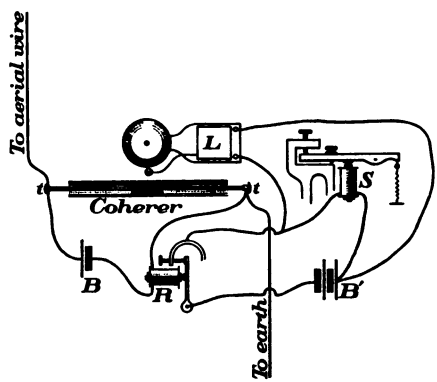

FIG. 348. — Diagram of receiving apparatus for wireless telegraphy.

in series instead of in parallel, or the sounder may be dispensed with, the bell acting as sounder and decoherer.

For illustrating wireless telegraphy in the school room, metal plates are often connected to each side of the spark gap of the induction coil of the sending apparatus, as in the Hertz oscillator (Fig. 346), and two similar plates are joined to the terminals, *t*, *t* (Fig. 348), of the coherer.

If these plates are not used, a vertical wire ten or twelve feet high takes the place of one of them in both sending and receiving instruments, and a wire connecting the terminal to the earth through a gas or water pipe takes the place of the other plate in each case.

A spark coil of an automobile may be used for the induction coil.

APPENDIX

VARIATION AND PROPORTION IN PHYSICS

In order to explain many phenomena in physics, it is necessary to make measurements and by means of them to find the relations existing between the various phenomena. These relations give rise to many physical laws which are often expressed in the form of a variation or a proportion. To illustrate these subjects, let us consider the wages, time, and amount earned by a laborer, representing the quantities respectively by w, t, and a.

I. When two quantities increase or decrease at the same rate, *i.e.* in the same ratio, one of them is said to vary as the other. It is obvious, for instance, that with fixed or constant wages, the amount earned by a man will increase or decrease in the same ratio as the time he works. If his time is doubled, or trebled, or halved, the amount earned will also be doubled, or trebled, or halved; and therefore we say the amount earned varies *directly* as his time. Using $\propto$, the symbol of variation, we express it as follows: —

$$(1) \quad a \propto t.$$

This can be expressed in another way. Since t and a change at the same rate, it is evident that the ratio $\frac{a}{t}$ must remain the same for different values of a and t, or, as it is expressed in mathematics, $\frac{a}{t} =$ a constant. Hence the law, *When the quotient of one variable divided by another is constant, each varies directly as the other.*

Every variation may be reduced to a proportion. If $a \propto t$, then $\frac{a}{t} = k$, a constant; and $\frac{a'}{t'} = k$ also, a' and t' being new values for amount earned and time. Since $\frac{a}{t}$ and $\frac{a'}{t'}$ both equal k, they are equal to each other. Hence $\frac{a}{t} = \frac{a'}{t'}$, or $a : t = a' : t'$.

We say, therefore, that the amount earned varies directly as the time the man works; or we may express the same thought by saying

the amounts a man earns are directly proportional to the times he works. Sometimes a law in physics is expressed in one way and sometimes in the other.

II. Again, let us compare the wages with the time, the amount earned being a fixed or constant quantity. It is evident that if a man's wages are increased, the time required to earn the fixed amount is decreased in the same ratio; if his wages are multiplied, the time is divided by the same quantity. In such a case we say the time varies *inversely* as the wages, and express it thus:—

$$(2) \quad t \propto \frac{1}{w}.$$

As in the other case, $t \div \frac{1}{w}$, or $t \times w$, equals a constant quantity. Hence the law: *When the product of two variables is constant, each varies inversely as the other.*

This variation may be put in the form of a proportion. Let $t =$ the time when the wages are w, and $t' =$ the time when the wages are w'. Then $t \times w = t' \times w'$ because both products are equal to the same constant. By taking two of these as extremes and the other two as means we have the proportion $t : t' = w' : w$, or, the times required to earn a fixed sum are inversely proportional to the wages received.

III. When one quantity varies *separately* as two others, it varies *jointly* as their product.

To illustrate this, let $t =$ the number of days required to dig a ditch; l, number of feet in the length of the ditch; and h, the number of hours per day the laborer works. It is evident that other things being equal the time required for the work will vary as the length of the ditch, or $t \propto l$. It is also plain that the number of days to do the work will increase in proportion as the number of hours worked per day diminishes, or $t \propto \frac{1}{h}$.

We wish to prove that if l and h both vary at the same time, t will vary as $l \times \frac{1}{h}$, that is, $t \propto \frac{l}{h}$, or that $t : \frac{l}{h} = t' : \frac{l'}{h'}$.

To do this, let us suppose that l and h vary separately, t first becoming t_1 when l becomes l', and then afterward t_1 changing to t' when h changes to h'. Expressing these changes by proportion, we have

$$t : l = t_1 : l', \text{ and } t_1 : \frac{1}{h} = t' : \frac{1}{h'}.$$

Multiplying these two proportions term by term and cancelling t_1 out of the first term of each ratio, we have

$$t : \frac{l}{h} = t : \frac{l'}{h'}, \text{ that is } t \propto \frac{l}{h}.$$

THE TOEPLER-HOLTZ MACHINE EXPLAINED

The Holtz machine (Fig. 237, page 358), as modified by Toepler and later by Voss, consists of two vertical glass plates, one of which is stationary while the other revolves in front of it. Upon the back of the fixed plate are two paper sectors, A and B, called *armatures*, each covering two disks of tin foil connected by a strip of tin foil. Upon the front of the revolving plate are a number of equidistant tin-foil disks called *carriers*, which are surmounted by brass buttons. Two brass rods, r and r' (Fig. 238), are fastened to the fixed plate, being connected to the tin-foil disks under the armatures; these curve around in front of the revolving plate, and carry at their ends tinsel brushes which rub upon the brass buttons as they pass by. The neutralizing rod bb, placed diagonally in front of the moving plate and opposite the armatures, has tinsel brushes, which touch the passing buttons, and also a number of sharp metallic points which are directed toward the plate, but do not touch it or the carriers.

On a horizontal diameter in front of the revolving plate and opposite the paper armatures are two collecting combs, CC', having sharp teeth pointing toward the plate. These are connected to conductors and the discharging rods KK.

Action of the Toepler-Holtz. — (Figure 238 is a diagram of the machine, the corresponding parts having the same letters as in Fig. 237.) In operation the armature A has at first a small initial charge — suppose it negative — which may be caused by friction of the brushes or otherwise. This charge acts by electrical influence upon the neutralizing rod bb, making its upper end negative and its lower end positive; and each carrier passing this neutralizing rod carries away a charge, at the

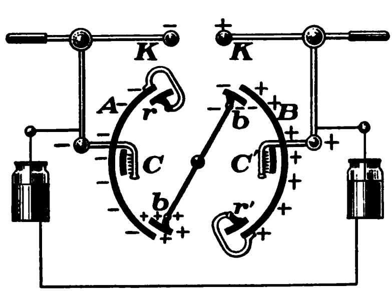

FIG. 238. — Diagram of the Toepler-Holtz machine.

top negative and at the bottom positive. These carriers as they pass the tinsel brushes r and r' give some of their charges to the armatures,—positive to B and negative to A. In this way the armatures A and B become strongly charged, and they soon take very little from the passing carriers, which go on and deliver their charges by the combs CC' to the conductors and the discharging rods, one side C receiving negative and C' positive. Both armatures, after the machine is started, act by influence on the neutralizing rod, one helping the other.

When the knobs of the discharging rods are separated, the two charges accumulate upon them until they are discharged by a spark passing between them. The capacity of these conductors and the intensities of the sparks are greatly increased by Leyden jars, usually attached to each side of the machine.

The Morse Alphabet

A – —	H – – – –	O – –	U – – —
B — – – -	I – –	P – – – – –	V – – – —
C – – –	J — — — –	Q – – — -	W – — —
D — – –	K — – —	R – – –	X – — – —
E –	L ——	S – – –	Y – – – –
F – — –	M — —	T —	Z – – – –
G — — –	N — –		

TABLE I. NATURAL SINES AND TANGENTS

Angle	Sine	Tangent	Angle	Sine	Tangent	Angle	Sine	Tangent
0	0.000	0.000	31	0.515	0.601	62	0.883	1.881
1	0.017	0.017	32	0.530	0.625	63	0.891	1.963
2	0.035	0.035	33	0.545	0.649	64	0.899	2.050
3	0.052	0.052	34	0.559	0.675	65	0.906	2.145
4	0.070	0.070	35	0.574	0.700	66	0.914	2.246
5	0.087	0.087	36	0.588	0.727	67	0.921	2.356
6	0.105	0.105	37	0.602	0.754	68	0.927	2.475
7	0.122	0.123	38	0.616	0.781	69	0.934	2.605
8	0.139	0.141	39	0.629	0.810	70	0.940	2.747
9	0.156	0.158	40	0.643	0.839	71	0.946	2.904
10	0.174	0.176	41	0.656	0.869	72	0.951	3.078
11	0.191	0.194	42	0.669	0.900	73	0.956	3.271
12	0.208	0.213	43	0.682	0.933	74	0.961	3.487
13	0.225	0.231	44	0.695	0.966	75	0.966	3.732
14	0.242	0.249	45	0.707	1.000	76	0.970	4.011
15	0.259	0.268	46	0.719	1.036	77	0.974	4.331
16	0.276	0.287	47	0.731	1.072	78	0.978	4.705
17	0.292	0.306	48	0.743	1.111	79	0.982	5.145
18	0.309	0.325	49	0.755	1.150	80	0.985	5.671
19	0.326	0.344	50	0.766	1.192	81	0.988	6.314
20	0.342	0.364	51	0.777	1.235	82	0.990	7.115
21	0.358	0.384	52	0.788	1.280	83	0.993	8.144
22	0.375	0.404	53	0.799	1.327	84	0.995	9.514
23	0.391	0.424	54	0.809	1.376	85	0.996	11.43
24	0.407	0.445	55	0.819	1.428	86	0.998	14.30
25	0.423	0.466	56	0.829	1.483	87	0.999	19.08
26	0.438	0.488	57	0.839	1.540	88	0.999	28.64
27	0.454	0.510	58	0.848	1.600	89	1.000	57.29
28	0.469	0.532	59	0.857	1.664	90	1.000	Infinity.
29	0.485	0.554	60	0.866	1.732			
30	0.500	0.577	61	0.875	1.804			

TABLE II. SIZE OF WIRE, ETC. AMERICAN GAUGE (B. & S.)

Gauge No.	Size				Pure Copper Wire			
					Weight		Resistance at 24° C. or 75° F.	
	Diameter in		Square of Diameter in Mils.	Area in Sq. In.	Lbs. per 1000 Ft.	Ft. per Lb.	Ohms per 1000 Ft.	Ohms per Lb.
	Mils.	Millim.						
0000	460.00	11.684	211600.0	.166191	639.33	1.56	.051	.0000798
000	409.64	10.405	167805.0	.131790	507.01	1.97	.064	.000127
00	364.80	9.266	133079.4	.104590	402.09	2.49	.081	.000202
0	324.95	8.254	105592.5	.082932	319.04	3.13	.102	.000320
1	289.30	7.348	83694.2	.065733	252.88	3.95	.129	.00051
2	257.63	6.544	66373.0	.052130	200.54	4.99	.163	.000811
3	229.42	5.827	52634.0	.041339	159.03	6.29	.205	.001288
4	204.31	5.189	41742.0	.032784	126.12	7.93	.259	.00205
5	181.94	4.621	33102.0	.025998	100.01	10.00	.326	.00326
6	162.02	4.115	26250.5	.020617	79.32	12.61	.411	.00518
7	144.28	3.665	20816.0	.016349	62.90	15.90	.519	.00824
8	128.49	3.264	16509.0	.012766	49.88	20.05	.654	.01311
9	114.43	2.907	13094.0	.010284	39.56	25.28	.824	.02083
10	101.89	2.588	10381.0	.008153	31.37	31.88	1.040	.03314
11	90.74	2.305	8234.0	.006467	24.88	40.20	1.311	.05269
12	80.81	2.053	6529.9	.005129	19.73	50.69	1.653	.08377
13	71.96	1.828	5178.4	.004067	15.65	63.91	2.084	.13321
14	64.01	1.628	4106.8	.003147	12.41	80.59	2.628	.2118
15	57.07	1.450	3256.7	.002558	9.84	101.63	3.314	.3368
16	50.82	1.291	2582.9	.002029	7.81	128.14	4.179	.5355
17	45.26	1.150	2048.2	.001609	6.19	161.59	5.269	.8515
18	40.30	1.024	1624.3	.001276	4.91	203.76	6.645	1.3536
19	35.89	.899	1252.4	.000984	3.78	264.26	8.617	2.2772
20	31.96	.812	1021.5	.000802	3.09	324.00	10.566	3.423
21	28.462	.723	810.1	.000636	2.45	408.56	13.323	5.443
22	25.35	.644	642.7	.000505	1.94	515.15	16.799	8.654
23	22.57	.573	509.5	.000400	1.54	649.66	21.185	13.763
24	20.10	.511	404.0	.000317	1.22	819.21	26.713	21.885
25	17.90	.455	320.4	.000252	.97	1032.96	33.684	34.795
26	15.94	.405	254.0	.000199	.77	1302.61	42.477	55.331
27	14.19	.361	201.5	.000158	.61	1642.55	53.563	87.979
28	12.64	.321	159.8	.000125	.48	2071.22	67.542	139.893
29	11.26	.286	126.7	.000100	.38	2611.82	85.170	222.449
30	10.03	.255	100.5	.000079	.30	3293.97	107.391	353.742
31	8.93	.227	79.7	.000063	.24	4152.22	135.402	562.221
32	7.95	.202	63.2	.000050	.19	5236.66	170.765	894.242
33	7.08	.180	50.1	.000039	.15	6602.71	215.312	1421.646
34	6.30	.160	39.7	.000031	.12	8328.30	271.583	2261.82
35	5.61	.143	31.5	.000025	.10	10501.35	342.443	3596.104
36	5.00	.127	25.0	.000020	08	13238.83	431.712	5715.36
37	4.45	.113	19.8	.000016	.06	16691.06	544.287	9084.71
38	3.96	.101	15.7	.000012	.05	20854.65	686.511	14320.26
39	3.53	.090	12.5	.000010	.04	26302.23	865.046	22752.6
40	3.14	.080	9.9	.000008	.03	33175.94	1091.865	36223.59

INDEX

Aberration, chromatic, 202.
  spherical, 269, 281.
Absolute temperature, 304.
Absolute units, 9.
Absolute zero, 305.
Absorption, 118.
  of heat, 328.
  of light, 216.
  of radiant energy, 328.
  selective, 330.
  spectrum, 250.
Accelerated motion, 20, 22–32.
Acceleration, 15, 20, 22.
  value due to gravity, 24, 68.
Achromatic lens, 282.
Achromatism, 241.
Activity, 72.
Actual velocity, 16.
Adhesion, 104, 118.
Advantage, mechanical, 82.
Agonic line, 373.
Air, liquid, 341.
  density of, 148, 161.
  pressure in, 129–133.
Air pump, 108.
Air resonator, 186.
Air thermometer, 296.
Alternating currents, 427.
Ammeters, 408.
Ampere, 381, 455.
Amplitude of vibration, 64, 163, 194.
Analysis of light, 239.
  of sound, 203.
Angle, critical, 233.
  of deviation, 237.
  of incidence, 226.
  of reflection, 226.
  of repose, 99.
  visual, 284.
Anode, 448, 466.
Aperture of a mirror, 266.
Apparatus M, 396.
Arc lamp, 461.
Archimedes, principle of, 152.
Armature, 400, 433.
Astigmatism, 286.
Astronomical telescope, 288.
Atmospheric pressure, 142.
Atoms, 101.
Atwood machine, 25.
Average velocity, 16.
Avoirdupois pound, 10.
Axis, of a lens, 276.
  of a mirror, 266.

Bar magnets, 370.
Barometer, 139–141.
  Fortin's, 141.
Baroscope, 152.
Batteries, 381–391.
  storage, 453.
Beats, 190.
Bell, electric, 401.
Bichromate cell, 388.
Boiling, 294, 337.
Boyle's law, 144.
Bright-line spectra, 250.
British Thermal Unit, 316.
Brittle, 115.
Brushes, 431.
Bunsen photometer, 223.
Buoyancy, 152.

Cake, 357.
Caloric, 306.
Calorie, 316.
Camera, 282.
Candle power, 224.
Capillarity, 123.
Capstan, 89.
Cathetal prism, 234.
Cathode, 448.
  rays, 466.
Caustic curve, 270.
Cells, electric, 387.
Celsius thermometers, 295.

Center of curvature, 266, 275.
of gravity, 50, 52.
of inertia, 51.
of mass, 51.
of oscillation, 64.
Centigrade thermometer, 293, 295.
Centimeter-Gram-Second System, 9.
Centrifugal force, 58.
Centripetal force, 58.
Change of motion, 31.
of velocity, 22.
Charles, law of, 303.
Chemical effects of current, 448.
Chest, steam, 312.
Chladni plates, 176.
Chromatic aberration, 282.
Circle of reference, 165.
Circuit, 383.
Coefficient of friction, 98.
Coherer, 471.
Cohesion, 104.
Cold, by expansion, 341.
by solution, 336.
by vaporization, 340.
Collecting rings, 431.
Color, of a body, 245.
of light, 244.
Colors, complementary, 247.
Commutator, 432.
Components, rectangular, 45.
Composition of forces, 37, 39.
of more than two forces, 41.
of motions and velocities, 47.
of parallel forces, 43.
Compound engine, 314.
Concurring forces, 39.
Condensation of a wave, 169.
Condenser, electrical, 365, 445.
of steam engine, 313.
of stereopticon, 286.
Condensing pump, 109.
Conduction of heat, 322.
Conductivity, 410.
Conductors, electrical, 350.
Conical pendulum, 164.
Conjugate foci, 269, 279.
Connecting rod, 313.
Conservation of energy, 77.
Constant acceleration, 22.
Constant force, 18, 22.
Continuous spectra, 249.
Convection, 322.
Converging pencil, 219.
Core, 399.
Cornea, 283.
Corpuscular theory, 213.
Coulomb, 381.
Crane, traveling, 90.
Crest of a wave, 169.
Critical angle, 99, 233.
Crookes radiometer, 329.
Crookes tubes, 466.
Crystalline lens, 284.
Current electricity, 349, 380.
Current strength, 381.
Currents, alternating, 427.
Foucault, 429.
induced by other currents, 437.
Curvature, center of, 266.
Cycle, 427.
Cyclones, 144.
Cyclonic area, 144.
Cylinder, 312.

Daniell cell, 389.
D'Arsonval galvanometer, 407.
Dead points, 313.
Declination, 373.
Declination needle, 374.
Decoherer, 472.
Defective vision, 285.
Deflection, 391.
Density, 12.
Derived units, 8.
Deviation, 235.
Dewar's apparatus for liquefying air, 342.
Dew-point, 345.
Diamagnetic, 372.
Dielectric, 355.
Differential pulley, 93.
Diffraction, 255.
interference by, 258.
Diffraction gratings, 259.
Diffusion, 110.
of liquids, 116.
Dip, 374.
Dipping needle, 374.
Direction, line of, 52.
Discharger, 367.
Discontinuous spectra, 249.
Discord, 192.
Dispersion, 235.
Dissonance, 192.
Distance of distinct vision, 285.
Diverging pencil, 219.

Drum armature, 433.
Dry cell, 389.
Ductile, 115.
Dynamic unit, 32, 70.
Dynamic theory of heat, 307.
Dynamics, 15.
Dynamo, 430.
Dyne, 32, 34.
Dyne centimeter, 71.

Ebullition, 337.
Eccentric, 313.
Echo, 182.
Eddy currents, 429.
Efficiency of machines, 83.
Effort in machines, 81.
Elasticity, 112.
  limit of, 113.
Elastic tension of gases, 106.
Electric arc, 460.
  bell, 401.
  lamp, 459.
  motor, 436,
  potential, 361.
  pressure, 385.
  telegraph, 402.
  waves, 469.
Electrical capacity, 364.
  charges, 349.
  condensers, 365.
  distribution, 359.
  resonance, 470.
Electricity, 349.
  current, 380.
Electrodes, 383.
Electrolyte, 449.
Electrolytic cell, 448.
Electrolysis, 450.
  laws of, 454.
  uses of, 455.
Electro-magnet, 398.
Electro-motive force, 385.
Electrons, 102.
Electrophorus, 357.
Electroplating, 450.
Electroscope, 352.
Electrostatics, 349.
Electrotyping, 451.
E.M.F. (electro-motive force), 385.
Energy, 73.
  conservation of, 77.
  forms of, 76.
  heat and, 306.
Energy of the electric current, 456.
  radiant, 214, 310.
  transference of, 77.
  transformation of, 76.
Engine, compound, 313.
  efficiency of, 314.
  gas, 314.
  hot-air, 310.
  steam, 310, 312.
  turbine, 315.
Equilibrant, 41.
Equilibrium, 50, 53.
Erg, 71.
Ether, 213.
Evaporation, 337.
Exhaust pipe, 313.
Exciter, 435.
Expansion, by heat, 297.
  cold by, 341.
Extra current, 441.
Eye, 283.
Eyepiece, 287.

Fahrenheit thermometer, 293, 295.
Falling bodies, 24.
Faraday's ice-pail experiment, 356.
Faraday's laws of electrolysis, 454.
Faraday's law of induction, 424.
Field coils, 430.
Field cores, 430.
Field, magnetic, 377.
Fifth, 196.
Films, interference by, 255.
Final velocity, 22.
Fixed pulley, 92.
Flame, singing, 177.
Flat, 196.
Floating bodies, 155.
Fluid, 103.
Fluids, pressure in, 124.
Fluoroscope, 468.
Flute pipes, 209.
Fly wheels, 313.
Focal length, 266, 273, 276.
Foci, conjugate, 269, 279.
  secondary, 278.
Focus of a mirror, 266.
Foot pound, 71.
Force, 18.
  absolute unit of, 32.
  a factor of work, 70.
  centrifugal, 58.
  centripetal, 58.

Force, constant, 18, 22.
electro-motive, 385.
in dynamic units, 34.
lines of, 378.
measured statically, 35.
measurement of, 30.
moment of, 85.
units of, 33.
Forced vibrations, 184.
Force pump, 150.
Forces, composition of, 37.
concurring, 39.
graphic representation of, 38.
molecular, 104, 112.
parallelogram of, 40.
resolution of, 37, 44.
Form, elasticity of, 113.
Fortin's barometer, 141.
Foucault currents, 429.
Franklin's experiment, 339
Fraunhofer lines, 253.
Free path, 105.
Freezing, 332.
Freezing point, 294.
Frequency, 163, 168.
Friction, 96.
coefficient of, 98.
heat caused by, 307.
Fringes, 257.
Fulcrum, 84.
Fundamental tone, 200.
Fundamental units, 8.
Fusion, 331.
heat of, 333.

Galileo's experiment with pendulum, 66.
Galileo's telescope, 289.
Galvanometers, 406.
Gas engine, 314.
Gases, adhesion between, 118.
elastic tension of, 106.
molecular motions in, 105.
spectra of, 252.
vibrations in, 177.
Gay-Lussac, law of, 103, 305.
Geissler tubes, 466.
Gold-leaf electroscope, 352.
Gram, 10.
value in dynes, 34.
Gramme ring, 434.
Graphic representation of forces, 38.
Gratings, diffraction, 259.
Gravitation, 48.
Newton's laws of, 49.
Gravitational unit, 32, 70.
Gravity, causes pressure, 129.
center of, 50.
specific, 13.
Gravity cell, 390.

Harmonic motion, 164.
partials, 200.
Harmonics, 200.
of a string, 207.
Hartl optical disk, 226.
Heat, 292.
and energy, 306.
development of, 307.
effects of electric current, 456.
expansion by, 297.
latent, 335, 339.
measurement of, 316.
nature of, 306.
of fusion, 333.
of vaporization, 359.
specific, 318.
transmission of, 322.
Helix, 399.
Helmholtz resonator, 203.
Hemispheres, Magdeburg, 131.
Hertz oscillator, 471.
Hofmann's apparatus, 451.
Hooke's law, 114.
Horse power, 72.
Horseshoe magnets, 370.
Hot-air engine, 310.
Humidity, 345.
Humor, aqueous, 284.
vitreous, 284.
Hydraulic press, 128.
Hydrometer, 158.
Hypermetropic, 285.

Ice, manufacture of, 343.
Illumination, 221.
Images, formation of, 259.
formed by lenses, 274.
Incandescent, 215.
Incandescent electric lamp, 459.
Incidence, 226.
Inclination, 374.
Inclined plane, 45, 93.
Induced charges, 354.
Induction, magnetic, 375.
coil, 443.

Induction, current, 424.
Inertia, 17.
center of, 51.
Insulators, electrical, 350.
Intensity of light, 221.
laws of, 222.
of sound, 192.
Interchangeable points, 65.
Interference, of light, 255.
of sound, 188.
wave, 172.
Intervals, musical, 196.
Ions, 384.
Iris, 283.
Isobaric lines, 143.
Isobars, 143.
Isochronous vibrations, 165.
Isoclinic lines, 375.
Isogonic lines, 373.

Jack screw, 95.
Joule, 71.
Joule's law of heat, 458.

Keynote, 197.
Kilogram, 10.
Kilogrammeter, 71.
Kilowatt, 72.
Kinetic energy, 73.
Kinetic friction, 96.
Kirchhoff's law, 329.

Lactometer, 159.
Laminated cores, 435.
Lamp, arc, 461.
electric, 459.
Lantern, 286.
Latent heat, 335, 339.
Leclanché cell, 388.
Legal unit, 8.
Length, unit of, 9.
Lens, crystalline, 284.
Lenses, 274.
Lenz, law of, 428.
Levers, 83.
Leyden jar, 366.
Lifting pump, 149.
Light, color of, 244.
intensity of, 221.
interference and diffraction of, 255.
nature of, 213.
reflection of, 216, 225.
refraction of, 227.
Light, transmission of, 215.
travels in straight lines, 220.
velocity of, 217.
Light waves, effect of matter on, 215.
measurement of, 244.
Lightning, 368.
Lightning rods, 368.
Limiting angle for friction, 99.
Limit of elasticity, 113.
Limpid, 116.
Lines of force, 378.
Liquid, 103.
in contact with a solid, 122.
surface of, 120.
Liquid air, 342.
Liquids, adhesion between, 118.
diffusion of, 116.
in connected vessels, 137.
molecular motions in, 115.
specific gravity of, 158.
velocity of sound in, 182.
Loadstone, 370.
Local action, 386.
Longitudinal vibrations, 163.
Longitudinal waves, 169.
Loop, 174.

Machines, efficiency of, 83.
law of, 80.
Magdeburg hemispheres, 131.
Magnetic axis, 371.
circuit, 399.
effects, 391.
field, 377.
induction, 375.
lines, 378.
meridian, 373.
needle, 370.
substance, 372.
susceptibility, 376.
transparency, 372.
Magnetism, 349, 369.
terrestrial, 373.
Magneto, 430.
Magnets, 369.
law of, 371.
Major, 196.
Malleable, 115.
Manometric flame, 201.
Mass, 9, 13.
center of, 51.
relation to weight, 12.
unit of, 10.

Mass, volume, and density, 13.
Matter, molecular structure of, 101.
  properties of, 15.
  states of, 103.
Measure, unit of, 7.
Measurement, of force, 30.
  of heat, 316.
  of temperature, 292.
  of work, 70.
Mechanical advantage, 82.
  equivalent of heat, 317.
  powers, 83.
  theory of heat, 307.
Mechanics, 15.
Melting, 331.
Melting point, 331.
Mercurial thermometer, 293.
Meridian, magnetic, 373.
Meter, 9.
Metric system, 8.
Micrometer screw, 96.
Microphone, 447.
Microscope, 287.
Mil-ammeter, 408.
Minor, 196.
Mirror, 225.
Mirrors, 261.
Mobile, 116.
Mobility, 103.
Molecular forces, 104, 112.
Molecular motions, 112, 115.
Molecular structure of matter, 101.
Molecules, 101, 102.
Moments, 85.
  law of, 87.
  measure of, 85.
Momentum, 31.
Motion, 15.
  accelerated, 20.
  change of, 31.
  friction of, 96.
  harmonic, 164.
  Newton's laws of, 17.
  of a pendulum, 63.
  periodic, 162.
  uniform, 16.
  vibratory, 162.
Motions, composition and resolution of, 47.
  in gases, 105.
  in liquids, 115.
  molecular, 112.
Motor, electric, 436.
Movable pulley, 92.
Multiple reflection, 264.
Multipolar machines, 431.
Musical instruments, 204.
  intervals, 196.
  scale, 195.
  sound, 195.
Myopic, 285.

National prototype meter, 9.
Needle, declination, 374.
  dipping, 374.
  magnetic, 370.
Nernst lamp, 460.
Neutral equilibrium, 53.
Newton's law of gravitation, 49.
  laws of motion, 17.
  rings, 255.
Nodes, 174.
Noises, 195.
Nonmagnetic, 372.
Normal, 226.
Nut, 94.

Objective, 287.
Octave, 196.
Ohm's law, 412.
Opaque, 216.
Optical instruments, 282.
Optical prism, 235.
Oscillation, center of, 64.
Oscillator, Hertz, 471.
Osmosis, 117.
Osmotic pressure, 117.
Overtones, 200.

Parallel, 416.
Parallel forces, 43.
Parallelogram of forces, 40.
Partials, 200.
P.D. (difference of potential), 364.
Pencil of light, 219.
Pendulum, conical, 164.
  laws of, 66.
  motion of, 63.
  simple, 62.
  theory of, 67.
  torsion, 163.
  uses of, 68.
Penumbra, 220.
Periodic motion, 162.
Permeability, 379.

Photometer, Bunsen, 223.
Photometry, 223.
Physics, 7.
Pigments, mixing, 247.
Pipes, 209.
Piston, 312.
Piston rod, 313.
Pitch of a screw, 94.
  of sound, 192, 195.
Plane mirrors, 261.
Plasticity, 115.
Point of incidence, 226.
Polarity, 371.
  of the helix, 399.
Polarization, 386.
Pole pieces, 430.
Poles of a cell, 383.
  of a magnet, 371.
Porosity, 102.
Ports, 312.
Potential, 361.
  difference of, 363.
Potential energy, 75.
  fall of, 414.
Pound, avoirdupois, 10.
Power, 72.
  of recovery, 112.
  in machines, 81.
Powers, mechanical, 83.
Presbyopic, 285.
Press, hydraulic, 128.
Pressure, 19.
  atmospheric, 142.
  below the surface, 135.
  caused by weight, 129.
  due to gravity, 129.
  in a horizontal plane, 135.
  in fluids, 124.
  osmotic, 117.
  transmission of, 126.
Primary bow, 243.
Primary coil, 438.
Prism, cathetal, 233.
  optical, 235.
Properties of matter, 15.
Pulley, 83, 91.
  law of, 92.
Pump, air, 108.
  condensing, 109.
  force, 150.
  lifting or suction, 149.
Pupil of the eye, 284.
Purple, visual, 284.
Radiant energy, 214.
  transformed, 310.
Radiation, 215.
  laws of, 327.
  of heat, 322.
Radiometer, 329.
Rainbow, 243.
Rarefaction of a wave, 169.
Ray of light, 218.
Réaumur thermometer, 295.
Receiver, 445.
Reciprocating engines, 515.
Rectangular components, 45.
Reed pipes, 209.
Reflection, multiple, 264.
  of light, 216, 225.
  of sound, 182.
Refraction, cause of, 231.
  index of, 230.
Relay, 403.
Repose, angle of, 99.
Resistance, 401.
  in machines, 81.
  laws of, 411.
Resolution, of forces, 37, 39, 44.
  of motions and velocities, 47.
Resonance, 183, 184, 185.
  electrical, 470.
Resonator, air, 186.
  Helmholtz, 203.
Rest, friction of, 96.
Resultant, 39.
Retentivity, 376.
Rigidity, 103.
Rings, Newton's, 255.
Rolling friction, 97.
Röntgen rays, 465, 467.

Saccharimeter, 159.
Saturated vapor, 344.
Scale, diatonic, 197.
  musical, 195.
  tempered, 199.
Sclerotic coat, 283.
Screw, 83, 94.
  law of, 95.
Second, 11.
Secondary axis of a lens, 276.
  bow, 243.
  coil, 438.
  foci, 278.
Selective absorption, 330.
Self-induction, 441.

Semipermeable, 117.
Semitone, 196.
Series, 416.
Series-wound dynamo, 435.
Series-wound motors, 437.
Set, 113.
Shadows, 220.
Shape, elasticity of, 113.
Sharp, 196.
Shunts, 417.
Shunt-wound dynamo, 435.
motors, 437.
Shuttle armature, 433.
Side valve, 312.
Sine of an angle, 230.
Singing flame, 177.
Siphon, 148.
Siren, 195.
Size, elasticity of, 113.
Slides, 286.
Soap bubble, colors of, 256.
ole, 357.
Solid, 103.
Solids, adhesion between, 118.
molecular motions in, 112.
specific gravity of, 157.
velocity of sound in, 182.
Solution, 117.
cold, 356.
Sonometer, 205.
Sound, analysis of, 203.
characteristics of, 192.
interference of, 188.
origin of, 175.
quality of, 200.
reflection of, 182.
transmission of, 178.
velocity of, 180, 182.
Sounder, telegraph, 403.
Sound waves, form of, 179.
Source of light, 221.
Space, a factor of work, 70.
Specific gravity, 13, 152, 156.
of liquids, 158.
of solids, 157.
Specific heat, 318.
Spectra, 249.
of gases, 252.
Spectroscope, 249.
Spherical aberration, 269, 281.
mirrors, 266.
Spheroidal state, 324.
Stability, 55.
Stable equilibrium, 53.
Standard unit, 8.
States of matter, 103.
Static friction, 26.
Stationary waves, 173, 174.
Steam chest, 312.
Steam engine, 310, 312.
Stereopticon, 286.
Storage battery, 453.
Strain, 15, 112.
Stress, 19, 113.
energy, of, 75.
Stringed instruments, 204.
Strings, laws of vibrating, 205.
Sublimation, 337.
Suction pump, 149.
Sulphion, 384.
Support of a body, 51.
Surface of a liquid, 120, 134.
Surface tension, 121.
Sympathetic vibrations, 185.
Synthesis of light, 240.
Systems of units, 8.

Tangent galvanometer, 406.
Telegraph, electric, 402.
Telegraphy, wireless, 465, 471.
Telephone, 445.
Telescope, astronomical, 288.
Galileo's, 289.
Temperature, 292.
absolute, 304.
Tempered scale, 199.
Tension, 19.
elastic, 106.
surface, 121.
transmitted, 91.
vapor, 106, 345.
Terrestrial magnetism, 373.
Thermal capacity, 318.
Thermometer, air, 296.
graduation of, 294.
mercurial, 293.
Thermoscope, 296.
Thermostat, 299.
Thread of a screw, 94.
Timbre, 192.
Time, unit of, 11.
Toepler-Holtz machine, 358.
Tone, fundamental, 200.
Torricelli's experiment, 140.
Torsion pendulum, 163.
Total internal reflection, 232.

Transference of energy, 77.
Transformation of energy, 76.
Transformer, 439.
Translucent, 216.
Transmission of heat, 322.
  of light, 215
  of pressure, 124, 126.
  of sound, 178.
Transmitted tension, 91.
Transmitter, 445.
  microphone as, 447.
Transparency, 372.
Transparent, 216.
Transposition, 198.
Transverse vibrations, 162.
  waves, 169.
Trough of a wave, 119.
Turbine engine, 315.

Umbra, 220.
Undercooling, 334.
Undulatory theory, 214.
Uniformly accelerated motion, 22.
Uniform motion, 16.
Unison, 196.
Unit, dynamic, 32.
  gravitational, 32.
  legal, 8.
  of length, 9.
  of mass, 10.
  of measure, 7.
  of time, 11.
  standard, 8.
Units, absolute, 9.
  derived, 8.
  fundamental, 8.
  of force, 63.
  of weight, 12.
  of work, 70.
  systems of, 8.
Unstable equilibrium, 53.
Upper partials, 200.
Useful work, 83.

Vacuum, Torricellian, 140.
Vapor, 103.
  saturated, 344.
  tension, 106, 345.
Vaporization, 337, 339.
Velocities, composition and resolution of, 47.
Velocity, 15.
  actual, 16.
Velocity, average, 16.
  change of, 22.
  final, 22.
  of light, 217.
  of sound, 180.
  of waves, 168.
Ventilation, 327.
Ventral segment, 174.
Vertex, of a mirror, 266.
Vibration frequency, 163.
  of air in pipes, 210.
Vibrations communicated to other bodies, 183.
  forced, 184.
  in gases, 177.
  isochronous, 165.
  sympathetic, 185.
Vibratory motion, 162.
Viscosity, 116.
Vision, defective, 285.
  distance of distinct, 285.
Visual angle, 284.
  purple, 284.
Vitreous humor, 284.
Volt, 385.
Voltaic cell, 380.
Voltmeters, 408.
Volume, 13.
  elasticity of, 113.

Wasted work, 83.
Water, electrolysis of, 451.
Watt, 72, 456.
Wave form, 166.
  front, 180.
  interference, 172.
  length, 167, 168.
  motion, 167.
  theory, 214.
Waves, interference of sound, 188.
  kinds of, 169.
Weather map, 144, 145.
Wedge, 83.
Weighing, 12.
Weight, 11.
  in machines, 81.
  laws of, 49.
  of a fluid causes pressure, 129.
  relation to mass, 12.
  units of, 12.
Weight arm of a lever, 84.
Wheatstone bridge, 415.
Wheel and axle, 83, 88.

Wheel and axle, law of, 89.
Wheel work, 90.
Wind instruments, 204, 209.
Windlass, 89.
Wireless telegraphy, 465, 471.
Work, 69.
  done by a machine, 83.
  measurement of, 70.
  units of, 70.

X-rays, 467.

Yard, 9.
Yoke, 430.
Young-Helmholtz theory, 248.

Zeiss binocular field glass, 290.
Zero, absolute, 304.

DESCRIPTIVE CATALOGUE OF HIGH SCHOOL AND COLLEGE TEXT-BOOKS

Published Complete and in Sections

WE issue a Catalogue of High School and College Text-Books, which we have tried to make as valuable and as useful to teachers as possible. In this catalogue are set forth briefly and clearly the scope and leading characteristics of each of our best text-books. In most cases there are also given testimonials from well-known teachers, which have been selected quite as much for their descriptive qualities as for their value as commendations.

¶ For the convenience of teachers this Catalogue is also published in separate sections treating of the various branches of study. These pamphlets are entitled: English, Mathematics, History and Political Science, Science, Modern Languages, Ancient Languages, and Philosophy and Education.

¶ In addition we have a single pamphlet devoted to Newest Books in every subject.

¶ Teachers seeking the newest and best books for their classes are invited to send for our Complete High School and College Catalogue, or for such sections as may be of greatest interest.

¶ Copies of our price lists, or of special circulars, in which these books are described at greater length than the space limitations of the catalogue permit, will be mailed to any address on request.

¶ All correspondence should be addressed to the nearest of the following offices of the company: New York, Cincinnati, Chicago, Boston, Atlanta, San Francisco.

AMERICAN BOOK COMPANY

124-138

152-159

297-302